Nomenclatures

***Atomic Number* (Z) :** The atomic number or "**Proton**" number equals the number of equals the number of protons in a nucleus.

***Mass Number* (A) :** Is the integer nearest to the exact atomic isotopic weight.

***Neutron Number* (N) :** The number of neutrons in any nucleus ; therefore ,

$$\mathbf{A = N + Z} \quad \text{for} \quad {}^{A}_{Z}X_N \ \ \textbf{nucleons}$$

***Unpaired Neutron Number* :** The number of neutrons which are in excess of the number of protons in a nucleus.

$$(\mathbf{A - 2Z}) = (\mathbf{N - Z}) \ \Rightarrow \ \text{for} \quad \textbf{most elements}$$

$$(\mathbf{A - 2Z}) = \mathbf{0} \ \Rightarrow \ \text{for} \quad \textbf{Light elements}$$

***Atomic Mass* :** The exact value of the mass of a neutral atom relative to the mass of a neutral atom of carbon isotope $^{12}\mathbf{C}$.

***Nucleon* :** Proton or neutron.

***Isotopes* :** Nuclei (nuclides) are having the same **Z** but different **A** and **N** .

$$\left({}^{1}_{1}H_0\,,{}^{2}_{1}H_1\,,{}^{3}_{1}H_2\right) \text{ or } \left({}^{4}_{2}He_2\,,{}^{3}_{2}He_1\right) \text{ or } \left({}^{16}_{8}O_8\,,{}^{15}_{8}O_7\,,{}^{17}_{8}O_9\right)$$

***Isobars* :** Nuclei (nuclides) are having the same **A** but different **Z** and **N** .

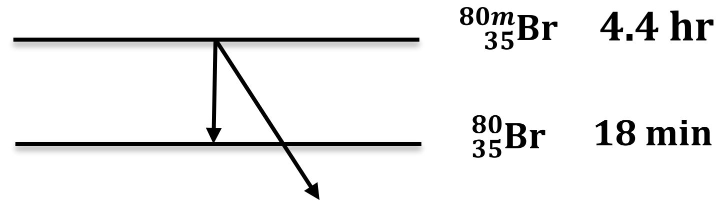
$$\left({}^{14}_{6}C_8\,,{}^{14}_{7}N_7\right)$$

***Isotones* :** Nuclei are having the same **N** but different **Z** and **A** .

$$\left({}^{14}_{6}C_8\,,{}^{15}_{7}N_8\,,{}^{16}_{8}O_8\right)$$

***Isomers* :** Nuclei are having the same **Z** , **N** and **A** but different in energy states.

The **4.4 hr** state is the **excited** state of $^{80m}\mathbf{Br}$ nucleus **,** and the life time of such a state is relatively long . Such a state is called **Isomeric state** or **metastable state**.



Isodiapheres **:** Nuclei are having the same value of $(\mathbf{N}-\mathbf{Z})$.

$$\left({}^{12}_{6}\mathbf{C}_{6}\ ,\ {}^{14}_{7}\mathbf{N}_{7}\ ,\ {}^{16}_{8}\mathbf{O}_{8}\right)$$

Mirror Nuclei **:** They are two isobaric nuclei (**same A**) but the **proton** number (**Z**) in one of them equals to the neutron number (**N**) in the other nucleus.

$$\left({}^{15}_{7}\mathbf{N}_{8}\ \Leftrightarrow\ {}^{15}_{8}\mathbf{O}_{9}\right),\left({}^{17}_{8}\mathbf{O}_{9}\ \Leftrightarrow\ {}^{17}_{9}\mathbf{F}_{8}\right)$$

Chapter "1"

Nuclear Properties

Charge , Radius , Mass , Angular Momentum , Magnetic Dipole Moment , Electric Quadruple Moment , Spin , Parity , Quantum Statistics , Excited States.

① Charge of Nuclei

The charge of a nucleus is

Equal to (**Ze**).

Where

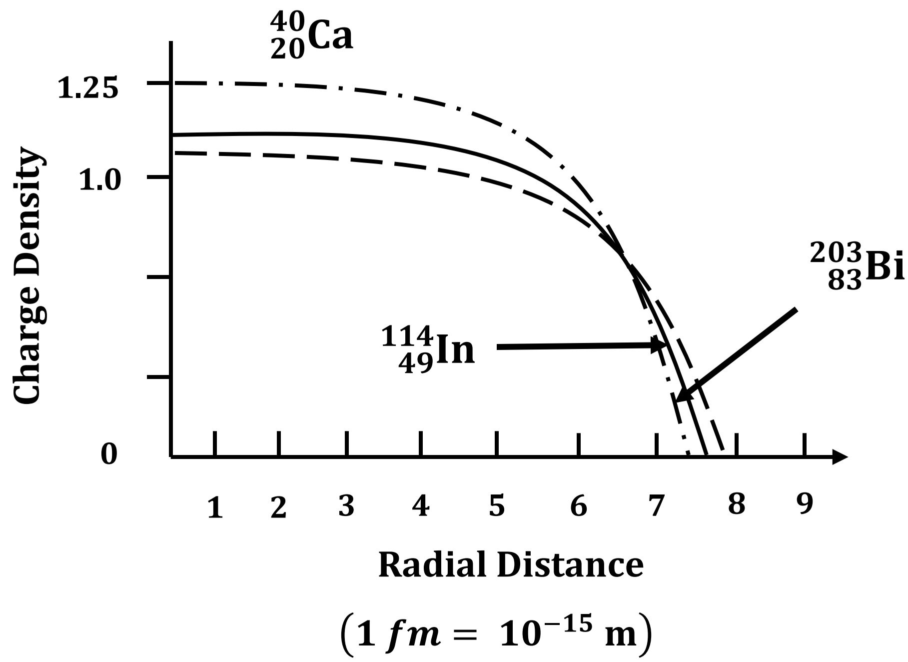
Z ⇒ proton number

e ⇒ elementary charge

e = 1.6×10^{-19} coulomb . (SI)

e = 4.8×10^{-10} esu

1 Coulomb (1C) = 3×10^{9} esu



It was found that the central parts of all nuclei are approximately the same density , i.e. " aside from surface effects , the density of nuclear matter is the same in all nuclei "

The Hofstadter Formula

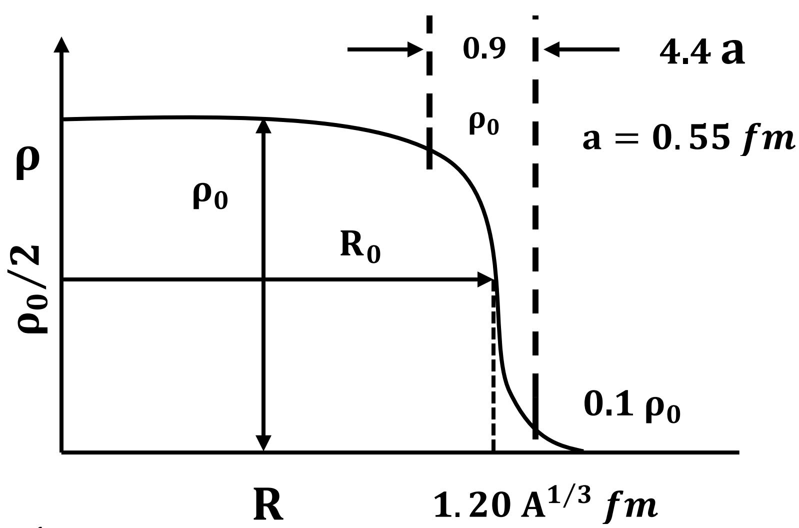
$$\rho = \frac{P_0}{1+\exp\left(\frac{r-R}{a}\right)}$$

$\rho_0 \cong 1.65 \times 10^{44}$ **nucleon/m^3**

$= 0.165$ **nucl./fm^3**

$1\ fm = 10^{-15}$ **meter**

$R = 1.20\ A^{1/3}\ fm$



② Radius of Nuclei

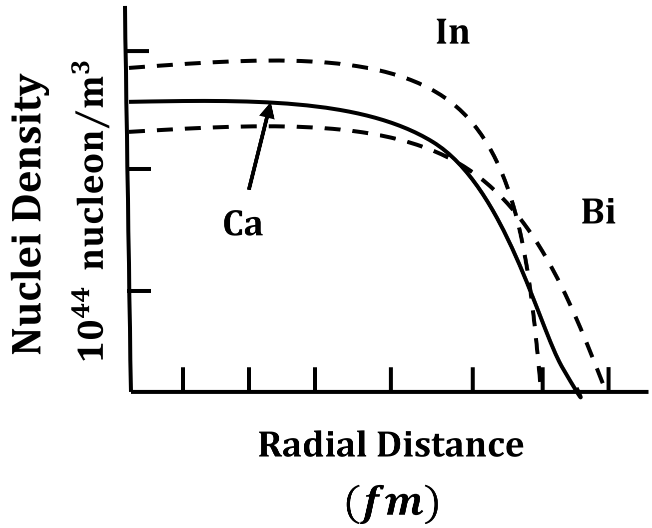
The nuclear volume is substantially proportional to the number of nucleons in a given nucleus. This means that nuclear matter is essentially in compressible and has a constant density for all nuclei.

$$\frac{4}{3}\pi R^3 \propto A \quad \text{or} \quad \text{const.} = \frac{A}{{}^4/_3\,\pi R^3}$$

$$R^3 \propto A \quad \Rightarrow \quad R = \text{const.}\, A^{1/3}$$

$$R = R_0\, A^{1/3}$$

$$R_0 \approx 1.25\, fm = 1.25 \times 10^{15}\ \text{m}$$



R – nuclear radius

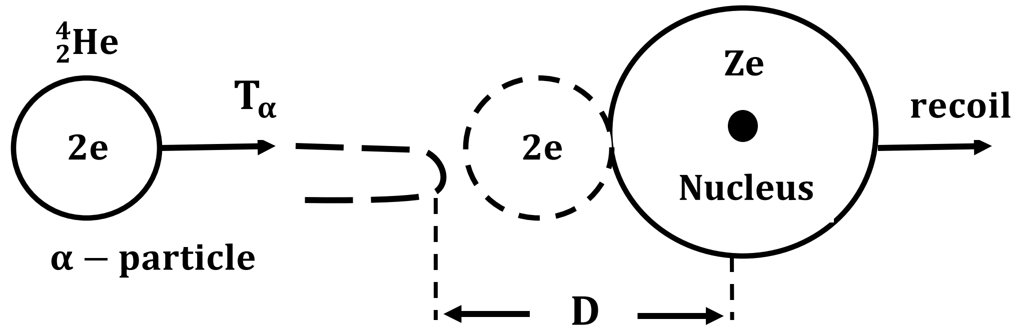
R_0 – Radius Const.

$$R_0 = \begin{cases} 1.4\ fm \text{ for nuclear particles scat.} \\ 1.2\ fm \text{ for electron scat. experiment} \end{cases}$$

Distance of Closest Approach :

By this method the radius of nuclei or (the nuclear size) can be roughly estimated.

* مسافة اقرب اقتراب هي اصغر مسافة تسطيع فيها جسيمة مشحونة من الاقتراب من نواة الهدف.



When an $\alpha -$ **particle** is far from given nucleus it has only kinetic energy (T_α).
It comes closest to the nucleus in a head – on collision.

At that point $\alpha -$ **particle** has only **potential** energy if the **recoil** of the nucleus is **neglected** because of the big mass of the nucleus. From conservation of energy we have :

$$K.E. = P.E.$$

$$T_\alpha = \frac{2e\,.\,Ze}{D} \quad ; \text{where}$$

2e ⇒ **Charge of α − particle**

Ze ⇒ **Charge of the nucleus**

D ⇒ **Distance of closest approach**

$$\therefore\ D = \frac{2Ze^2}{T_\alpha} \quad \text{(in esu)}$$

Or

$$D = \frac{1}{4\pi\epsilon_0} \times \frac{2Ze^2}{T_\alpha} \quad \text{(in SI units)}$$

$$\frac{1}{4\pi \in_0} = \frac{1}{4\pi\,(8.9\ \times\ 10^{-12})} = \frac{C^2}{N-\ m^2}$$

Example **:** calculate the distance of closest approach for $\boldsymbol{\alpha}$ **− particle** with **K.E = 25 MeV**, interacting with $^{238}_{92}\mathbf{U}$ nucleus.

Solution **:**

In the (esu)

$$D = \frac{2Ze^2}{T_\alpha} \Rightarrow D = \frac{2 \times 92\,(4.8 \times 10^{-10})^2}{25\ \text{MeV} \times 1.6 \times 10^{-6}\ \text{erg/MeV}}$$

Because **1 MeV = 1.6 × 10^{-6} erg**

∴ **D ≈ 10^{-12} cm ≅ 10 fm** **1 fm = 10^{-13} cm**

In the SI units

$$D = \frac{1}{4\pi\in_0} \times \frac{2Ze^2}{T_\alpha} = \frac{1}{4\pi\,(8.9 \times 10^{-12})} \times \frac{2 \times 92\,(1.6 \times 10^{-19})^2}{25 \times (1.6 \times 10^{-13}\ \text{J})}$$

$$\cong 0.0089 \times 11.77 \times 10^{-38} \times 10^{12} \times 10^{13} = 0.1 \times 10^{-13} = 10 \times 10^{-15}\ \text{m}$$

∴ **D = 10 fm** **1 fm = 10^{-15} m**

③ **Mass of Nuclei**

The nucleus contains about **99.975%** of the mass of an atom.

$$\acute{M}_{nuc.} = M_{atom} - [zm_e - B_e(z)]$$

$\acute{M}$ ⇒ mass of the nucleus or nuclear mass

M_{atom} ⇒ atomic mass

m_e ⇒ electron rest mass

$B_e(z)$ ⇒ total binding energy of all electrons

$B_e(z)$ ⇒ $15.73\ Z^{7/3}$ **eV**, which is can be neglected

$$m_e = \begin{cases} 9.1 \times 10^{-31}\ kg \\ 5.48 \times 10^{-4}\ amu \\ 0.511\ MeV \end{cases}$$

For the heaviest elements ($z \approx 100$), the total electron binding energy approaches $B_e(z) \approx 1\ MeV$, even this value negligible compared with the binding energy of the nucleus.

Conversion of Units تحويل

$$1\ amu = \frac{1}{2} \times \frac{12}{N_A}$$

amu ⇒ atomic mass unit

N_A ⇒ Avogadro's number

$N_A = 6.025 \times 10^{23}\ atom/mol$

$$W = \frac{N \times A}{N_{av}}$$

$\therefore\ 1\ amu = 1.66 \times 10^{-27}\ kg$

$1\ amu = 1.66 \times 10^{-29}\ gm$

$1\ amu = 931.5\ MeV$

راجع كتاب د. منيب
صفحة (50) موضحة

The energy equivalent in **Joules** can be estimated from **Einstein's mass – energy relationship**

$E = mc^2$

$E = 1\ kg \times (3 \times 10^8)^2 = 9 \times 10^{16}\ J = 9 \times 10^{23}\ erg$

$1\ amu = 1.66 \times 10^{-27} \times 9 \times 10^{16} = 1.494 \times 10^{-10}\ J \cong 1.5 \times 10^{-10}\ J \cong 1.5 \times 10^{-3}\ erg$

Electron Volt (eV) **:** is the energy acquired (**gained**) by a particle of charge $e = 1.6 \times 10^{-19}\ C$ when it is accelerated through a potential difference of **1 Volt.**

$\mathbf{1\ eV = 4.8 \times 10^{-10} \times \frac{1}{300} = 1.6 \times 10^{-12}\ erg}$ (in **esu** units)

$\mathbf{1\ eV = 1.6 \times 10^{-19}\ C \times 1\ Volt = 1.6 \times 10^{-19}\ J}$ (in **SI** units)

$\mathbf{1\ MeV = 10^{6}\ eV = 1.6 \times 10^{-6}\ erg = 1.6 \times 10^{-13}\ J}$

<u>Example</u> **:** calculate the mass of electron in **MeV** units.

<u>Solution</u> **:** the electron mass **=** $\mathbf{9.1091 \times 10^{-31}\ kg}$

$$\mathbf{m_e = 5.48597 \times 10^{-4}\ amu}$$

Since **1 amu = 931.5 MeV**

$\therefore\ \mathbf{m_e = 5.48597 \times 10^{-4} \times 931.5\ MeV =}$

Packing Fraction & Binding Energy

The packing fraction can be regarded as a small **correction** which relates the isotopic mass **M** to the **mass number A** , i.e. the ratio between **Mass defect** and **A**

$$\textbf{Packing Fraction}\ \mathbf{P} = \frac{\mathbf{M\ (A,Z) - A}}{\mathbf{A}} = \frac{\mathbf{Mass\ defect}}{\mathbf{Mass\ number}}$$

Mass defect = (**M – A**) **Note :** (**M – A**) can be **positive** or **negative**

Therefore, the isotopic mass ${}^{A}_{Z}\mathbf{M}_{\mathbf{N}}$ can be calculated if the packing fraction (**P**) is known :

$$ {}^{A}_{Z}\mathbf{M}_{\mathbf{N}} = \mathbf{A\ (1 + P)}$$

Total Binding Energy :

Is defined as the amount of work we would have to supply in order to **dissociate** completely a nucleus into its component nucleons.

$$\mathbf{B.E.} = \left[\mathbf{Zm_p + Nm_n} - {}^{A}_{Z}\mathbf{\acute{M}_N}\right]\mathbf{c^2} \quad \Rightarrow \quad \mathbf{E = \Delta mc^2}$$

Where $\mathbf{m_p}$, $\mathbf{m_n}$ and ${}^{A}_{Z}\mathbf{\acute{M}_N}$ are the masses of proton , neutron and bare nucleus respectively. Note that the **B.E.** is a positive quantity we can without appreciable **error** , compute the nuclear **B.E.** using the atomic masses or (**weight**) $\mathbf{\acute{M}_s}$

$$\mathbf{B.E.} = \left[\mathbf{ZM_H + Nm_n} - {}^{A}_{Z}\mathbf{M_N}\right]\mathbf{c^2}$$

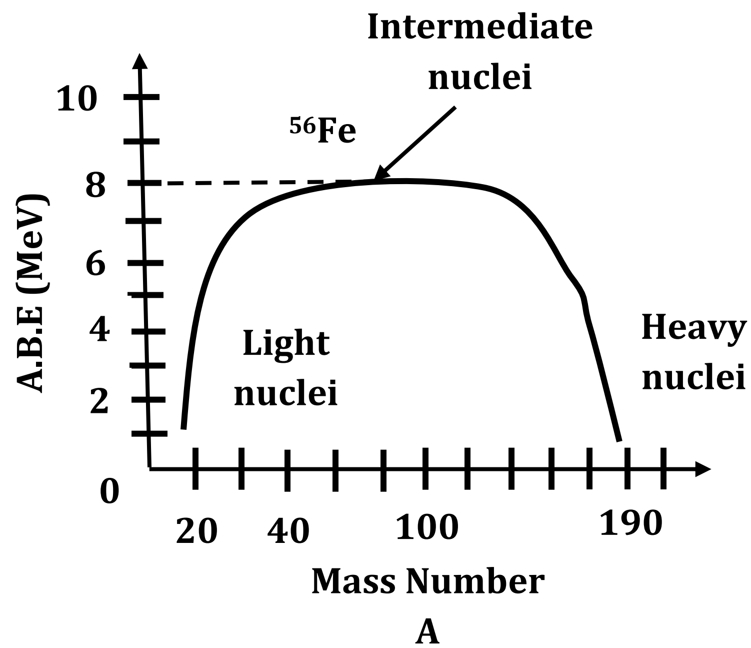
Average Binding Energy per Nucleon

Is the total **B.E.** divided by the number of nucleons.

$$\mathbf{A.B.E.} = \frac{(\mathrm{B.E.})_{tot.}}{A}$$

$$\mathbf{A.B.E.} = \overline{\mathbf{B.E.}} \cong \mathbf{8.5\ MeV/nucleon}$$

from the carve , the most stable nuclei is the intermediate nuclei.



Example **:** Calculate the total and the **A.B.E.** for deuterons and for **α − particle** in **MeV** units.

$$(\mathbf{B.E})_{tot.} = \left[ZM_H + Nm_n - {}^{A}_{Z}M_N\right]c^2$$

① For deuteron

$$\left({}^2_1H_1 \text{ or } {}^2_1D_1\right) = [1 \times 1.008143 + 1 \times 1.008983 - 2.014736]\ \text{amu}$$

$$(\mathbf{B.E})_{tot.} = 2.017126 - 2.014736 = 0.00239\ \text{amu}$$

Since **1 amu = 931.5 MeV**

$$\therefore (\mathbf{B.E})_{tot.} = 0.00239 \times 931.5 = 2.226\ \text{MeV}$$

$$\mathbf{A.B.E.}\ (\text{for } {}^2H) = \frac{\text{Total B.E}}{A} = \frac{2.226}{2} = 1.113\ \text{MeV/nucleon}$$

For α − particles ($^4_2He_2$)

$$(\mathbf{B.E})_{tot.} = [2 \times 1.008143 + 2 \times 1.008983 - 4.003872]\ \text{amu}$$

$$= [4.03425 - 4.003872] = 0.03038\ \text{amu}$$

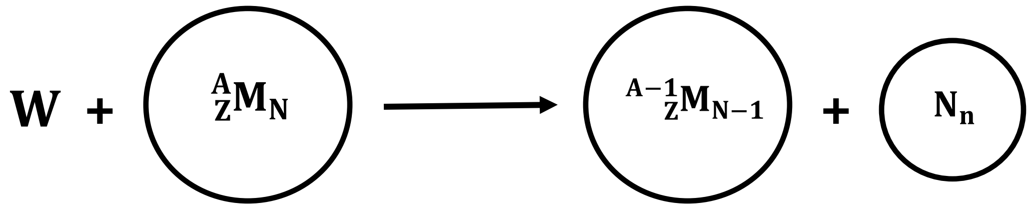
$$(\mathbf{B.E})_{tot.} = 0.03038 \times 931.5 = 28.29\ \text{MeV}$$

$$\mathbf{A.B.E.} = \frac{28.29}{4} = 7.06\ \text{MeV/nucleon}$$

Separation Energy :

The work or the amount of **energy** that is needed to separate (**remove**) **proton , neutron** or **α − particle** from a nuclei is called the **Separation Energy "S"** , which is the energy released when such a particle is **captured** by a nucleus.

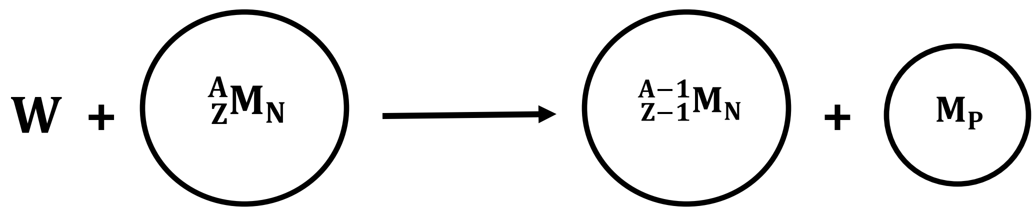
For Neutrons



$$S_n = \left[{}^{A-1}_{Z}M_{N-1} + M_n - {}^{A}_{Z}M_N\right]c^2 = \frac{\Delta B.E.}{\Delta N}$$

$$S_n = B\left({}^{A}_{Z}X_N\right) - B\left({}^{A-1}_{Z}X_{N-1}\right)$$

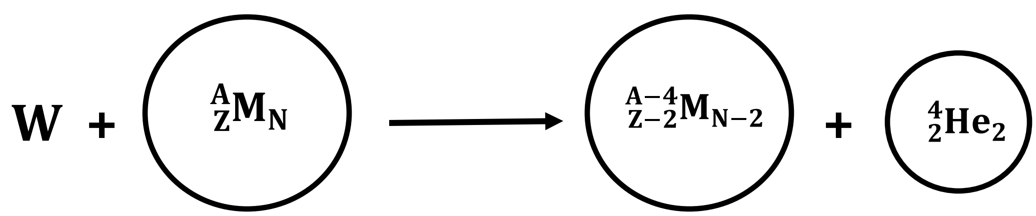
For Protons



$$S_P = \left[{}^{A-1}_{Z-1}M_N + M_P - {}^{A}_{Z}M_N\right]c^2$$

$$S_P = B\left({}^{A}_{Z}X_N\right) - B\left({}^{A-1}_{Z-1}X_N\right)$$

For α - particle



$$S_\alpha = \left[{}^{A-4}_{Z-2}M_{N-2} + {}^{4}_{2}M_2 - {}^{A}_{Z}M_N\right]c^2$$

$$S_\alpha = B\left({}^{A}_{Z}X_N\right) - B\left({}^{A-4}_{Z-2}X_{N-2}\right) - B\left({}^{4}_{2}He_2\right)$$

Note

① that Δ**B. E.** is the variation in the binding energy of the "**Last**" neutron in a group of nuclides.

② The separation energy is the increment in total binding energy when one nucleon or (**α - particle**) are added to the lower isotope or nuclides.

Total Nuclear Angular Momentum (Nucleus Spin)

We can imagine a nucleus as consisting of nucleons (**protons** and **neutrons**) moving around the center of mass in certain orbits (independent – particle model or shell model). Each nucleon has orbital angular momentum $\boldsymbol{\ell}$ and spin **S**. The coupling of these quantum numbers give the total angular momentum **j**.

∴ $\mathbf{j} = \boldsymbol{\ell} + \mathbf{S}$ $\quad \boldsymbol{\ell} = \pm {}^{1}/_{2}$ (since the spin of each nucleon equals ½ in units of ħ)

The total angular momentum of a **nucleus** containing (**A**) **nucleons** would be then the vector sum of the angular momenta of **all** nucleons **:**

$$\sum_{i>1} \mathbf{j}_i = \sum_{i>1} \boldsymbol{\ell}_i + \sum_{i>1} \mathbf{S}_i$$

Therefore $\quad \mathbf{J = L + S} \quad$ where **L** is the orbital momentum

S is the spin of **all** nucleons

J is the Nuclear Spin

The nuclear spin **J** represents a rotational motion which is absolute value equals to **:**

$$|\mathbf{J}| = \hbar\sqrt{\mathbf{J(J+1)}} \qquad \text{i.e. } \mathbf{J}^2 = \hbar^2\,\mathbf{J(J+1)}$$

and the projection of (J) on the **Z – axis**,

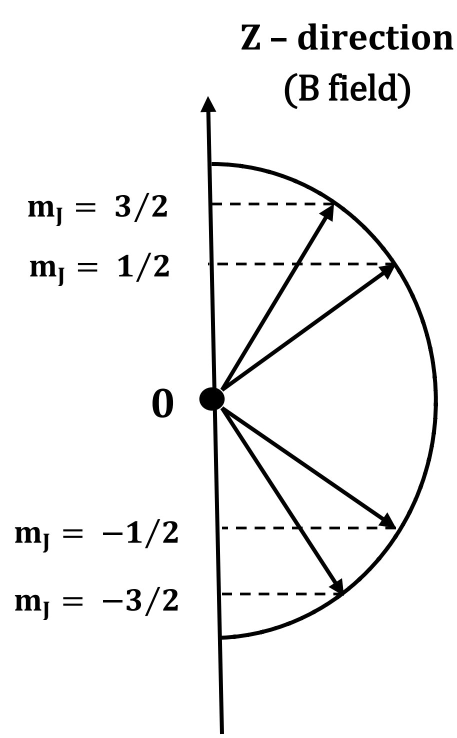
the direction of the external magnetic field

(**B**) , is $\mathbf{J_Z}$ where :

$\mathbf{J_Z = m\hbar}$

$\mathbf{m = J, (J-1), (J-2), \ldots, -(J-2),}$

$\mathbf{-(J-1), -(J)}$



A nucleus with spin (**B**) can **only** take (**2J + 1**) orientations in an external Magnetic field (**B**). The reduced Plank's constant **ħ** :

$$\hbar = \frac{\mathbf{h}}{\mathbf{2\pi}} = \frac{\mathbf{6.6256 \times 10^{-34}\ J.s}}{\mathbf{2\pi}} = \mathbf{1.055 \times 10^{-34}\ J.sec}$$

The total Angular quantum number **J** is :

1) **J = 0 for the ground state of Even – Even nuclei**

   (e.g. $^{4}_{2}He_2$, $^{12}_{6}C_6$, $^{16}_{8}O_8$ .....)

2) **J = half – integer $\left({}^{1}/_{2}\ ,{}^{3}/_{2}\ ,{}^{5}/_{2}\ ,\dots\right)$ for odd – A nuclei.**

   (e.g. $^{3}_{2}He$, $^{15}_{7}N_8$, ^{17}O .....)

3) **J = integer** (0 , 1 , 3 , ...) **for Even – A nuclei.**

   (e.g. $^{14}_{7}N_7$, $^{10}_{5}B_5$, $^{6}_{3}Li_3$ .....)

∗ Nuclei which its spin is not Zero ($J \neq 0$) has a magnetic moments.

∗ Nuclei which its spin equals Zero ($J = 0$) has not a magnetic moments.

Example **:** calculate the number of orientations of spins in an external magnetic field for the followings :

Solution **:** The number of orientations = **2J + 1** .

J = 0 ; No. of orientations = 1 ; i.e. J = 0

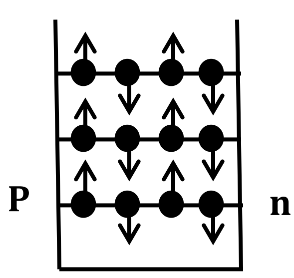
J = 1 ; No. of orientations = 3 ; i.e. 1 , 0 , -1

J = 2 ; No. of orientations = 5 ; i.e. 2 , 1 , 0 , -1 , -2

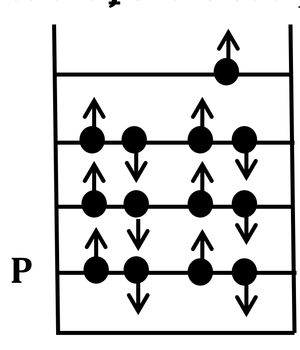
J = $^{3}/_{2}$; No. of orientations = 4 ; i.e. $^{3}/_{2}\ ,{}^{1}/_{2}\ ,-{}^{1}/_{2}\ ,-{}^{3}/_{2}$

J = $^{5}/_{2}$; No. of orientations = 6 ; i.e. $^{5}/_{2}\ ,{}^{3}/_{2}\ ,{}^{1}/_{2}\ ,-{}^{1}/_{2}\ ,-{}^{3}/_{2}\ ,-{}^{5}/_{2}$

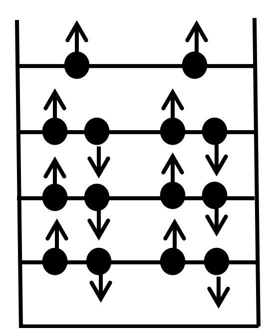
The measured values of the nuclear spins can tell us a great deal about the nuclear structure . For example , **Even Z** , **Even N** nuclei **<u>all</u>** have spin – 0 ground state, this is an evidence for the **<u>nuclear pairing force</u>** . The ground state spin of an **odd – A** nucleus **<u>must be</u>** equal to the **<u>j</u>** of the odd proton or neutron.



$^{12}_{6}C_6$ (J = 0)



$^{13}_{6}C_7$ (J = $^{1}/_{2}$, $^{3}/_{2}$, ...)



$^{14}_{7}N_7$ (J = **0 , 1 , 2** , ...)

Parity

Along with the nuclear spin (J) , the parity is also used to label nuclear state ;
($\mathbf{0}^+$, $\mathbf{2}^-$, $\mathbf{1/2}^-$, $\mathbf{3/2}^+$, …. **etc**). There is no direct relation between parity and spin .

The parity is a classification of wave functions into two groups , those of Even parity (+) and those of odd parity (−) . The parity of **isolated** system is a constant of its motion and **cannot** be changed by any **internal** process . Only if radiation or a particle **enters** or **leaves** the system (nucleus) , and hence the system is no longer isolated , can its parity change.

A wave function representing a single particle is said to have **positive** (+) **even parity** if it does not change sign by reflection through the origin , and **negative** (−) **parity** if it does change sign.

$$\Psi\,(x , y , z) = +\,\Psi\,(-x , -y , -z) \quad \Rightarrow \quad \textbf{Even parity (positive)}$$

$$\Psi\,(x , y , z) = -\,\Psi\,(-x , -y , -z) \quad \Rightarrow \quad \textbf{Odd parity (negative)}$$

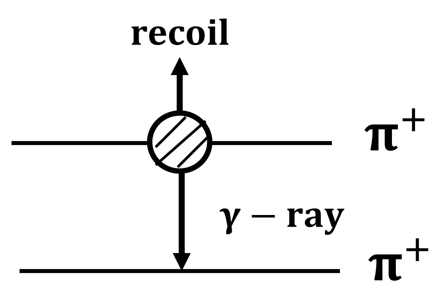
Where $\Psi\,(x , y , z)$ is a wave function describing a system (**nucleus**)

$|\Psi|^2 = \Psi\Psi^*$ is the probability of finding a particle at the position (**x , y , z**).

Parity is an interesting property because of the law conservation of parity. Suppose that an nucleus in an excited state is described by a wave function with **Even** parity of it emits a gamma ray and goes over to a lower energy state the system

"recoiling nucleus + δ - ray " must continue to have

Even parity.



This imposes some restrictions on the **δ - ray emission** process , the conservation of parity had led to some **selection rules**. A particle with an **even** value of (ℓ)
the motion has even parity , and with odd (ℓ) has odd parity , $\mathbf{p} = (-1)^{\ell}$

L ⇒ orbital quantum number

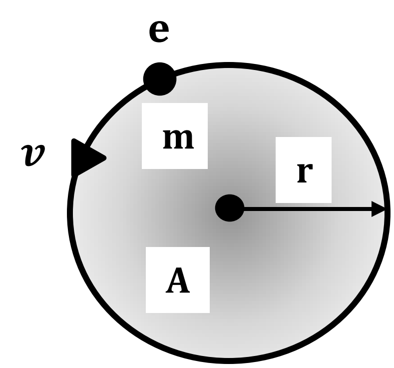
P ⇒ parity

Nuclear Magnetic Dipole Moment

Any charged particle moving in a closed circular path produced a magnetic field which can be described , at large distances , as due to a magnetic dipole located at the current loop.

$|\mu| = i\,A$; where **i is the current** , **A the area of the loop**

Therefore the **protons** (and not the neutrons) within nuclei should produce **extra** nuclear magnetic field **:**



$|\mu| = \left(\frac{v}{2\pi r}\right)(e)(\pi r^2)$

Where i = frequency × charge

$= \left(\frac{v}{2\pi r}\right)(e)$

$A = \pi r$

$\therefore\ |\mu| = \frac{evr}{2}$; $v \Rightarrow$ the velocity of the particle with charge **e** (**proton** for example)

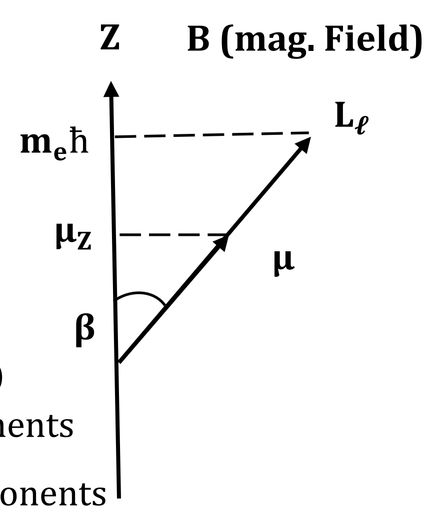
$r \Rightarrow$ the radius of the orbit.

The orbital angular momentum of this particle (with mass **m**) is then equals to **:**

$$L_\ell = mvr \ \Rightarrow\ L = mvr$$

Therefore $$|\mu| = \left(\frac{e}{2m}\right)L = g \times L_\ell$$

Where **g** is the gyromagnetic ratio



$$g = \left(\frac{\mu}{L}\right) = \frac{evr/2}{mvr} = \frac{e}{2m}$$

$$\therefore\ g = \frac{e}{2m}$$

Actually , neither L_ℓ nor its collinear μ their ratio (**g**) can be measured accurately in many types of experiments the relation $\left(g = \frac{e}{2m}\right)$ will also **correct** for the components of L_ℓ and μ in any direction , for instant in the direction of a magnetic field (**B**) , the z - direction therefore ; $g = \frac{\mu_Z}{(L)_z} = \frac{\mu_Z}{m_\ell\hbar}$ $\therefore\ \mu_Z = \left(\frac{e\hbar}{2m}\right)m_\ell$ $\mu_Z = g m_\ell\hbar$

$$\mu_Z = \left(\frac{e}{2m}\right)m_\ell\hbar$$

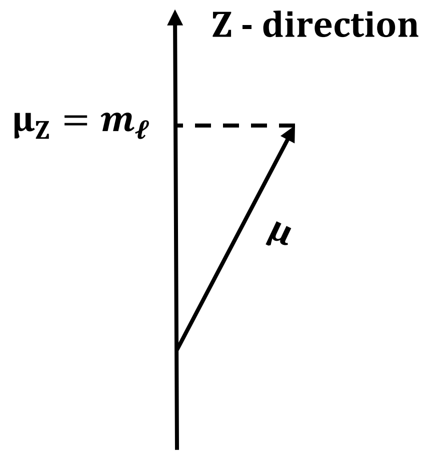
$\left(\frac{e\hbar}{2m}\right)$ is called the **Nuclear Magnetron** $\mu_N = \left(\frac{e\hbar}{2m}\right) = 5.0505 \times 10^{-27}\ J \cdot m^2/Wb.$

$\mu_N = 3.1525 \times 10^{-8}\ eV/Tesla$

Therefore $\mu_Z = m_\ell\, \mu_N$ or $\mu_Z = m_\ell$ in unit of **Nuclear Magnetron** μ_N

This means that the **Z – component** of magnetic dipole moment (μ_Z) resulting from orbital motion off a proton is (m_ℓ) nuclear magnetron.

The total magnetic Dipole Moment for a nucleus is the **sum** of moments associated with the spins of protons and neutrons and the moments associated with the orbital motion of the protons (the orbital motion of neutrons do not contribute to the Mag. Dipole Moment of the nucleus) because the neutron is chargeless particle.



$$\mu_{tot.} = \mu_{sp} + \mu_{sn} + \mu_{\ell p}$$

The magnetic moments are **:**

$\mu_e = -1.001145358 \quad \mu_E \Rightarrow$ **for electron**

$\mu_p = +2.79275 \quad \mu_N \Rightarrow$ **for proton**

$\mu_n = -1.9135 \quad \mu_N \Rightarrow$ **for neutron**

* العزم المغناطيسي ثنائي القطبة تتكون الحركة المدارية للبروتونات و لا علاقة للنيترونات في تكوينه لهذا

$$\mu = \frac{e\hbar}{2m} \cdot \ell$$ العدد الكمي المداري ℓ

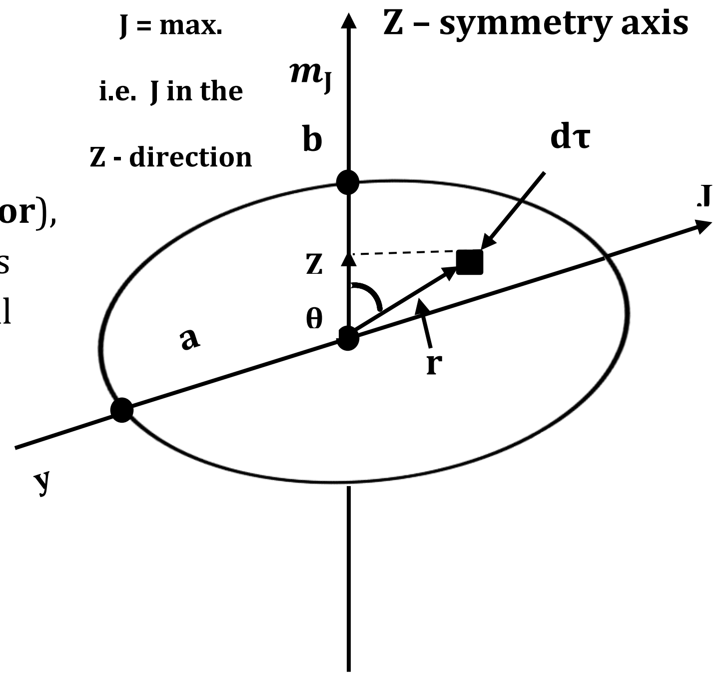
**** كذلك ينشأ عزم مغناطيسي ذاتي تتشارك في تكوينه الحركة البرمية للبروتونات و النيترونات حيث أن**

$$\mu = \mu_p + \mu_n$$

Nuclear Electric Quadruple Moment

The nuclear electric quadruple moment is the **deviation** from spherical symmetry distribution of **protons** in the nucleus.

If nucleus is imagined to be as a uniformly charged ellipsoid of rotation which its diameter is (**2b**) along the symmetry axis (the z – direction or the direction of **J vector**), the quadruple moment referred to this axis will be (if the nucleus moves with spherical symmetry then $z^2 = x^2 = y^2 = \frac{1}{3}r^2$)



$$Q_0 = \frac{1}{e}\int \rho(3z^2 - r^2)\, d\tau$$

And since **z = r cos θ** , then

$$Q_0 = \frac{1}{e}\int \rho(3r^2\cos^2\theta - r^2)\, d\tau = \frac{1}{e}\int \rho r^2(3\cos^2\theta - 1)\, d\tau$$

Where **ρ** ⇒ is the density of nuclear charge in volume element **dτ** at point **(z , r)**

$$\rho = \frac{Ze}{\frac{4}{3}\pi a^2 b} \quad \Rightarrow \quad \text{for ellipsoid}$$

Integrating over the charged system (nucleus) we obtain **:**

$$Q_0 = \frac{2}{5} Z\,(b^2 - a^2) \cong \frac{6}{5} Z\, R_{av.}^2 \left(\frac{\Delta R}{R}\right)$$

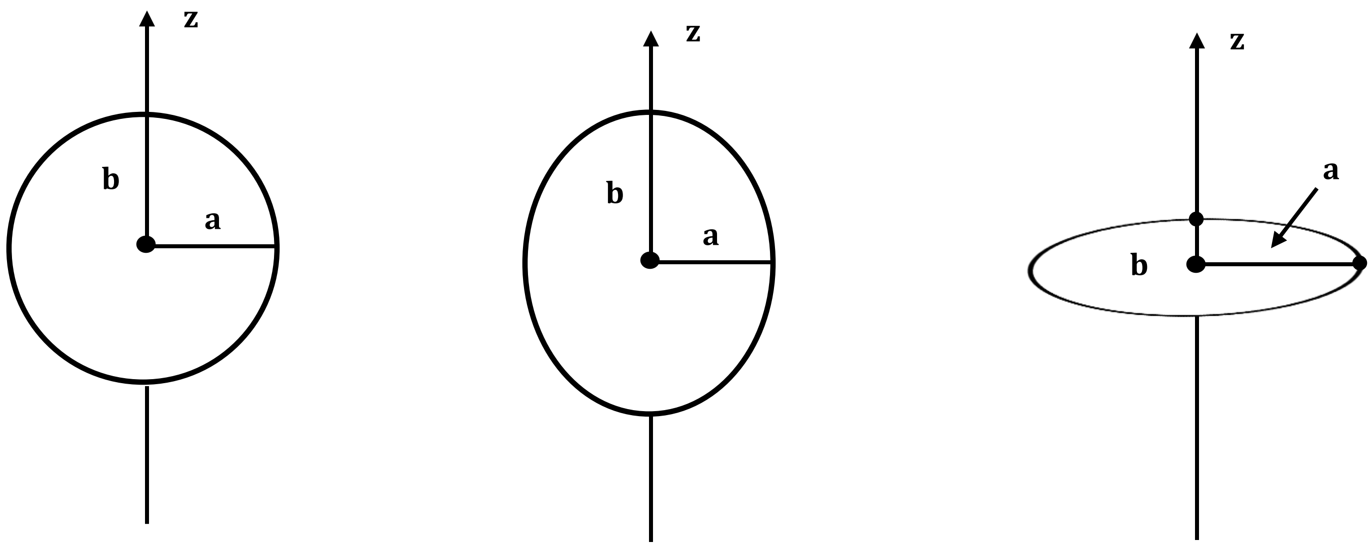
ΔR ⇒ the deviation from the average in the direction of the symmetry axis.

$R_{av.}$ ⇒ average nuclear radius.

band a ⇒ is the **major** and **minor** half – axis.

$\frac{4}{3}\sqrt{\frac{\pi}{5}}\left(\frac{\Delta R}{R_{av.}}\right) = \beta$ is the deformation parameter

$R_{av.} \cong R_0\, A^{1/3}$ although this relation is not exact , it is customary to be taken.



$a = b ; Q_0 = 0$

Spherically distribution

$b > a ; Q_0 > 0$

Prolate distribution

$b < a ; Q_0 < 0$

Oblate distribution

when $\quad$ **a = b** $\quad$; $\quad Q_0 = 0$ $\quad$; i.e. **protons are spherically distributed.**

when $\quad$ **b > a** $\quad$; $\quad Q_0 > 0$ $\quad$; i.e. Q_0 **is positive** (**prolate distribution**).

when $\quad$ **b < a** $\quad$; $\quad Q_0 < 0$ $\quad$; i.e. Q_0 **is negative** (**oblate distribution**).

Q ⇒ lies between $\mathbf{10^{-30}}$ to $\mathbf{10^{-28}\ m^2}$, which is the order of $\mathbf{R^2}$ of the nucleus and measured in **BARNS** (**1 barn = 1b =** $\mathbf{10^{-28}\ m^2}$).

The relation between the deformation parameter ($\boldsymbol{\beta}$) and the quadruple moment is:

$$\mathbf{Q_0 = \frac{3}{\sqrt{5\pi}}\, R^2_{av.}\, Z\beta\,(1 + 0.16\,\beta)} \quad ; \quad \boldsymbol{\beta \approx 0.3}$$

$\mathbf{Q_0}$ is known as the **<u>intrinsic</u>** quadruple and observed only when the nucleus at rest.

ان العزوم المتولدة في النواة عزم مغناطيسي ثنائي القطبية أو عزم كهربائي رباعي القطبية , و لا يتولد عزم مغناطيسي احادي أو رباعي القطبية , أو عزم كهربائي ثنائي القطبية , و لماذا لا تمتلك النواة على هذه العزوم في حين عزوما اخرى معينة

ذلك يتحدد بمبدأ **تناظر التماثل** , ان تماثل العزوم الكهربائية هو $(-1)^{\ell}$: هي ℓ تمثل مركبة العزم , اما تماثل العزوم المغناطيسية هو $(-1)^{\ell+1}$ و انه لا يوجد عزوما كهربائية و مغناطيسية في نفس الوقت و نفس الرتبة الجدول اناه يوضح الحالة

مهم

المرتبة ℓ	نوع العزم 2^{ℓ}	تسمية العزم	تماثل كهربائي $(-1)^{\ell}$	تماثل مغناطيسي $(-1)^{\ell+1}$	العزوم الممكنة
0	**1**	احادي القطبية	زوجي (+)	فردي (−)	كهربائي احادي القطبية
1	**2**	ثنائي القطبية	فردي (−)	زوجي (+)	مغناطيسى ثنائي القطبية
2	**4**	رباعي القطبية	زوجي (+)	فردي (−)	كهربائي رباعي القطبية
3	**8**	ثنائي القطبية	فردي (−)	زوجي (+)	مغناطيسى ثنائي القطبية

يتبين أن العزوم الكهربائية و ثنائية القطبية ان تكون من خواص النوى كم أن العزوم المغناطيسية احادية القطبية و رباعية القطبية لا يمكن أن تمتلكها النوى **(لنفس السبب)**. و عليه فان العزوم الكهربائية رباعي القطبية هو الذي يمكن أن يعطينا فكرة عن شكل النواة.

<u>Statistics of Nuclear Particles</u>

Another important property of nuclei statistics and arises from considerations of the symmetry properties of wave functions. Statistics property has to do with the effect on the wave function Ψ of **<u>interchanging all</u>** the coordinates (three spatial **x** , **y** , **z** and one spin s) of **<u>two</u>** identical particles . Every particles in nature **<u>must</u>** obey one of the **<u>two</u>** types of statistics **;** either **Bose – Einstein** statistics (**symmetric**) or the **Fermi – Dirac** (**antisymmetric**)

① <u>Fermi – Dirac Statistics</u>

According to this statistics the wave function **<u>changes</u>** sign when all coordinates of two identical particles are interchanged.

$$\Psi(x_1 , x_i , x_j , \dots , x_n) = -\Psi(x_1 , x_j , x_i , \dots , x_n)$$

The anitsymmetric of the wave function **<u>restricts</u>** the number of particles per quantum state to **<u>one</u>** , that is the **Pauli Exclusion Principle.** Protons , neutrons , electrons and all **odd – A nuclei** obey **Fermi – Dirac** statistics ; therefore , they are called **Fermions** (**Fermi Particles with spin $s = 1/2$ integer**)

② <u>Bose – Einstein Statistics</u>

A system which its wave function is "**symmetric**" is said to follow **Bose – Einstein Statistics.** Interchange of **<u>two</u>** identical **Bose – Einstein** particles , leave the wave function for their system **<u>uncharged</u>**

$$\Psi(x_1 , x_i , x_j , \dots , x_n) = +\Psi(x_1 , x_j , x_i , \dots , x_n)$$

Bose – Einstein particles do not follow the **Pauli** exclusion principle. Two or more such particles may be in the same quantum state. The photons and **α – particles** obey **Bose – Einstein Statistics** and **<u>all</u>** nuclei of **Even – A** mass number

Mass Number (A)	**Angular Momentum (J)**	**Statistics**	**Particles**
Odd	**J = 1/2 , 3/2 , 5/2 , ….**	**Fermi – Dirac**	**P , n , e^- , e^+, v , $\bar{v}$ and all odd – A nuclei (^{3}He , ^{9}B , ^{15}O)**
Even	**J = 0 , 1 , 2 , ….**	**Bose – Einstein**	**γ – ray , α – particle and all Even – A nuclei (^{12}C , ^{16}O , ….)**