Electron Effective Mass

The movement of an electron in a lattice will, in general, be different from that of an electron in free space. In addition to an externally applied force, there are internal forces in the crystal due to positively charged ions or protons and negatively charged electrons, which will influence the motion of electrons in the lattice. We can write where F,,,I, *F,,,.* and *Fin*, are the total force, the externally applied force, and the internal forces, respectively, acting on a particle in a crystal. The parameter a is the acceleration and *m* is the rest mass of the particle
where the acceleration a is now directly related to the external force. The parameter ***m*.*** called the ***effective mass,*** takes into account the panicle mass and also takes into account the effect of the internal forces. In a semiconductor material, we will be dealing with allowed energy bands that are almost empty of electrons and other energy bands that are almost full of electrons. To begin, consider the case of a free electron whose E versus k curve was show in Figure 3.7. Recalling Equation (1.28). the energy and momentum are related $E = p^2/2m = ħ^2 k^2/2m$, where m is the mass of the electron. The momentum an wave number k are related by p = ***Rk.*** If we take the derivative of Equation (3.28 with respect to k, we obtain

$$\frac{dE}{dk} = \frac{ħ^2 k}{m} = \frac{ħp}{m} - - - - -\rightarrow (1)$$

Relating momentum to velocity

$$\frac{1}{ħ}\frac{dE}{dk} = \frac{p}{m} = v - - - - -\rightarrow (2)$$

where ***v*** is the velocity of the particle. The first derivative of E with respect to k is related to the velocity of the particle. If we now take the second derivative of E with respect to k, we have

$$\frac{d^2E}{dk^2} = \frac{ħ^2}{m} - - - - - -\rightarrow (3)$$

We may rewrite Equation (3) as

$$\frac{1}{ħ^2}\frac{d^2E}{dk^2} = \frac{1}{m} - - -\rightarrow (4)$$

The second derivative of E with respect to k is inversely proportional to the mass of the particle. For the case of a free electron, the mass is a constant (nonrelativistic effect), so the second derivative function is a constant. We may also that $\frac{d^2E}{dk^2}$ is a positive quantity, which implies that the mass of the electron is also a positive quantity.
If we apply an electric field to the free electron and use Newton's classical equation of motion, we can write.

$$F = ma = -eE - - - - -\rightarrow (5)$$

where a is the acceleration, E 1s the applied electric field, and **e** is the magnitude of the electronic charge. Solving for the acceleration, we have

$$a = \frac{-eE}{m} -----\rightarrow (6)$$

The motion of the free electron is in the opposite direction to the applied electric field
because of the negative charge

$$\hbar^2 / \frac{d^2E}{dk^2} = m^* ----\rightarrow (7)$$

H.W: Derive the effective mass m^*

$$\frac{dE}{dt} = -F.v_e ------\rightarrow (1)$$

$$E = \hbar\,\omega -----\rightarrow (2)$$

$$P = \hbar k -----\rightarrow (3)$$

$$(\,v\ in\ one\ dimension)v = \frac{d\omega}{dk}$$

$$(\,v\ in\ three\ dimension) \quad v_e = \nabla_p.\omega = \frac{1}{\hbar}\,\nabla_k E -----\rightarrow (4)$$

$$\frac{dE}{dt} = -F.\frac{1}{\hbar}\nabla_k E ---\rightarrow dE = -F.\frac{1}{\hbar}\nabla_k E\ dt -----\rightarrow (5)$$

$$dE = \nabla_k E.dk -----\rightarrow (6)$$

$$F\,.\frac{1}{\hbar}\nabla_k E\ dt = \nabla_k E.dk ----\rightarrow (7)$$

$$\nabla_k E\left[.\frac{1}{\hbar}F.dt-.dk\right] = 0 ----\rightarrow (8)$$

$$\nabla_k E \quad \neq 0 ---\rightarrow (9) because\ k\ value\ is\ dependent\ on\ field\ and\ not\ constant$$

$$So\ \left[.\frac{1}{\hbar}F.dt-.dk\right] = 0 ----\rightarrow F = \hbar\frac{dk}{dt} ----\rightarrow (10)$$

$$a = \frac{dv}{dt} = \frac{1}{\hbar}grad_k\frac{dE}{dt} = \frac{1}{\hbar}grad_k(F.v) = \frac{1}{\hbar^2}grad_k(F.grad_k E) --\rightarrow (11)$$

Equation 11is newton law in motion

$$a = \left(\frac{1}{m}\right)F = \frac{1}{m^*}\,F ----\rightarrow (12)$$

$$by\ comparing\ tow\ equation\ we\ get \quad \frac{1}{m^*} = \frac{1}{\hbar^2}.grad_k(\,grad_k E) = \frac{1}{\hbar^2}.\frac{d^2E}{dk^2}$$

$$m^* = (\hbar^{-2}\frac{d^2E}{dk^2})^{-1} = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$