Second method depend on density state D(E)

$$n_o = \int D_c(E) f_F(E) dE ------\rightarrow (1)$$

Or the density of electrons and holes is related to the density of state function
and Fermi-Dirac distribution function

$n(E)\,dE = g_c(E)\, f_F(E)\, dE$

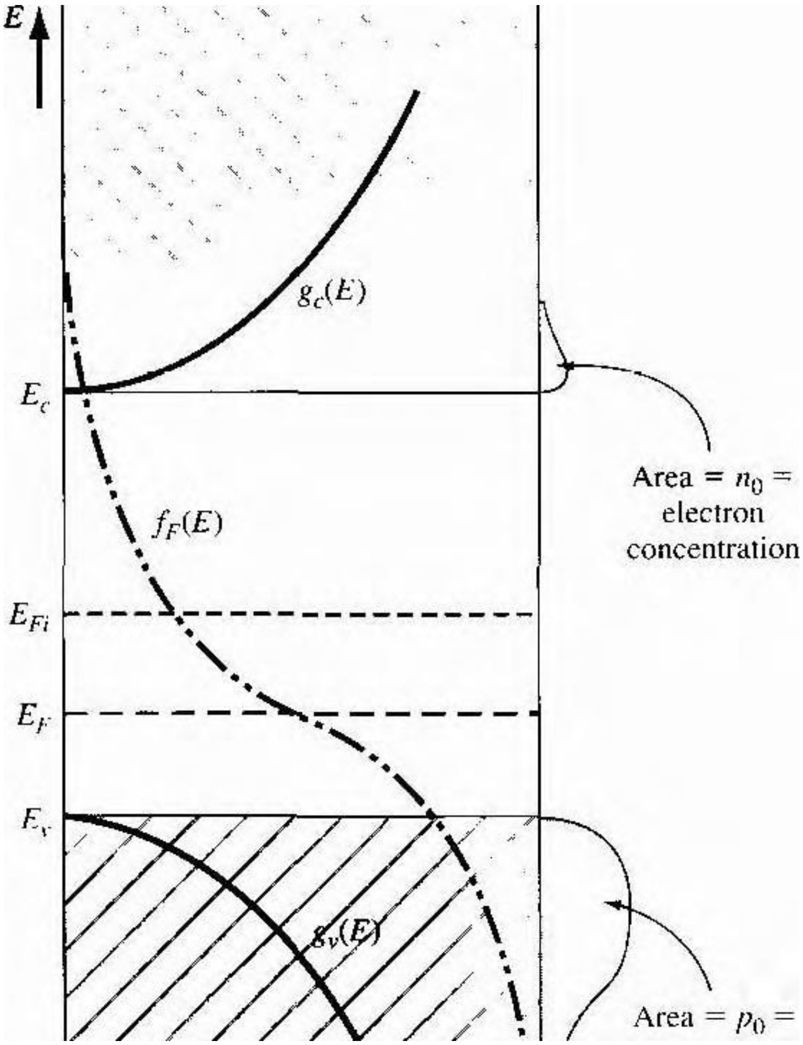
$n(E)dE$: density of electrons in CB at energy levels between E and $E+dE$

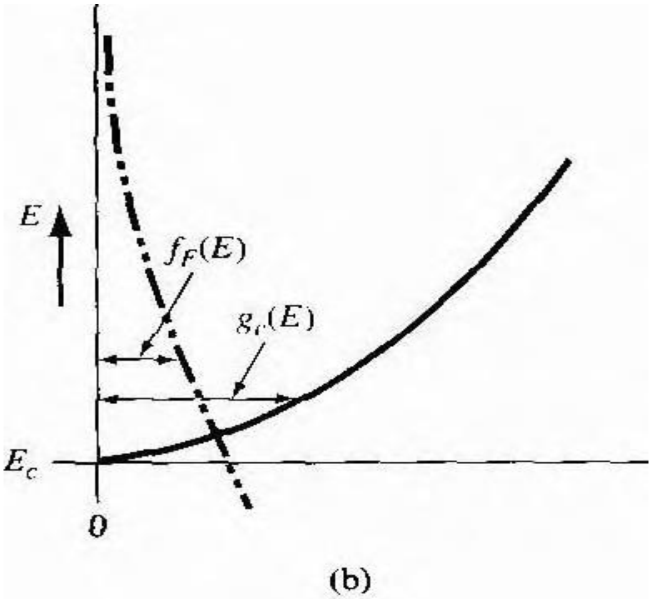
$p(E)\,dE = g_v(E)[1- f_F(E)]\, dE$

$p(E)dE$: density of holes in VB at energy levels between E and $E+dE$As state in conduction band haven't same energy but have E value of the beginning and increases upwardly to the top band .to calculate the number of electrons

$$n_o = \int_{E_c}^{top} D_c(E) f_F(E) dE ------\rightarrow (2)$$

The lower limit of integration is E_c and the upper limit of integration should be t he top of the allowed conduction band energy. However, since the Fermi probability function rapidly approaches zero with increasing energy as indicated in Figure 4.la we can take the upper limit of integration to be infinity.





(b)

Figure 4. (a) Density of states functions, Fermi-Dirac probability function, and areas representing electron and hole concentrations for the case when ***E*,** is near the mid gap energy; **(b)** expanded view near the conduction band energy and (c) expanded view near the valence band energy

that the number of electrons in the conduction band is equal to the number of holes in the valence band.

The different between electron in metal and semiconductor that in metal electron no affective strongly by atom attractive but in semiconductor the electrons placed under attractive of atom , so we replaced mass of free electron by effective mass m^*. ($E=\frac{n^2h^2}{8mL^2}$)

$$n_c=\frac{4\pi}{h^3}(2m_e^*)^{3/2}\int_{E_c}^{\infty}\frac{(E-E_c)^{\frac{1}{2}}}{\exp\left(\frac{E-E_f)}{K_\beta T}\right)+1}.d(E)------\rightarrow(3)$$

At $E-E_f/K_\beta T\gg1$

$$n_c=\frac{4\pi}{h^3}(2m_e^*)^{3/2}\int_{E_c}^{\infty}(E-E_c)^{\frac{1}{2}}\exp\left(\frac{E_f-E)}{K_\beta T}\right)dE----\rightarrow(4)$$

Suppose $E-E_c=x$ -------------→(5)

$$n_c=\frac{4\pi}{h^3}(2m_e^*)^{3/2}exp\left(\frac{E_f-E_c)}{K_\beta T}\right)\int_0^{\infty}x^{\frac{1}{2}}\exp-(\frac{x}{K_\beta T})\,dx------\rightarrow(6)$$

$$\int_0^{\infty}x^{\frac{1}{2}}\exp-(\frac{x}{K_\beta T})\,dx=(K_\beta T)^{\frac{3}{2}}\frac{\pi^{\frac{1}{2}}}{2}------\rightarrow(7)$$

Substitute 7 in 6 we get

$$n_c=2(\frac{2\pi m_e^*K_\beta T}{h^2})^{3/2}exp\left(\frac{E_f-E_c)}{K_\beta T}\right)---\rightarrow(8)$$

As the same (H.W)

$$n_v=2(\frac{2\pi m_h^*K_\beta T}{h^2})^{3/2}exp\left(\frac{E_v-E_f)}{K_\beta T}\right)---\rightarrow(9)$$

The part $2(\frac{2\pi m_e^*K_\beta T}{h^2})^{3/2}$is constant because it is change very small with Temperature

$$N_c=2(\frac{2\pi m_e^*K_\beta T}{h^2})^{\frac{3}{2}}\quad----N_v=2(\frac{2\pi m_h^*K_\beta T}{h^2})^{3/2}$$

$$n_c = N_c\, exp\left(\frac{E_f - E_c)}{K_\beta T}\right) - -\!-\!\!\rightarrow (10$$

$$n_v = N_v exp\left(\frac{E_v - E_f)}{K_\beta T}\right) - -\!-\!\!\rightarrow (11)$$