Nuclear Models

In the absence of detailed theory of nuclear structure , attempts have been made to correlate nuclear data in terms of various nuclear models . Several models have been proposed , each based on a set of simplifying assumption and useful in a limited way.

Each model can explain a small range of the experimental data but fails when applied to data outside this range.

The nuclear models proposed to describe the interaction between nucleons are :

① **The shell Model**

② **Liquid Drop Model**

③ **The collection Model**

④ **The optical Model**

① ***<u>The shell Model</u>*** :

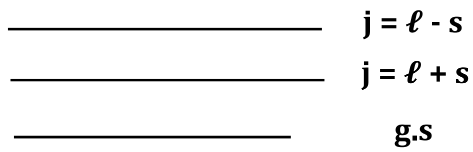
In this model it is assumed that the interaction between nucleons is very little or weak interaction ; therefore , this model can be called **<u>Independent particles model</u>**.

In 1932 Chadwick discovered the neutron and opened the way for the development of models of nuclear structure. The shell model is **<u>one</u>** of the most important and useful model. By this model the **<u>spin</u>** and the **<u>parity</u>** (J^{π}) predicted and explained also the stability of nuclei near the magic numbers for $\boldsymbol{N}$ and $\boldsymbol{Z}$, which are **2 , 8 , 20 , 28 , 50 , 82 , 126 .** In the shell model or spin – orbit coupling , we have, ① for the same (ℓ) , each state can accommodate $\mathbf{2(2\ell + 1)}$ nucleons , and $(\mathbf{2j + 1})$ nucleons

② for the same (ℓ) , the $\mathbf{j = \ell + \frac{1}{2}}$ state (i.e. parallel spin and orbital number $\ell \upuparrows \mathbf{s}$)

is lying deeper or more lightly bound , the $\mathbf{j = \ell - \frac{1}{2}}$ state where $\ell \uparrow\downarrow \mathbf{s}$.

③ and even number of identical nucleons (proton or neutron) are having the same ℓ and **j** will always couple to give even parity , zero spin , and zero magnetic moment.



According to Pauli exclusion principle each state is permitted to contain a maximum of (**2j + 1**) identical particles . This consideration leads to the following values for the spins and parities of nuclear ground state.

① **Even – even nuclei** is having

$$\mathbf{J^{\pi} = 0^{+}} \Rightarrow \text{ No exception to this rule}$$

② For **odd – A nuclei** (odd **Z** or odd **N**)

$$\mathbf{J = \frac{1}{2} \text{ integer }} \left(\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \text{------} \right)$$

And

$$\pi = (-1)^{\ell}$$

There are some exceptions to this rule

③ **Odd – odd nuclei** , the total angular momentum **J** is the vector sum of odd **N** and odd **Z.**

$$|J_n - J_p| \leq J \leq |J_n + J_p|$$

and the parity ($\boldsymbol{\pi}$) will be the product of the proton and neutron

$$\pi = (-1)^{\ell_p} \times (-1)^{\ell_n} \Rightarrow (-1)^{\ell_p + \ell_n}$$

① **Even – even nuclei**

$$J^{\pi} = 0^{+}$$

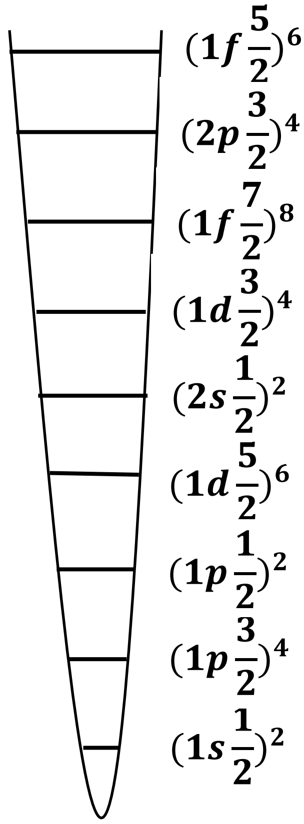
② **Odd – A nuclei** $J^{\pi} = \frac{1}{2}$ **integer**

$$\pi = (-1)^{\ell}$$

③ **Odd – odd nuclei** $\Rightarrow$ **even – A**

$$|J_n - J_p| \leq J \leq |J_n + J_p|$$

$$\pi = (-1)^{\ell_p + \ell_n}$$



For example :-

$${}^{16}_{8}O_{8} \quad \frac{Z=8}{N=8}\left|\frac{(1s\frac{1}{2})^2\,(1p\frac{3}{2})^4\,(1p\frac{1}{2})^2\ \ \textit{Not contribute}}{(1s\frac{1}{2})^2\,(1p\frac{3}{2})^4\,(1p\frac{1}{2})^2\ \ \textit{Not contribute}}\right.$$

عدد النيكلونات لكل

مستوى **(2j +1)**

$$\therefore J^{\pi} = 0^{+}$$

$${}^{17}_{8}O_9 \quad \frac{Z=8}{N=9} \Bigg| \frac{(1s\frac{1}{2})^2\ (1p\frac{3}{2})^4\ (1p\frac{1}{2})^2 \quad \textit{Not contribute}}{(1s\frac{1}{2})^2\ (1p\frac{3}{2})^4\ (1p\frac{1}{2})^2\ (1d\frac{5}{2})^1 \quad \textit{Yes contribute}}$$

$$J = 5/2 \text{ and } \pi = (-1)^{\ell} = (-1)^2 = +$$

$$J^{\pi} = \frac{5}{2}^{+}$$

$${}^{16}_{7}N_9 \quad \frac{Z=7}{N=9} \Bigg| \frac{(1s\frac{1}{2})^2\ (1p\frac{3}{2})^4\ (1p\frac{1}{2})^1 \quad \textit{Yes contribute}}{(1s\frac{1}{2})^2\ (1p\frac{3}{2})^4\ (1p\frac{1}{2})^2\ (1d\frac{5}{2})^1 \quad \textit{Yes contribute}}$$

$$\left|5/2 - \frac{1}{2}\right| \leq J \leq \left|5/2 + \frac{1}{2}\right| \Rightarrow 2 \leq J \leq 3$$

$$J = 2 \text{ or } 3$$

$$\pi = (-1)^{\ell_p + \ell_n} = (-1)^{2+1} = (-1)^3 = -$$

$$\therefore\ J^{\pi} = 2^{-},\ 3^{-}$$

① ***Liquid Drop Model (L D M)*** :-

The binding energy of the nucleus consisting of **Z** protons and **N** = (**A** – **Z**) neutrons , in the first order , well described by (**L D M**). The model was first proposed by **WeiZsäcker** and **Bethe** , describes the nucleus as a charged droplet of nuclear **incompressible** "**Liquid**" of **constant density** with **strong nuclear force** holding the drop together and coulomb interaction pushing it apart. i.e.

اسس النموذج لحساب طاقة الربط النووية

$$B(Z\,,A) = a_v A - a_s A^{2/3} - a_c \frac{Z\,(Z-1)}{A^{1/3}} - a_{asy} \frac{(N-Z)^2}{A} \pm \delta \quad \text{—— } *$$

Volume term	Surface term	Coulomb term	asymmetry term	Pairing term

∗ The volume term or the nuclear volume is then directly proportional to the number of constituent nucleons , the binding energy is proportional with $A \Rightarrow B = a_v A$, i.e. the volume term accounts for the ***B.E*** of all protons and neutrons as if they were surrounded by infinite nuclear matter . It does not depend on **Z** , since the strong nuclear force acts on neutrons and protons alike. It is proportional to the volume of the nucleus.

∗ The surface term ($a_s A^{2/3}$) : the surface term corrects the ***B.E*** for these nucleons close to the nuclear surface which do not feed an attractive nuclear force on all sides. It is analogous to a surface tension and proportional to the nuclear surface area.

The nuclear surface area α with R^2 or with $A^{2/3}$ because $R = R_0 A^{1/3} \Rightarrow R^2 \propto A^{2/3}$

* $a_c \frac{Z(Z-1)}{A^{1/3}}$: The coulomb tem accounts with the coulomb repulsion between **Z** protons in the nucleus.

For heavy nuclei it is usually approximated

$$a_c \Rightarrow -\frac{3}{5} \cdot \frac{e^2}{4\pi E_0} \cdot \frac{Z(Z-1)}{R}$$

$$-\frac{3}{5} \cdot \frac{e^2}{4\pi E_0 R_0} \cdot \frac{Z(Z-1)}{A^{1/3}} = -a_c \frac{Z(Z-1)}{A^{1/3}}$$

---

* $a_{asy} \frac{(N-Z)^2}{A} = a_{asy} \frac{(A-2Z)^2}{A}$: The asymmetry term accounts with the difference between protons , neutrons and the Pauli principle. In stable nuclei , the **N = Z** or $Z = \frac{A}{2}$ but when **N > Z** will be considered radioactive nuclei (Not stable)

$$* \pm\delta \Rightarrow \pm 34\, A^{-3/4} \begin{cases} +34\, A^{-3/4} \rightarrow \text{even – even} \\ 0 \rightarrow \text{odd – A} \\ -34\, A^{-3/4} \rightarrow \text{odd – odd} \end{cases}$$

The pairing term accounts with the tendency of nucleons to form pairs ; which are more strongly bound than unpaired nucleons

Eventually $\Rightarrow$ a_v = **15.5 MeV** , a_s = **16.8 MeV** ,
a_c = **0.62 – 0.72 MeV** , a_{asy} = **23 MeV**

و بما أن طاقة الربط تساوي

$$\mathbf{B(Z\,,A)} = Z_{mp} + N_{mp} - {}^{A}_{Z}\mathrm{M}_{N} \text{ ——— } **$$

و بدمج المعادلتين * و ** يمكن ان نستنتج علاقة شبه تجريبية للكتلة

مهم →

$${}^{A}_{Z}\mathrm{M}_{N} \text{ or } \mathbf{M(Z\,,A)} = Z_{mp} + N_{mp} - a_v A + a_s A^{2/3} + a_c \frac{Z\,(Z-1)}{A^{1/3}} + a_{asy} \frac{(N-Z)^2}{A} + (0 \pm 34\,A^{-3/4})$$

③ *Collective Models* :-

Also called **unified Model** , description of atomic nuclei that incorporate aspects of both shell nuclear model and liquid drop model to explain certain magnetic and electric properties that neither of the two separately can explain.

In the shell model the **nuclear model levels** are calculated on the basis of **single nucleon** (proton or neutron) moving in a potential field produced by all the other nucleons. Nuclear structure and behavior are explained by considering single nucleons beyond a passive nuclear core composed of paired protons and paired neutrons that fill groups of energy levels or shells.

In the Liquid Drop Model nuclear structure and behavior are explained on the basis of statistical contributions of all nucleons (much as the molecules of a spherical water contribute to the overall energy and surface tension).

In the **Collective Model** , high energy states of the nucleus and a certain **magnetic** and **electric** properties are explained by the motion of the nucleons outside the **closed shells** (full energy levels) combined with motion of the paired nucleons in the core.

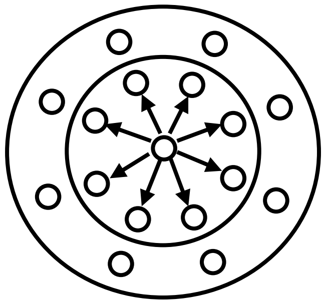
The tide positively charged protons **constitutes** a current that intern contributes to a magnetic properties of the nucleus. The increase in nuclear deformation that occurs with the increase in the number of unpaired nucleon accounts with the measured **electric quadruple moment** which may be considered a measure of how much the distribution of electric charge in the nucleus departs from spherical symmetry. That is because the nucleons inside the nucleus from a direct and focused pressure on the nuclear surface.

As a result , the nucleus can be deformed and take non-spherical and permanent shape ; therefore , the surface can vibrate according to the concept of the liquid drop when an external pressure is exerted on it and the excitation of the nucleus can be seen as superficial vibrations or elastic vibrations , rotational motion or any form of mass movement.

In brief for $A < 150$ nuclei can be described by model based on vibration that occurs around spherical equilibrium shape.

for $150 < A < 190$ nuclei can be represented as a non-spherical rotational and system

Thus , the vibration and rotational moments represent the two main classes of collective nuclear movement.



The semi – empirical relation of mass :-

العلاقة شبه التجريبية للكتلة :

ان أهمية المعادلة شبه التجريبية للكتلة لا يتمثل في أنها ستساعدنا على التنبؤ بأنها ظاهرة جديدة او غريبة في الفيزياء النووية لكن بالأحرى ممكن اعتبارها على أنها أول محاولة لتطبيق النماذج النووية بحيث نستطيع أن نفهم السلوك المنظم للخواص النووية (طاقة الربط) , كما يمكن وضع معادلة بسيطة تعبر عن طاقة الربط **B(Z , N)** او الكتلة المكافئة **M(Z , N)** بدلالة عدد من الحدود و العوامل التي تؤثر على طاقة الربط انطلاقا من نموذج قطرة السائل على اعتبارها غير قابلة للانضغاط الذي يستند على اساس أن حجم النواة يتزايد مع زيادة (**A**) و ان الحدود التي تؤثر بطاقة الربط هي :

حد الحجم و حد السطح و حد قوة كولوم التنافرية و حد التناظر و يسمى احيانا باللاتناظر و جد الازدواج و هناك يتعلق بالقشرات النووية اي :

مهم

حفظ

$$B(Z, N) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sy} \frac{(N-Z)^2}{A} \pm \delta$$

الحد الاول يمثل حد الحجم و فيه a_v = **15.5 MeV**

الحد الثاني يمثل حد السطح و فيه a_s = **16.8 MeV**

الحد الثالث حد قوة كولوم التنافرية a_c = **(0.62 – 0.72)**

الحد الرابع (حد التناظر) a_{sy} = **23 MeV**

$\delta = \mp 34 A^{-3/4}$ **MeV**

$\delta = +$	for	**even – even**	**nuclei**
$\delta = 0$	for	**odd**	**nuclei**
$\delta = -$	for	**odd – odd**	**nuclei**

اما المعادلة شبه التجريبية للكتلة

$$B(Z, N) = \left[Zm_p + Nm_n - {}^A_Z M_N\right]c^2$$

$${}^A_Z M_N = Zm_p + Nm_n - \left(\frac{B(Z, A)}{c^2}\right)$$

Example **:-** Calculate the energy of pairing term and symmetry term for $^{136}_{57}La_{79}$, $^{135}_{57}La_{78}$

For $^{135}_{57}La_{78}$ → odd nucleus ⇒ $\boldsymbol{\delta = 0}$

$$E_{sy} = a_{sy}\frac{(N-Z)^2}{A} = 23\,\frac{(78-57)^2}{135} = 23\,(3.2) = 73.6 \text{ MeV}$$

For $^{136}_{57}La_{79}$ → $\boldsymbol{\delta = -\delta = -34\,A^{-3/4} = -34(136)^{-3/4} = ?}$

$$a_{sy} = a_s\frac{(N-Z)^2}{A} = 23\,\frac{(79-57)^2}{136} = 88 \text{ MeV}$$

Example :- Using semi – empirical mass formula to reveal the (Q – value) (release energy) in nuclear fission that is positive for heavy nuclei? as $^{238}_{92}\mathrm{U}$

Solution :

$$\mathrm{B(Z\,,A)} = \left[Zm_p + Nm_n - {}^{A}_{Z}M_N\right]c^2$$

$$= [92\,(1.007825) + 146\,(1.008665) - 238.05]c^2$$

$$= 180 \text{ MeV}$$

$$\mathrm{B(Z\,,N)} = a_v\,A - a_s\,A^{2/3} - a_c\,\frac{Z(Z-1)}{A^{1/3}} - a_{sy}\,\frac{(N-Z)^2}{A} \pm \delta$$

ملاحظة : ولكن بالسؤال يقول استخدم العلاقة شبه التجريبية اذن الحل يكون باستخدام المعادلة

لذا ارجو اكمال الحل