Chapter "2"

Radioactivity

The radioactivity divided into two types : the naturally occurring radioactivity , in which minerals containing **Uranium** and **Thorium** decay , and the artificially produced radioactivity through nuclear reactions. In this chapter we explore the physical laws governing the production and decays of radioactive materials.

The Radioactive Decay Law

The radioactivity of both , naturally occurring or artificially produced materials , decreases with time according to an exponential law , known as **Radioactive Decay Law.**

The decay process is statistical (**random**) in nature , that it is impossible to predict when any given atom (**nucleus**) will disintegrate (**decay**).

If **N** radioactive nuclei is present at time (**t**) and if **no new nuclei** is introduced into the sample , then the change in the number of decaying nuclei **dN** in a time interval **dt** is proportional to **N** , i.e.

$$-\left(\frac{dN}{dt}\right) \propto N \quad ; \quad -\left(\frac{dN}{dt}\right) = \lambda N \quad .... ①$$

The minus sign ($-$) means that (N) decreases as time (t) increases , and (λ) is a constant called the **disintegration** or **Decay Constant** , which is the probability that any radioactive nucleus will decay in unit time ; therefore ,

$$\lambda = -\left(\frac{dN/dt}{N}\right) \qquad , \quad \left(\frac{dN}{dt}\right) \text{ is the activity of the sample}$$

Activity $A = \frac{dN}{dt} = \lambda N$, **integrating this equation we obtain :**

$$\int \frac{dN}{dt} = -\int \lambda N$$

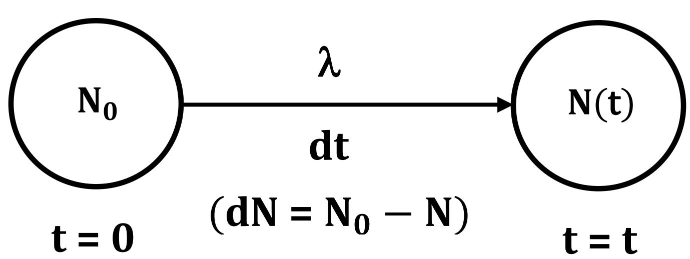


Fig. (1) , Decay Law

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_{t=0}^{t} \lambda \, dt \quad .... ②$$

The probability of decay of a nucleus (**atom**) is independent of the age that nucleus (**atom**) , then (λ) is independent of (**t**).

And is constant (λ = **constant**) , we can integrate **equ.** ②

$$\therefore\ \mathbf{N(t) = N_0\, e^{-\lambda t}} \quad .... \text{③} \quad \Rightarrow \textbf{The decay law of radioactivity}$$

$\mathbf{N_0}$ ⇒ **the number of radio nuclides at time t=0 .**

Multiplying by (λ) , we obtain that the activity ($\mathbf{A = \lambda N}$) equals to **:**

$$(\lambda N) = (\lambda N_0)e^{-\lambda t} \quad \text{or} \quad A = A_0\, e^{-\lambda t} \quad .... \text{④}$$

Where $\mathbf{A_0 = \lambda\, N_0}$ **is the activity of the sample at time t=0 .**

Note that the activity ($\mathbf{A = \lambda N}$) tell us only the number of disintegration / second ($\mathbf{A = dN/dt}$) **;** it says <u>nothing</u> about the <u>kind</u> (α , β or γ) of radiations emitted or about their energies. According to **equ.** (**4**) we have

$$\left(-\frac{dN}{dt}\right) = \lambda N = \lambda\, N_0\, e^{-\lambda t} = \left(-\frac{dN}{dt}\right)_0 \lambda e^{-\lambda t}$$

Taking the logarithm of base (**10**) we find **:**

$$\log_{10}\left(-\frac{dN}{dt}\right) = \log_{10}\left(-\frac{dN}{dt}\right)_0 - \lambda t \log_{10} e$$

Taking the natural logarithm we obtain **:**

$$\ln\left(-\frac{dN}{dt}\right) = \ln\left(-\frac{dN}{dt}\right)_0 - \lambda t \quad .... \text{⑤}$$

we can thus measure the activity as a function of time by counting the number of decays in a <u>sequence</u> of short time intervals (Δ**t**). Plotting these data on a semi log paper (**that is ln A** or **ln** ($-$ **dN/dt**) vs. **t**) , we obtain a straight line of slope ($-\lambda$). Figure **2** shows the exponential decay of activity. (**a**) Linear plot. (**b**) semi log plot. From which one can determine the half – life $\left(T_{1/2} = \frac{\ln 2}{\lambda}\right)$ of a radioactive decay.

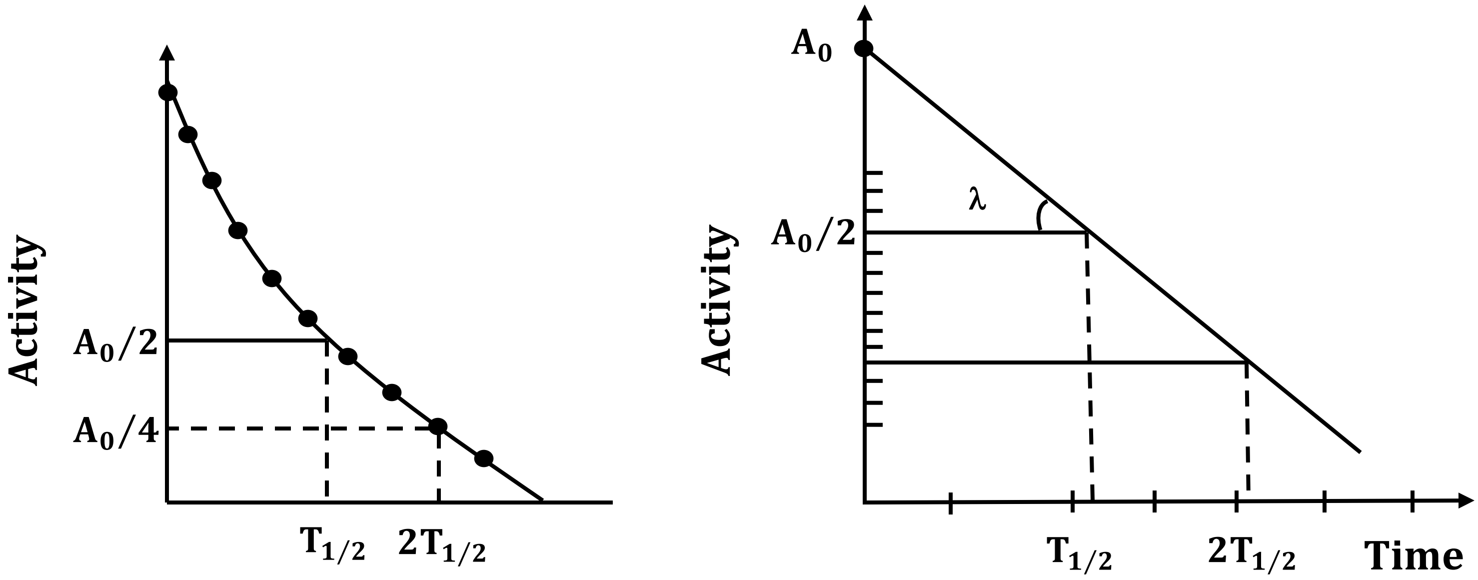


Fig. (2). The exponential decay of activity. (a) Linear Plot (b) Semi log

The Half – life $T_{1/2}$ (or half – priod)

The half – life ($\mathbf{T_{1/2}}$) is the time required for radioactive nuclei to disintegrate **exactly** to one – half its initial number $\mathbf{N_0}$ (or its initial activity $\mathbf{A_0} = \lambda\, \mathbf{N_0}$)

Therefore , after time $\mathbf{t = T_{1/2}}$, $\mathbf{N = \frac{N_0}{2}}$

At any time $\mathbf{t}$: $\mathbf{N = N_0\, e^{-\lambda t}}$ $\Rightarrow$ **Radioactive decay law**

At any time $\mathbf{t = T_{1/2}}$: $\mathbf{\frac{N_0}{2} = N_0\, e^{-\lambda T_{1/2}}}$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\therefore \quad \ln 1/2 = -\lambda\, T_{1/2}$$

$$\ln 2 = \lambda\, T_{1/2}$$

$$\therefore \quad T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The half – life ($\mathbf{T_{1/2}}$) can be found from the slope of a plot (**ln A**) against time (**t**) , fig. (**2 , b**)

Mean Lifetime τ

The mean lifetime or average time of radioactive nuclei is the total lifetime of **all** nuclei divided by the number of nuclei present initially ($\mathbf{N_0}$)

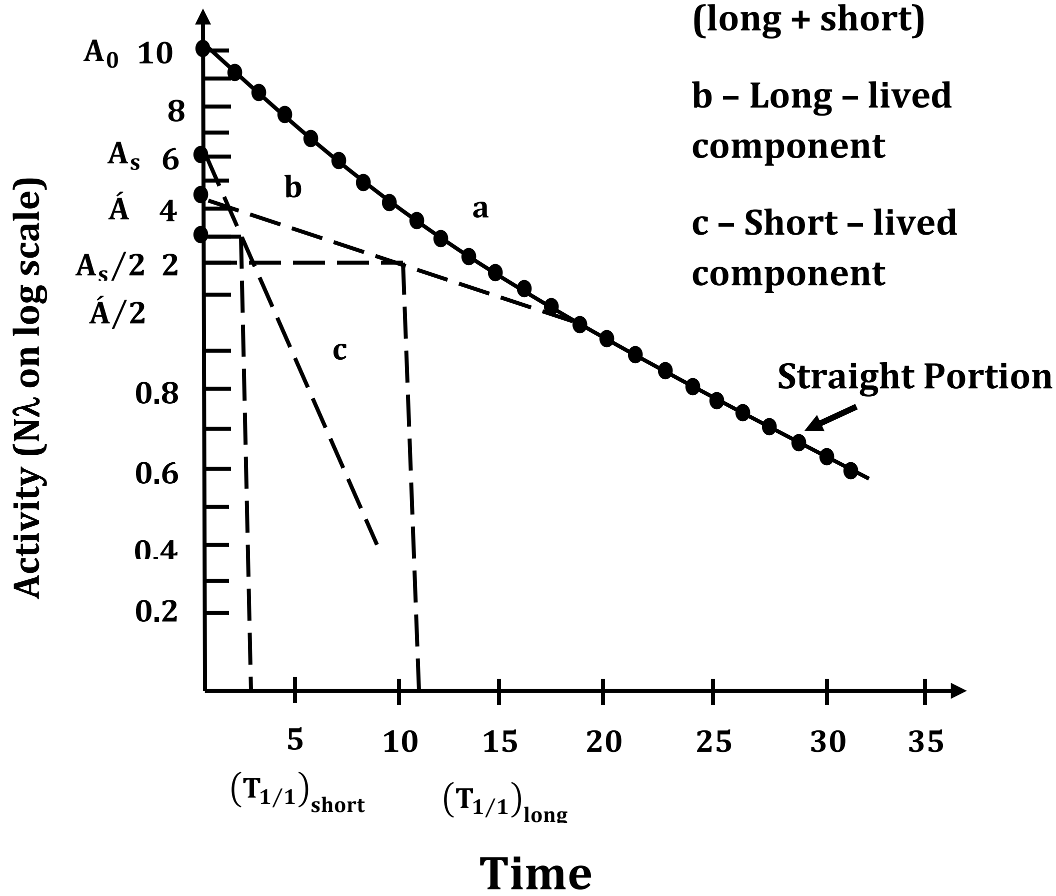
$$\tau = \frac{\int_0^\infty (-dN)t}{N_0} = \frac{-\int_0^\infty tN\lambda\, dt}{N_0} = \frac{-\int_0^\infty N_0\, e^{-\lambda t}\, dt}{N_0}$$

$$\tau = \int_0^\infty t\lambda\, e^{-\lambda t}\, dt = \frac{1}{\lambda}$$

$$\therefore \quad \tau = \frac{1}{\lambda} \quad \text{or} \quad \tau = \frac{1}{\frac{0.693}{T_{1/2}}} = \frac{T_{1/2}}{0.693} = 1.44\, T_{1/2}$$

$$\tau = 1.44\, T_{1/2} \quad \text{and} \quad T_{1/2} = 0.693\, \tau$$

Mixture of Radioactive Samples



If two or more radioactive isotopes are mixed together , then the observed activity (**curve a**) is the sum of the two (**long** and **short**) activities. If the activities are independent on each other , then the various activities can be distinguished and the separate half – lives can be determined. After a sufficient **long – time** only the **long – lived** activity will remain , and the **half – life** (T_{long}) can be determined from the straight portion. If this straight (**b**) portion is extrapolated back to (**t=0**) and the values of activity given by this line are subtracted from the total activity curve (**a**), the curve that remains (**c**) will represent the decay of **short – lived component** (**s**).

This method of measurement is useful only for **half – live** that are neither <u>too short</u> nor <u>too long</u>. The **half – life** must be <u>short</u> enough that we can see the sample decaying , for **half – lives** too <u>much greater</u> than a human lifetime , we would <u>not</u> be able to observe reduction in activity. For such cases , we can measure $\mathbf{dN/dt}$ (which is the activity $\mathbf{A = \lambda N}$) and by determining the number of atoms (**N**) by weighing a sample (**w**) which its chemical composition is accurately known , where

$\mathbf{N = \frac{w(g)\times NA}{A} = \frac{w\times 6.02\times 10^{23}}{A}}$ and since $\left(-\frac{dN}{dt}\right) = \lambda N$, we can determine the decay constant (λ) and then the **half – life** $\mathbf{T_{1/2}}$ which is equal to $\mathbf{T_{1/2} = \frac{0.693}{\lambda}}$