Penetration Depth

London suppose that at T< T_c a part value equal to $(n_s\ /n)$from all number of conduction electrons (n) can be contributed in super current in matter . The value ns called electrons density of superconductor in matter. This value n_s approaching n (completely electrons density) when temperature decrease to become less than T_c and n_s decreases to become zero when T increases to reach T_c. London suppose when electrical field intensity E applied on superconductor matter by period time then super conduction electrons moves according this equation

$m_s\, dv_s\, /dt = -eE$ ——————→ (1)

V_s: velocity of super conduction electrons, m_s : electron mass From equation 1 we see the first force effect at this type of electrons is the force which result from electrical field. The scatter force unfounded in equation because the super conduction electrons did not scattered or collided. Current density from super conduction electrons is

$J_s = -eV_s n_s$ ———————→ (2)

$dJ_s\, /dt = -e(\, dV_s\, /dt\,)n_s$ —————————→ (3)

Substitute 1 in 3 we get

$dJ_s\, /dt = (n_s e\, /m_s\,)E$ ——————→ (4) Or

$J_s = (n_s e^2\, /m_s\,)E$ —————→ (5) First London Eq

To calculate the value of penetration depth of solid material we suppose stable state (J=0) , this mean that electrical field in equation 5 equal zero, so in in this state normal current dose not pass throw superconductor and the current only super conduction electrons , If we substitute J_s=0, E=0, in Maxwell equation.

$\dot{\bar{B}} = \bar{\nabla} \times \bar{E}$ − − − − − − −−→ (6) We get

$\dot{B} = 0$ − − − − −−→ (7)

Equation 7 means magnetic field be constant and this contradicts( ) يناقض Meissner effect, because B is constant regardless (بغض النظر) about temperature, But Messniers phenomenon explain that at increasing temperature the magnetic field penetrate matter at T=T_c so

$\dot{\bar{B}} = -\left(\frac{m_s}{n_s e^2}\right) \bar{\nabla} \times \bar{J_s}$ − − − −−→ (8) second London equation

We can write Maxwell equation denoted by Js

$\bar{\nabla} \times \bar{B} = \mu_o \times \bar{J_s}$ —————→ (9)

μo: magnetic permeability

take curl of two hand equation 9 we get

$$\bar{\nabla} \times \bar{\nabla} \times \bar{B} = -\nabla^2 \bar{B} \text{ − − − −−→ (10)}$$

If we substitute 8 in 10

$$\nabla^2 \bar{B} = \left(\frac{\mu_o n_s e^2}{m_s}\right) \bar{B} - - - - - - - \rightarrow (11)$$

As the same method we get

$$\nabla^2 \overline{J_s} = \frac{\mu_o n_s e^2}{m_s}) \overline{J_s} - - - - - - \rightarrow (12)$$

By solving the two equation if x > 0 we get

$$\bar{B} = B_{ext} \exp\left(-\frac{x}{\lambda_L}\right) - - - \rightarrow (13)$$

$$\frac{1}{\lambda_L^2} \bar{B} = \frac{\mu_o n_s e^2}{m_s} - - - \rightarrow (14)$$

$$\nabla^2 \bar{B} = \frac{1}{\lambda_L^2} \bar{B} - - - (15)$$

$$\lambda_L = (\frac{m_s}{\mu_o n_s e^2})^{\frac{1}{2}} - - \rightarrow (16)\ London\ pentration\ depth$$

We found experimentally that λ variant with temperature so that when T increases penetration depth λ increases as becomes infinity at $T = T_c$ and variant with T as follow

$$\lambda = \frac{\lambda_o}{\sqrt{1 - (\frac{T}{T_c})^4}} - - - \rightarrow (17)\ geniral\ equation$$

λ_o = penetration depth at zero Kelvin (o K^o)