Nuclear Reaction

A nuclear reaction in a process in which a change in the composition and / or of a target nucleus after its bombardment with charge particle or ϒ-ray. From the study of nuclear reaction the following determined

1) ***<u>The Reaction Mechanism</u>*** : by comparison the experimental data obtained about the reaction mechanism (nuclear models) and the predictions of the theories about it.

2) ***<u>The Nuclear Energy Levels</u>*** : the decay scheme , with all information about each state (energy, spin, parity, M, Q , ……….) constitute the most valuable data for testing of nuclear structure (i.e. Nuclear models) when a given target nucleus interact with a particle of sufficient Kinetic energy. There are many ways for the nuclear reaction to proceed.

$$a + x \rightarrow y + b \text{ or } x(a,b)y$$

this mean that particle a interacts nucleus x and produce a nucleus Y and particles a and b may be elementary particle i.e. proton, neutron, electron, deutron, α – particle or ϒ - ray.

$a + x \rightarrow x + a$ => elastic scattering

$a + x \rightarrow x^* + a$ => inelastic scattering

$a + x \rightarrow y + b$ => transmutation

$a + x \rightarrow z + \gamma$ => reaction (capture reaction)

* ***Elastic Scattering $a + x \rightarrow x + a$*** :-

In elastic scattering the total K.E. of system (incident particle + target nucleus) in the same before and after the collision. Some K.E. of the incident particle transferred to the nucleus, which is left M the same internal nuclear state as before the collision.

Inelastic Scattering ($a + x \rightarrow x^* + a$) :-

The target nucleus in inelastic scattering in raised into an excited state x^* , and the total Kinetic energy of the system is **decreased** by the amount of excitation energy given to the target nucleus.

Nuclear transmutation ($a + x \rightarrow y + b$)($a + x \rightarrow z + \gamma$)

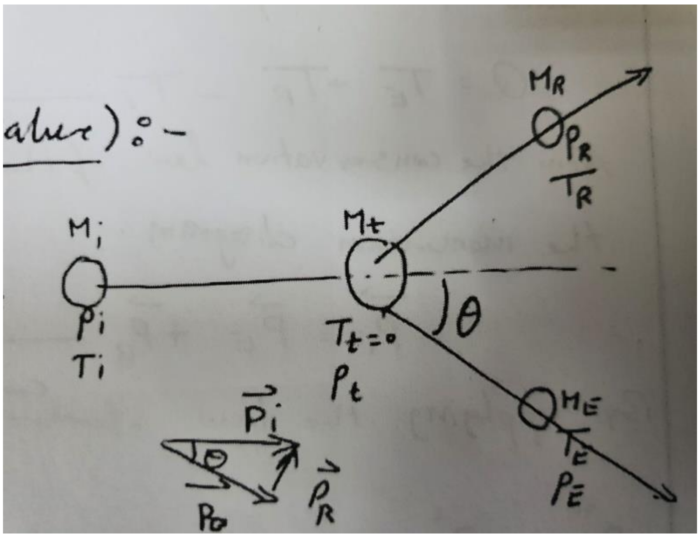
In nuclear transmutation reaction the product nuclei (Y and Z) may be formed in their ground state or in an excited state. The excited product nucleus usually decays to the g-state by the emission of γ-rays in a nuclear reaction of transmutation for scattering process, the number of protons (Z) and the number of neutrons (N) are conserved.

$$^{2}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He_{1} + ^{1}_{0}n$$

$$^{4}_{2}He_{2} + ^{14}_{7}N \rightarrow ^{17}_{8}O_{9} + ^{1}_{1}P$$

Reaction Dynamics (Q – Value)

The important part of a nuclear reaction experiments of the energy released, the Q - Value



From the particle – particle (nucleus) reaction, the relationship between particle energies and Q – value can be calculated. An incident particle mass $\mathbf{M_i}$ and Kinetic energies $\mathbf{T_i}$ interacted with a target nucleus with mass $\mathbf{M_t}$ and K.E. $\mathbf{T_t} = 0$ (L – system)

After reaction, a particle with mass $\mathbf{M_E}$ and K.E. of $\mathbf{T_E}$ and a recoil nucleus mass with $\mathbf{M_R}$ and K.E. of T_R are emitted we consider here the non-relative case, in which the K.E. of each particle in small compared with its rest $\mathbf{Mc^2}$; therefore

$$M_ic^2 + T_i + M_tc^2 + T_t = M_E\,c^2 + T_E + M_R\,c^2 + T_R$$

and since $\mathbf{T_t} = 0$

$$M_ic^2 + T_i + M_tc^2 = M_E\,c^2 + T_E + M_R\,c^2 + T_R$$

The Q – value of the reaction is the differences between the final and the initial kinetic energies.

$$Q = (T_E + T_R) - (T_i + T_t)$$

$$\text{Or } \; Q = (M_i + M_t)c^2 - (M_E + M_R)c^2 \quad \ldots\ldots\ldots\ldots \; **$$

And the L-system

$$Q = T_E + T_R - T_i \;\; \ldots\ldots\ldots \quad 1$$

From the conservation law of the momentum we have from the momentum diagram :

$$\vec{P}_i = \vec{P}_E + \vec{P}_R \;\; \ldots\ldots\ldots \; 2$$

By applying the law cosine to the momentum triangle

$$P^2_R = P^2_i + P^2_E - 2P_iP_E \cos\theta \quad \ldots\ldots\ldots \; 3$$

And since $P^2 = 2MT$, $P = \sqrt{2MT}$ then :

$$2M_RT_R = 2M_iT_i + 2M_ET_E - 2\sqrt{2Mi\,Ti}\,\sqrt{2MeTe}\cos\theta$$

$$\therefore \; T_R = \frac{Mi}{Mr}T_i + \frac{Me}{Mr}T_E - 2\,\frac{\sqrt{MiTiMeTe}}{Mr}\cos\theta$$

Substituting in eq. (1), obtain :

$$Q = T_E + T_R - T_i = T_E + \frac{Mi}{Mr}T_i + \frac{Me}{Mr}T_E - 2\,\frac{\sqrt{MiTiMeTe}}{Mr}\cos\theta - T_i$$

$$* \; Q = (1 + \frac{Me}{Mr})T_E - (1 - \frac{Mi}{Mr})T_i - 2\cos\theta\,\frac{\sqrt{MiTiMeTe}}{Mr} \quad \ldots\ldots\ldots \; 4$$

This is conventional form of the Q – value, to determine it (Q – value) of the nuclear reaction. These the K.E. , T_i, T_E and the angle θ, are all measured in laboratory system of coordinate, this equation is independent of the mechanism of the reaction (compound nucleus, direct reaction or nuclear fission). In calculation the Q – value of the masses (i.e. A) can be used without significant error.

$$Q = (T_E + T_R) - (T_i + T_t)$$

$$\text{Or} \quad Q = (M_i + M_t)c^2 - (M_E + M_R)c^2 \quad \ldots\ldots\ldots\ldots \quad **$$

And the L-system

$$Q = T_E + T_R - T_i \quad \ldots\ldots\ldots \quad 1$$

From the conservation law of the momentum we have from the momentum diagram :

$$\vec{P}_i = \vec{P}_E + \vec{P}_R \quad \ldots\ldots\ldots \quad 2$$

By applying the law cosine to the momentum triangle

$$P^2_R = P^2_i + P^2_E - 2P_iP_E\cos\theta \quad \ldots\ldots\ldots \quad 3$$

And since $P^2 = 2MT$, $P = \sqrt{2MT}$ then :

$$2M_RT_R = 2M_iT_i + 2M_ET_E - 2\sqrt{2Mi\,Ti}\,\sqrt{2MeTe}\cos\theta$$

$$\therefore \; T_R = \frac{Mi}{Mr}T_i + \frac{Me}{Mr}T_E - 2\,\frac{\sqrt{MiTiMeTe}}{Mr}\cos\theta$$

Substituting in eq. (1), obtain :

$$Q = T_E + T_R - T_i = T_E + \frac{Mi}{Mr}T_i + \frac{Me}{Mr}T_E - 2\,\frac{\sqrt{MiTiMeTe}}{Mr}\cos\theta - T_i$$

$$* \; Q = \left(1 + \frac{Me}{Mr}\right)T_E - \left(1 - \frac{Mi}{Mr}\right)T_i - 2\cos\theta\,\frac{\sqrt{MiTiMeTe}}{Mr} \quad \ldots\ldots\ldots \quad 4$$

This is conventional form of the Q – value, to determine it (Q – value) of the nuclear reaction. These the K.E. , T_i, T_E and the angle θ, are all measured in laboratory system of coordinate, this equation is independent of the mechanism of the reaction (compound nucleus, direct reaction or nuclear fission). In calculation the Q – value of the masses (i.e. A) can be used without significant error.

Exoergic and Endorgic reaction :-

Nuclear reaction is divided energetically into two classis :

Exoergic has a positive Q – value (**$Q > 0$, positive**) i.e. the K.E. of the products (**$T_E + T_R$**) exceeds that of the inputs particle (**$T_i + T_t$**) ; therefore

$T_E + T_R > T_i + T_t$ $\Rightarrow$ for (**+ Q**).

Endorgic reaction has negative Q – value (**$Q < 0$, negative**) and an excess kinetic energy should be given to the incident particle in order to make the reaction energetically positive.

$T_E + T_R < T_i + T_E$ $\Rightarrow$ for (**- Q**).

A) ***Exoergic reaction (+ Q)*** :

In exoergic reaction we focus our attention on the energy **T_E** and the direction θ on the incident energy **T_i** is gradually increased.

1) ***Zero – bombarding ($T_i \cong 0$)*** :

This reaction possible only with neutrons (**Z = 0**) as in the interaction of thermal neutron (**E_n = 0.025 eV**) with **^{10}B** as follow :

$$^{10}_{5}B + {}^{1}_{0}n \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He_2$$

If **$T_i \cong 0$** , then **T_E** from eq. (**4**) will be

$$T_E = \frac{Mr}{Me+Mr} Q \quad \Rightarrow \quad (Q > 0)$$

the Kinetic energy **T_E** is the same for all angles,

i.e. is independent on (θ), and the angular distribution is isotropic.

Example :- prove that **Q = 2.8 MeV** for the **$^{10}B\ (n\ ,\ \alpha)\ ^{7}Li$** reaction :

Solution :

(H.W.)

$$Q = (M_i + M_t)c^2 - (M_E + M_R)c^2$$

B) Endoergic reaction ($Q < 0$) :-

for every nuclear reaction with a positive Q – value , the inverse has a negative Q – value of exactly equal absolute magnitude

$$^{13}_{6}C + {}^{1}_{1}H \rightarrow {}^{10}_{5}B + {}^{4}_{2}He + Q \quad \Rightarrow Q = -4\ MeV$$

1) Zero – bombarding energy :-

When Q – value is negative and $T_i \rightarrow 0$ then the reaction **cannot occur** because reactions induced by charge particles are **threshold reactions** and the energy of the incident particle must be more than the threshold energy **($T_i > T_{threshold}$)**

i.e. the energy of the emitted particle T_E will be not negative value

$$T_E = \frac{Mr}{Me+Mr} Q$$

Since **Q** is negative, then T_E is negative also.

2) The Threshold Energy :-

The smallest value of bombarding energy at which the reaction can take place (**can occur**) is called the threshold energy **$(T_i)_{threshold}$**

$$T_i = -Q\left[\frac{M_E + M_R}{M_E + M_R - M_i - \frac{M_i M_E}{M_R}\sin^2\theta}\right] \quad \dots\dots\dots\; 5$$

If the product particle M_E is observed at $\theta = 0$ (i.e. $\sin\theta = 0$), then T_i has its minimum value, which is the **<u>threshold energy</u>** i.e.

$$T_i)_{threshold} = -Q\left[\frac{M_E + M_R}{M_E + M_R - M_i}\right] \quad \dots\dots\dots\; 6$$

From the general relationship between **<u>Q</u>** and the rest masses , which is :

$$(M_i + M_t)C^2 = (M_E + M_R)C^2 + Q$$

$$(M_i + M_t) = (M_E + M_R) + \frac{Q}{C^2}$$

And then $M_t \gg \frac{Q}{C^2}$

$M_i + M_t \cong M_E + M_R \quad \Rightarrow$ therefore ∴

طاقة العتبة $T_i)_{threshold} = -Q\left(\frac{M_i + M_t}{M_t}\right) \quad \dots\dots\dots\; 7$

As the bombarding energy is raised the emitted particle M_E appear at $\theta > 0$

For elastic scattering , $Q = 0$, $M_E = M_i$ and $M_t = M_R$ ∗
(***لا يوجد فقدان في الطاقة للنظام***)

For inelastic scattering , the struck nucleus is left in an excited ∗ level after the collision

$Q = -E_{ex}$ where $E_{ex} \Rightarrow$ **is the excitation energy** given to the target M_t and $M_E = M_i$, $M_R = M_t$

**** Theories of Nuclear Reaction*** :-

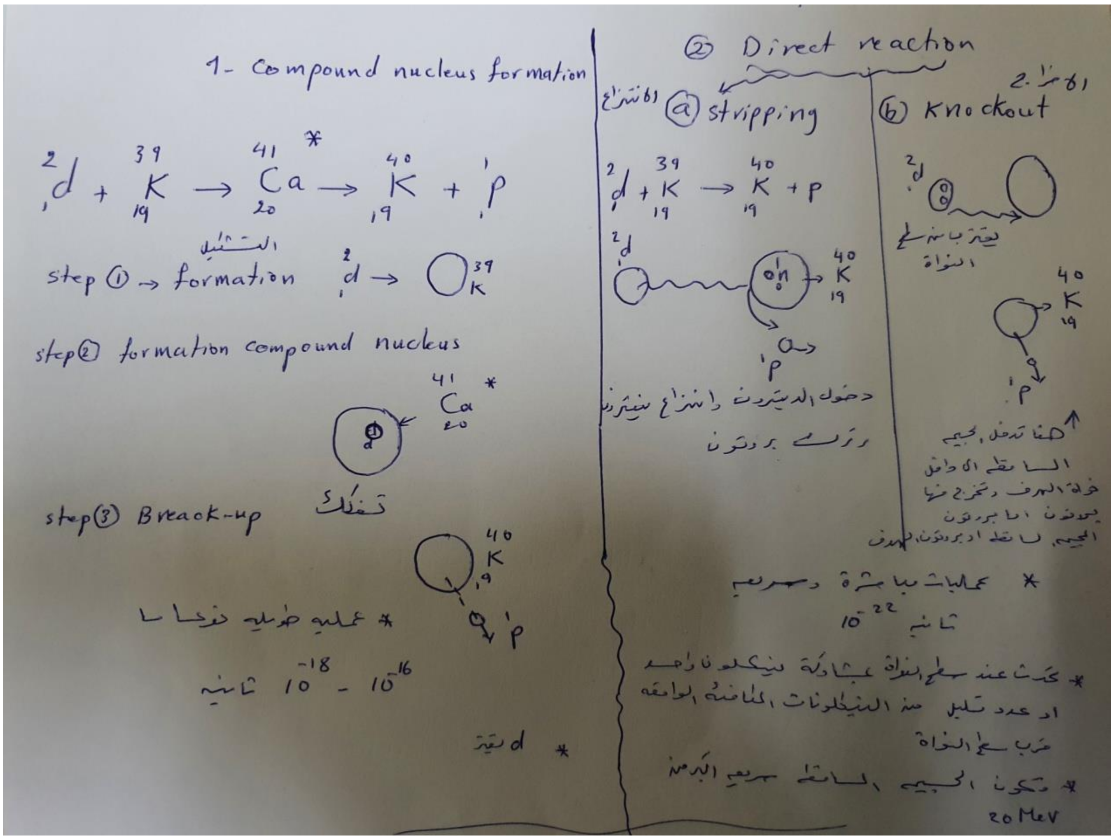
Reaction induced by particle , such as **α** , **^{1}P** , **^{2}d** are classified as :

1) Compound nuclear Reaction

تفاعلات النواة المركبة

2) Direct reaction

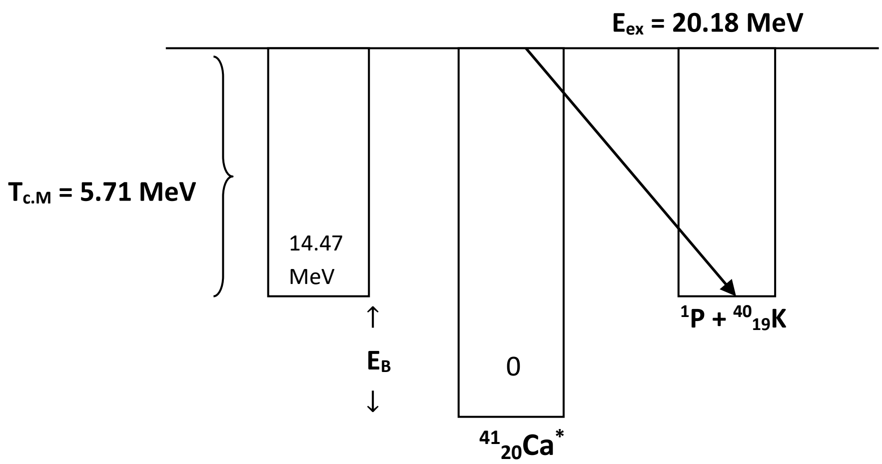
التفاعلات المباشرة



1) Compound Nucleus :

For bombarding energies below (**0.1 – 1 MeV**), nuclear reaction generally proceed through the compound nucleus mechanism. The theory of nuclear reactions through compound nucleus first introduced by **Neils Bohr** in **1936** according to Bohr idea's , when bombarding particle has a **sufficiently low energy** it will be absorbed.

By the target nucleus and the kinetic energy as well as the binding energy of the incoming particle will represent the excitation energy of the compound nucleus's



$T_{C.M}$ ⇒ is the Kinetic energy of the center of mass coordinates

E_{ex} ⇒ excitation energy of the compound nucleus

E_B ⇒ Binding energy

$$\mathbf{E_{ex} = E_B + T_{em}}$$

$$\mathbf{E_{ex} = 14.47 + 5.71 = 20.18\ MeV}$$

$$T_{c.M} = \frac{T_{Labratory}}{1+\frac{m}{M}}$$

m ⇒ mass of the incident particle

M ⇒ Mass of the target nucleus

$T_{Lab.}$ ⇒ K.E. of the particle in L - system

This excitation energy (**E_{ex}**) will be (after collision) rapidly shared between the nucleons of the compound nucleus ; therefore , none of them will have sufficient energy to escape immediately the compound nucleus will then stay together

($\mathbf{10^{-16}}$ - $\mathbf{10^{-18}}$) sec until by chance enough energy is concentrated on one nucleon or an assembly of nucleon's , and after that the escape will happen. The reaction through a compound nucleus's formation is the target as two process

The formation and ∗∗ break-up process , it is assumed that ∗ the time lapse between the two event's is sufficiently long ($10^{-16} – 10^{-18}$) sec . The compound nucleus does not break-up until it has forgotten how it is formed .

The second stop $\Rightarrow$ the decay by particle emission is simply the decay of nucleus in a highly excitation

$${}^{1}_{1}P + {}^{63}_{29}Cu$$

$${}^{4}_{2}He_{2} + {}^{60}_{28}Ni$$

$${}^{64}_{30}Zn^{*}$$

$${}^{63}_{30}Zn + {}^{1}_{0}n$$

$${}^{62}_{29}Cu + {}^{1}P + {}^{1}_{0}n$$

$${}^{62}_{30}Zn + 2\ {}^{1}_{0}n$$

<u>2) Direct reaction (10^{-22}) sec</u> :-

<u>a) Stripping reaction</u> :-

The stripping process represent the second reaction mechanism . The <u>**neutron**</u> and <u>**proton**</u> in the deuteron (${}^{2}_{1}d$) (are bound with binding energy = 2.23 MeV) it is ; therefore , quite likely that when the deuteron comes close to the target nucleus either the neutron or the proton may be captured by the target nucleus while the other particle's continues on its path relatively undisturbed. Stripping process belong to the direct reaction class. The direct reaction is an instantaneous process that usually take place at the <u>**nuclear surface**</u>.

b) Knock – out reaction :-

This reaction is the second type of the direct reaction.

In knock – out reaction the initial particle inter the nucleus and then knock – out another nucleus without forming compound nucleus. In given the example, the deuteron $^{2}_{1}\mathbf{d}$ inter the target nucleus $(^{39}_{19}K)$ and a proton is knock – out from the nucleus.

This is again an instantaneous process and quite

different in the theory of the compound nucleus.

Resonances in the Formation of the compound nucleus :-

In order to form a compound nucleus in nuclear reaction the incident particle must penetrate the coulomb barrier and also the nucleus surface (nuclear surface tension) . Excited state of compound nucleus or the excitation energy is always determined by the masses of the colliding nuclei plus the incident kinetic energy in CM coordinate.

$$E_{ex} = E_B + T_{cm}$$

If the excitation energy just equal to one of the excited state of the compound nucleus , the we would expect resonance forming of the compound nucleus and a large reaction probability i.e. large cross-section

* عند دراسة التفاعلات المباشرة (Direct reaction) فان الحالات المترابطة (Bound states) يمكن ملاحظة ان معدل تلك الحالات طويل نسبيا و بهذا سيكون عرضها قليل $(\Gamma = \frac{\hbar}{\tau})$ داخل ضمن المسافات الاعتيادية (D) التي تفصل بين المستويات المترابطة , و هكذا يمكن القول بأنها حالات منفصلة (discrete) .

اما بالنسبة للنواة المركبة فيمكن تقسيم حالات الاثارة فيها , حسب طاقة الجسيمة الساقطة الى صنفين رئيسيين عندما تكون طاقة القصف كبيرة تتهيج النواة الى حالات عالية و يكون معدل اعمارها قصيرة عرضها (Γ) طويل .

و هو اكبر من المسافات D ($\Gamma \gg D$) فتتداخل مع بعضها مكونة بذلك **طيفا مستمرا** .

اما عندما تكون طاقة القصف قليلة فعند ذلك تتكون حالات منفصلة في منطقة النواة المركبة تعرف **بالحالات شبه المترابطة** (quasi bound state) و يكون معدل عمرها طويل نسبيا و عرضها صغير نسبيا و تسمى بالحالات الرنينية .

Breit – Wigner Formula :-

Breit – Wigner derived a theoretical formula for the cross – section of the resonance processes. If the incident particle has zero angular momentum (J = 0 , i.e. α – particle) then the formula in

$$\sigma(a,b) = \frac{\lambda^2}{4\pi} \frac{\Gamma_a \Gamma_b}{(E_a - E_0)^2 + (\frac{\Gamma}{2})^2} \Rightarrow \text{for } (J = 0)$$

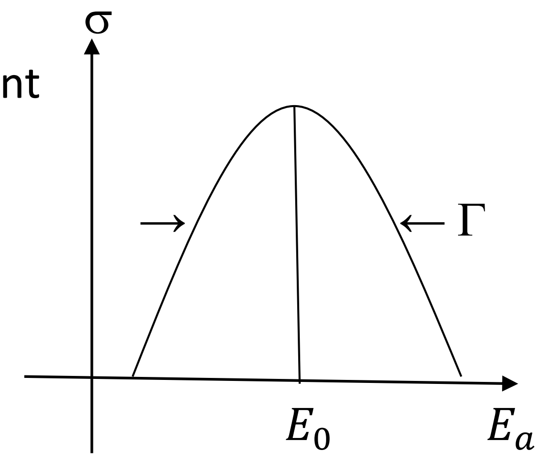
Where , λ is the de Broglie wave length of the incident particle

$$\lambda = \frac{h}{mv}$$

$E_a \rightarrow$ energy of the incident particle

$E_0 \rightarrow$ energy of the peak of the resonance

$\Gamma \rightarrow$ is the width of the peak



If the angular momentum of the incident particle is not zero (J ≠ 0) , then

$$\sigma(a,b) = \frac{\lambda^2}{4\pi} \left[\frac{2J_c + 1}{(2J_a + 1)(2J_t + 1)}\right] \frac{\Gamma_a \Gamma_b}{(E_a - E_0)^2 + (\frac{\Gamma}{2})^2}$$

Where , $J_c \Rightarrow$ is the **angular momentum** of the compound nucleus.

$J_a \Rightarrow$ the **spin** of the bombarding particle (a)

($J_a = \frac{1}{2}$ for nucleons)

$J_t \Rightarrow$ the spin of the target nucleus.

Although the Breit – Wigner formula is strictly valid only **near a resonance** it may applied in the thermal energy region in the absence of resonance if it is assumed that the energy $\boldsymbol{E_0}$ is the resonance energy nearest to the thermal region
[the energy in the thermal region in the (n,γ) reaction $\cong$ **0.025eV**]

In this case all the factors in the Breit – Wigner formula is constant compared with Γ_n which is proportioned to the neutron velocity ***v*** and λ which ***n*** **proportional** to $\frac{1}{v}$
$(\lambda = \frac{h}{mv} \propto \frac{1}{v})$. Therefore the formula for the (n,γ) reaction then reduced to

$$\sigma(n,\gamma) = \frac{constant}{v} \propto \frac{1}{v}$$

This mean that cross – section of (n,γ) is inversely proportional to neutron velocity ***v*** and this is called the $\frac{1}{v}$ law.

The Breit – Wigner formula consist of three (3) factors.

① $\frac{\lambda^2}{4\pi} \Rightarrow$ is the probability of forming the compound nucleus.

② $\Gamma_a\,\Gamma_b \Rightarrow$ are the partial the widths of levels i.e. the probability for definite types of decays.

③ $\frac{1}{(E_a - E_0)^2 + (\frac{\Gamma}{2})^2} \Rightarrow$ is the resonance factor when $E_a = E_0$

the cross – section (σ) has its greatest value i.e. resonance cross – section.

Q1)

The Compton scattering of protons with **7.5 MeV** by target of $^{7}_{3}Li$. determine

1) The energy of protons that elastic scattered with **90°**

2) The energy of proton that in inelastic scattered with **90°** when the nucleus target rises to the level of the first excited state?

Q2)

بالنسبة للتفاعل **^{9}Be (p,d) ^{8}Be** قيمة **Q** تساوي **0.5595 MeV** استخدم هذه القيمة بالإضافة الى الكتل المعروفة و الدقيقة لكل من **^{9}Be , d , ^{1}H** لإيجاد كتلة **^{8}Be** ؟

m(^{1}H)= 1.007277 u , m(d) = 2.014102 u , m(^{9}Be) = 9.012182 u

Q3)

بالنسبة للتفاعلات الماصة للطاقة جد قيمة **Q** و طاقة العتبة , على فرض أن الجسيمة الخفيفة هي التي تسقط على الهدف الأثقل و الساكن

1) **^{7}Li + P $\rightarrow$ ^{7}Be + n**
2) **^{12}C + P $\rightarrow$ n + ^{12}N**

m(p) = 1.007277 u , m(n) = 1.008665 u , m(^{7}Li) = 7.016003 u ,
m(^{7}Be) = 7.016928 u , m(^{12}C) = 12.000 u , m(^{12}N) = 12.018613 u

Q4)

عرف طاقة العتبة بالنسبة لتفاعلات الماصة , أثبت العلاقة الخاصة بطاقة العتبة , وهل للتفاعلات الباعثة للطاقة طاقة عتبة ؟

Q5)

ماذا يحدث في كل نوع من التفاعلات الاتية , الاستطارة المرنة و غير المرنة , و تفاعلات التحول النووي , اكتب المعادلات الخاصة بكل نوع ؟

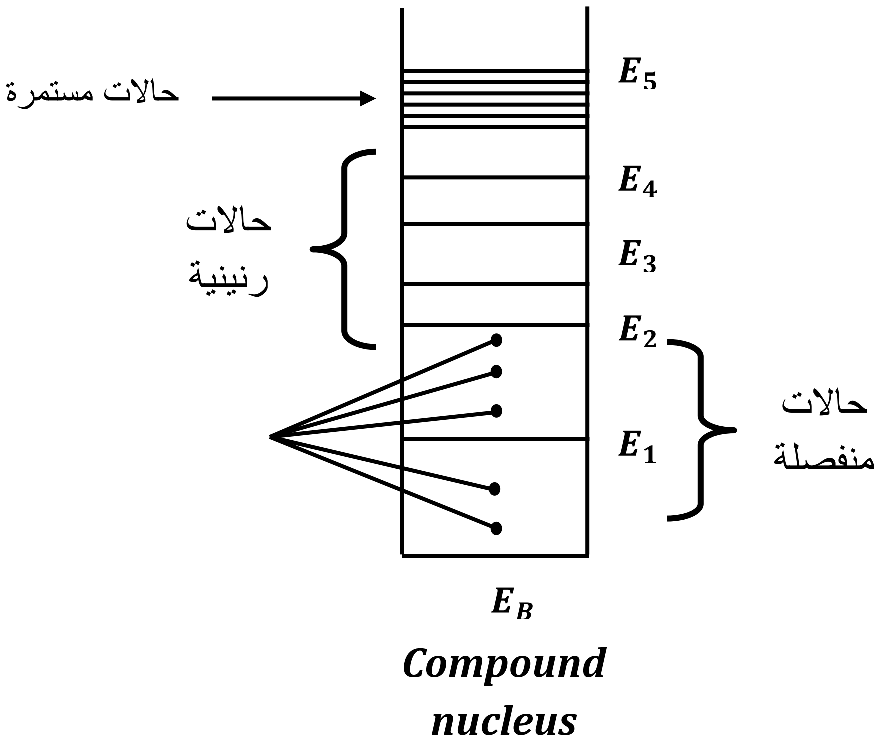
$$\because\ T_{cm} = \frac{T_{Lab.}}{1+\frac{m}{M}}$$

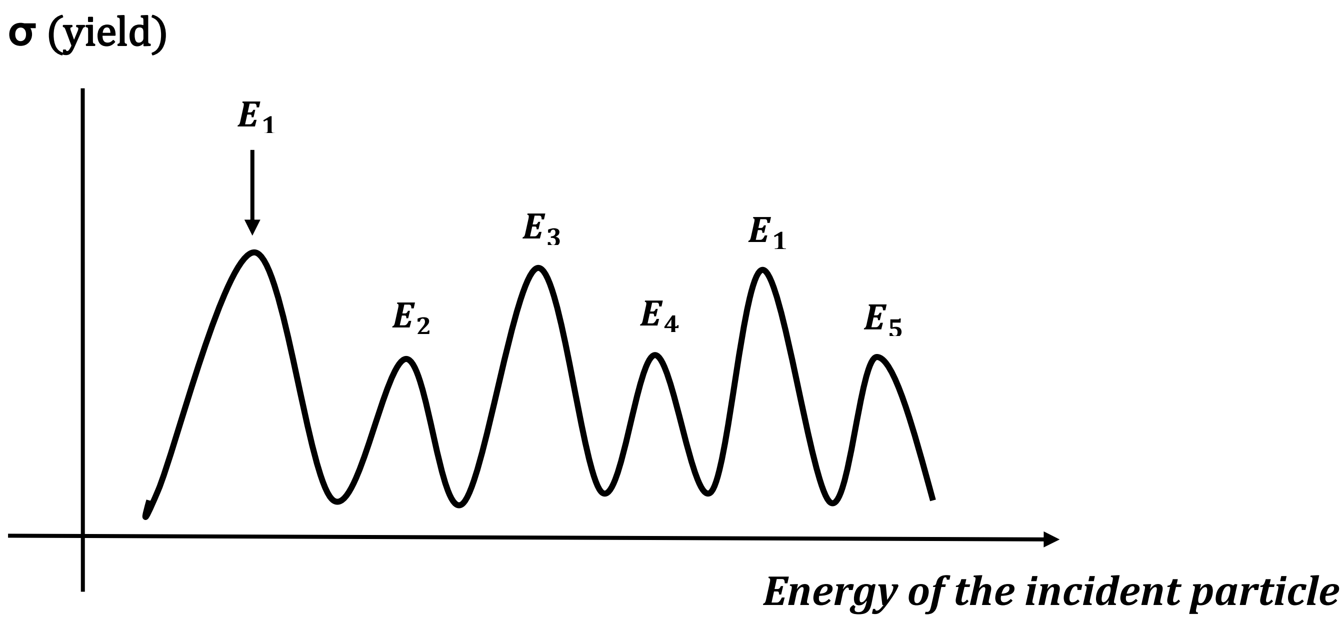
فمثلا $^{2}_{1}d$ سقط ب 5 MeV في نظام المختبر على هدف $^{39}_{19}K$ ما مقدار $E_{ex.}$

$$\because\ T_{cm} = \frac{5}{1+\frac{2}{39}} = ?$$

نجد قيمتها في قوانين $\Rightarrow E_B \rightarrow$ طاقة ربط الجسيم الساقط $B.E$ من فص (1)

$$E_{ex} = E_B + T_{cm} = \text{-----}\ \ \text{MeV}$$





i.e. where : $E_{ex} = E_B + T_{cm} = E_{resonance}$

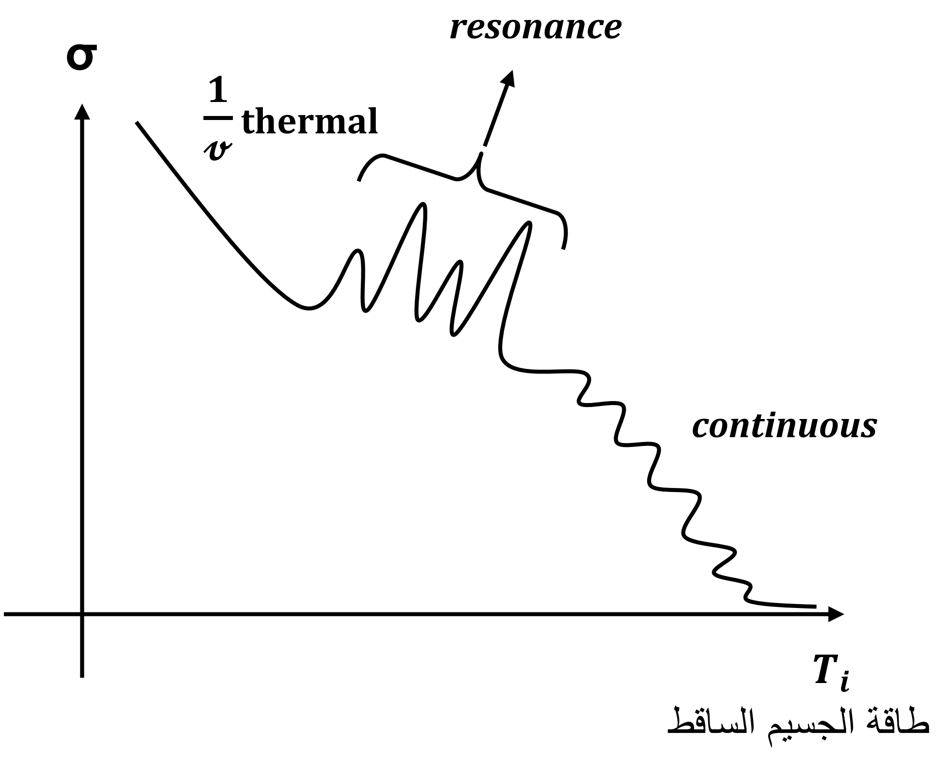
* عندما يسقط جسيم يمتلك طاقة تهيج $E_{ex} = E_B + T_{cm}$ اكبر من طاقة الربط E_B تشكل طاقة رنينية

----

* ***<u>Nuclear Reaction Cross – Section</u>*** :-

To study nuclear reaction in details , it is necessary to have a quantitative measure of the probability of a given reaction. This quantity , which is called the Cross – Section , which can be measured experimentally and calculated in such a way that the theoretical and experimental values can be compared.

Te theories of the cross – section for nuclear reaction in which a compound nucleus formed divided into two broad classifications



① At a low bombarding energies of the excited level of the compound nucleus are discrete and may be widely separated. Here the reaction cross – sections are described by a resonance theory.

② At a higher bombarding energies the excited levels in the C.M are more closely spaced , broader and practically overlapped. Here the reaction cross – section is described by continuous theory.

Cross – Section for nuclear reactions are often expressed in barn , each barn is equal to $\mathbf{10^{-24}}\ \boldsymbol{cm^2}$ or $\mathbf{10^{-28}}\ \boldsymbol{m^2}$

1 barn = $\mathbf{10^{-28}}\ \boldsymbol{m^2}$