

$$\psi(x, t) = u(x)\phi(t)e^{ikx}e^{-\frac{E}{\hbar}t}$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0 \longrightarrow 1$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E]\psi(x) = 0 \longrightarrow 2 \quad \text{at } 0 < x < a \quad V(x) = 0 \quad \text{Region I}$$

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V_o]\psi(x) = 0 \longrightarrow 3 \quad \text{at } -b < x < 0 \quad V(x) = V_o \quad \text{Region II}$$

Put :  $\alpha^2 = \frac{2mE}{\hbar^2} \longrightarrow \alpha = \sqrt{\frac{8\pi^2m}{h^2}} \longrightarrow \alpha = \frac{\sqrt{2mE}}{\hbar}$

$$\beta^2 = \frac{2m}{\hbar^2}[V_o - E]$$

So equation 2 and 3 become

$$\frac{d^2\psi(x)}{dx^2} + \alpha^2[E]\psi(x) = 0 \longrightarrow 4 \quad \text{at } 0 < x < a \quad V(x) = 0 \quad \text{Region I}$$

$$\frac{d^2\psi(x)}{dx^2} - \beta^2\psi(x) = 0 \longrightarrow 5 \quad \text{at } 0 < x < a \quad V(x) = V_o \quad \text{Region I}$$

$$\psi(x) = U(x)e^{ikx}$$

This traveling-wave solution represents the motion of an electron in a single-crystal material. The amplitude of the traveling wave is a periodic function and the parameter  $k$  is also referred to as a wave number.

We solved all the equation in two region and find

$$\frac{\beta^2 + \alpha^2}{2\alpha\beta} \sin(\alpha a) \sinh(\beta b) + \cos(\alpha a) \cosh(\beta b) = \cos k(a + b) \longrightarrow 12$$

$V_o$  b : barrier height ( Strength)

If  $V_o \longrightarrow \infty$  that means  $b \longrightarrow 0$  so  $\sinh(\beta b) = \beta b$  ,  $\cosh(\beta b) = 1$

Equation 12 become

$$\frac{(\beta^2 + \alpha^2) \beta b}{2\alpha\beta} \sin(\alpha a) + \cos(\alpha a) = \cos ka \longrightarrow 13$$

$$\beta^2 + \alpha^2 = \frac{2m}{\hbar^2}[V_o - E] + \frac{2m}{\hbar^2}E = \frac{2m}{\hbar^2}V_o \longrightarrow 14$$

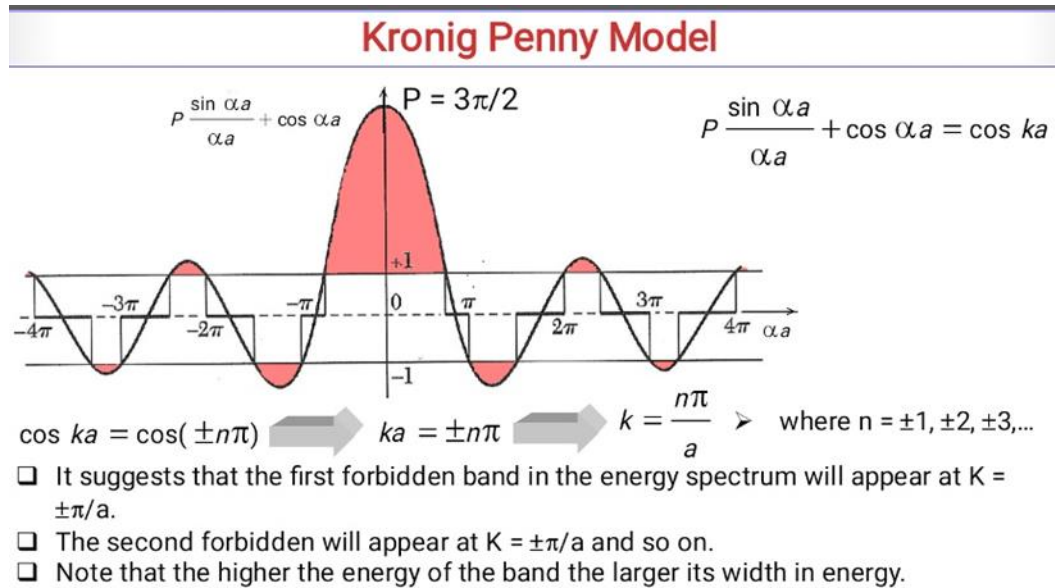
Substitute eq 14 in 13 we get

$$\frac{mV_o}{\hbar^2} \frac{ab \sin(\alpha a)}{\alpha a} + \cos(\alpha a) = \cos ka \longrightarrow 15$$

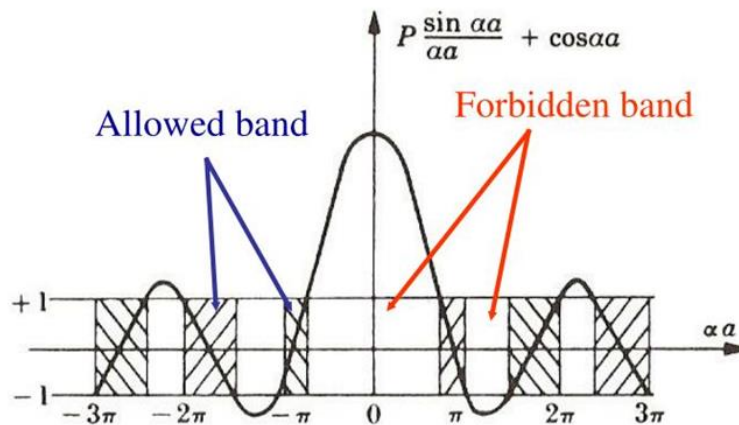
$$P \frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a) = \cos ka \quad \text{---} \rightarrow 16$$

Where P is scattering power and  $P = \frac{mV_0 ab}{\hbar^2}$

The equation 16 is analysis by drawing graph between  $P \frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a)$  versus  $\alpha a$  see Fig (1, 2, 3)



Fig(1): A plot of  $\alpha a$  versus  $P \frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a)$



Fig(2): A plot showed allowed band and forbidden band

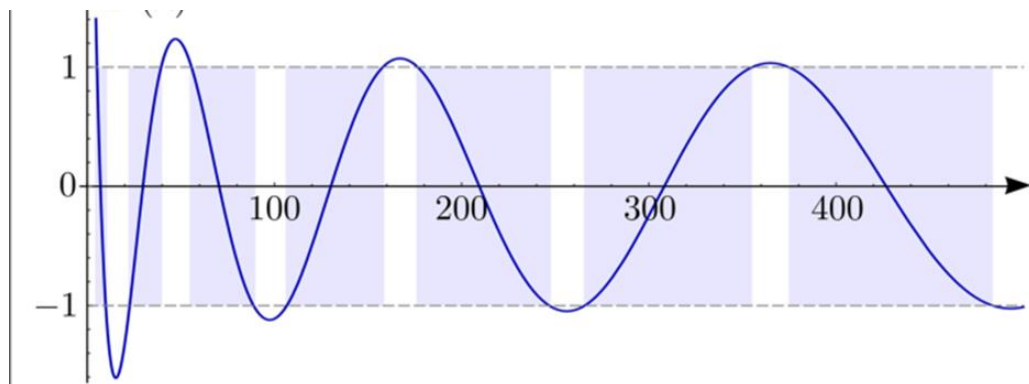


Fig (3) : A plot shows that as increasing  $\alpha a$  increases allowed band