Chapter "3"

Interaction of Nuclear Radiation with Matter

Nuclear radiations can be divided into **three** types :

① **Charged particles (P , d , α - particles , ...).**

② **Uncharged particles (neutrons , $^0_0\upsilon$, $\tilde{\upsilon}$).**

③ **Electromagnetic radiation (γ - rays , X – rays , ...).**

The interaction and detection of the **three** types will be treated separately. The intensity (**number of detected events per unit time**) and the energy (**kinetic**) of radiation is determined.

I. Interaction of Charged Particles with Matter

When a charged particle passes through neutral atoms interacts mainly by means of the **Coulomb force** with the **electrons** in the atom and loss only **few** electron volts in ionization and excitation of the atoms.

Energy Loss by Collision

For the energy loss calculation we assume that the heavy charged particle will collide with a free electrons of the atom , so the loss will be equal to the gain of kinetic energy of the electron and this can be estimated as follows :

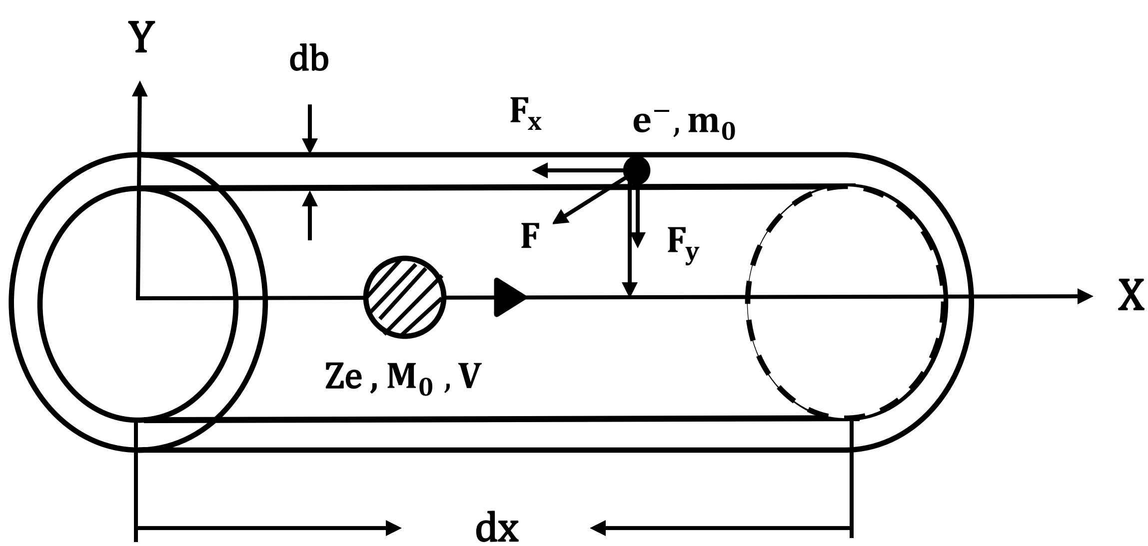

Fig. 1

b $\Rightarrow$ impact parameter which is the smallest distance at which the centers of the particles (heavy charged particle and the electron) would pass each other , if there were no force between the particles.

The impact equations for the electron (in the **x – y** coordinations) are **:**

$$\int \mathbf{F_x}\ \mathbf{dt} = \mathbf{0} \quad \text{--------} \quad ①$$

$$\int \mathbf{F_y}\ \mathbf{dt} = \mathbf{P_e} \quad \text{--------} \quad ②$$

Note that the velocity of the heavy particle is practically unaffected by the interaction.

Where $\vec{\mathbf{F}} = \vec{\mathbf{F}}_\mathbf{x} + \vec{\mathbf{F}}_\mathbf{y}$, **F** $\Rightarrow$ is the **coulomb force** exerted on the electron

$\mathbf{P_e}$ $\Rightarrow$ is the momentum imparted (**given**) to the electron (**only the y component is non-zero**)

The momentum imported to the electron can be obtained by an application of **Gauss's law** of electrostatics.

$$\oint \mathbf{E} \,.\, \mathbf{dS} = \frac{\mathbf{1}}{\cancel{4}\boldsymbol{\pi}\epsilon_0} \times \cancel{4}\boldsymbol{\pi}\ \mathbf{q} = \frac{\mathbf{q}}{\epsilon_0} \qquad \textbf{(in SI units)}$$

Where **E =** the **electric field** at the surface of any closed volume surrounding a chrage **q** (**q = Ze** , the charge of the incident particle). $\mathbf{E} = \frac{\mathbf{F_y}}{\mathbf{e}}$

dS = is an element of surface

$$\mathbf{dS} = \mathbf{2\pi\ b.dx}$$

$$\therefore \quad \oint \mathbf{E} \,.\, \mathbf{dS} = \int \left(\frac{\mathbf{F_y}}{\mathbf{e}}\right) \times \mathbf{2\pi\ b.dx} = \frac{\mathbf{1}}{\mathbf{4\pi}\epsilon_0} \times \mathbf{4\pi\ Ze}$$

$$\therefore \quad \int \frac{\mathbf{F_y}}{\mathbf{e}}\ \mathbf{dx} = \frac{\mathbf{1}}{\mathbf{4\pi}\epsilon_0} \cdot \mathbf{Ze} \cdot \frac{\mathbf{2}}{\mathbf{b}}$$

Since $\mathbf{dx} = \boldsymbol{v}\ \mathbf{dt}$ then

$$\int \mathbf{F_y}\ \mathbf{dt} = \frac{\mathbf{Ze^2}}{\mathbf{4\pi}\epsilon_0} \cdot \frac{\mathbf{2}}{\mathbf{b}\ \boldsymbol{v}} = \mathbf{P_e} \ \text{, therefore} \ \ \mathbf{P_e} = \left(\frac{\mathbf{e^2}}{\mathbf{4\pi}\epsilon_0}\right) \frac{\mathbf{2Z}}{\mathbf{b}\ \boldsymbol{v}} \quad \text{--------} \quad ③$$

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$$\left(\frac{e^2}{4\pi \in_0}\right) = 1.44 \text{ MeV.fm.}$$

and the energy **gained** by the electron and **lost** by the heavy particles

$$T = \frac{P_e^2}{2m_0} = \left(\frac{e^2}{4\pi\in_0}\right)^2 \frac{2Z^2}{m_0\, b^2\, v^2} \quad \text{--------} \quad ④$$

Total number of electrons within a distance b to $b + db$ for a path length dx is equal to **:**

Total number $= n\,Z\,.\,2\pi b\, db\,.\,dx$, where **nZ** number of electrons per unit volume in stopping material , i.e. in **n** atoms
Z = number of electrons in each atom.
n = number of atoms per volume (atoms/volume).

Each of these electrons will gain an amount of energy given by **eq.** ④ , i.e. the heavy particle will lose total energy per unit length equal to **:**

$$\left(-\frac{dT}{dx}\right) = \left(\frac{e^2}{4\pi \in_0}\right)^2 \int_{b_{min}}^{b_{max}} nZ.\,2\pi\, db\, \frac{2Z^2}{m_0 b^2 v^2}$$

$$\left(-\frac{dT}{dx}\right) = \left(\frac{e^2}{4\pi\in_0}\right)^2 \frac{4\pi\, Z^2.nZ}{m_0 v^2} \ln\frac{b_{max}}{b_{min}} \quad \text{--------} \quad ⑤ \rightarrow \quad \textbf{prove} \circledast$$

This is an approximate expression , because for nearly **head – on – collision** $\theta = 180°$ and the electrons are not **free** but bound to atoms.

To calculate b_{max} **and** b_{min} **:**

For b_{max} **:** The impact time Δt must not be **longer** than the period of rotation of an electron in its orbit in order that energy will be transferred to **atomic electron.**

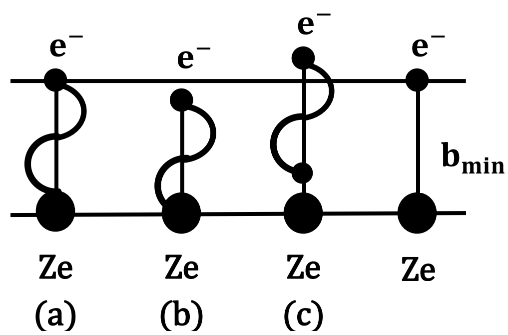
$$\Delta t_{max} \approx \frac{1}{\nu} \approx \frac{b_{max}}{v} \qquad \text{where } \nu = \textbf{frequency of rotation}$$

$$\therefore\ b_{max} = \frac{v}{\nu} \quad \text{--------} \quad ⑥$$

speed

For b_{min} : The minimum impact parameter $\mathbf{b_{min}}$ is limited by the uncertainty principle , because the electron cannot be located with respect to the heavy particle more closely than its **de Broglie** wavelength.

In (**a**) the wavelength $\lambda = \mathbf{b_{min}}$, in (**b**) $\lambda > \mathbf{b_{min}}$ the electron inters into the nucleus , in (**c**) $\lambda < \mathbf{b_{min}}$, i.e. impact parameter $\mathbf{b} > \mathbf{b_{min}}$



From the uncertainty principle $\Delta x . \Delta p \approx \frac{\hbar}{2} \cdot (\Delta x = b_{min})$

$$\therefore\ b_{min} \approx \frac{\hbar}{2} \approx \frac{\hbar}{2 m_0\, v}$$

$$\left(-\frac{dT}{dx}\right) \approx \left(\frac{e^2}{4\pi\, \epsilon_0}\right)^2 \frac{4\pi\, Z^2 \cdot nZ}{m_0 v^2}\ \ln \frac{v/\upsilon}{\dfrac{\hbar}{2 m_0\, v}}$$

$$\left(-\frac{dT}{dx}\right) \approx \left(\frac{e^2}{4\pi \epsilon_0}\right)^2 \frac{4\pi\, Z^2 \cdot nZ}{m_0 v^2}\ \ln \frac{2 m_0\, v^2}{I_{ave.}} \quad \text{--------} \ (7)$$

Where $I_{average} = \hbar\upsilon$ is the mean ionization and excitation potential of atoms in the stopping material . in practice $I_{ave.}$ is regarded as an empirical constant , with a value in (**eV**) of the order of **10 z .** in air , $I_{ave.} = \mathbf{86\ eV}$, while for $^{27}_{13}\mathbf{Al}$, $I_{ave.} = \mathbf{163\ eV}$ **.**

The correct theoretical relationship between range and energy including relativistic corrections were done in **1930** by **Hans Bethe** sometimes this relation called **"Stopping Power"** of the materials.

$$\left(-\frac{dT}{dx}\right) = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi Z^2 \cdot nZ}{m_0 v^2}\left[\ln\frac{2m_0 v^2}{I_{ave.}} - \ln\left(1-\frac{v^2}{c^2}\right) - \frac{v^2}{c^2}\right] \text{--------} ⑧$$

Since **n = number of atoms / volume** $\quad N = \frac{w \cdot N_A}{A} = \frac{\rho \times v \times N_A}{A}$

$\therefore\ n = \frac{N}{V} = \frac{\rho \times N_A}{A}$ and since $\beta = \frac{v^2}{c^2}$

Therefore **Bethe** formula will be in the form **:**

$$\left(-\frac{dT}{dx}\right) = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi Z^2 \times \rho \times N_A \times Z}{m_0 c^2 \cdot \beta \cdot A}\left[\ln\frac{2m_0 c^2}{I_{ave.}} - \ln(1-\beta^2) - \beta^2\right] \text{--------} ⑨$$

Where $\left(\frac{e^2}{4\pi\epsilon_0}\right) = \mathbf{1.44\ MeV \cdot fm.}$ Z , ρ and A are the **atomic number density and the atomic weight of the stopping material .** m_0c^2 = **the rest energy of the electron.** $mc^2 = 0.511\ \text{MeV}$ **.**

Experimentally , the energy loss $\left(-\frac{dT}{dx}\right)$ is determined by the number of **"ion pairs"** (+ and −) produced along the path of the particle **:**

Δx . Δp ≥ ħ **uncertainty principle**

$$\therefore\ \frac{b_{max}}{b_{min}} = \frac{m_0 v^2}{\hbar\, \upsilon}$$

⊛ In head – on elastic collision

Conversion of energy and momentum in a head – on – elastic collision between a heavy particle of mass **M** and an electron of mass **m** (which we assume to be at rest for simplicity) gives for the loss in kinetic energy of the particle.

عندما $b = 0$ مهم مهم $\boxed{\Delta T = T\left(\frac{4m}{M}\right)}$ ——⊛ قانون عام فقدان الطاقة بالنسبة للتصادم

A head – on – collision gives the maximum energy transfer to the electron . in glancing collision between the electron and the heavy particle , the heavy particle is deflected by a negligible angle , and so the particle follows very nearly a straight – line path.

حيث

m = كتلة الجسيم الساقط (الجسيم المشحون) المتفاعل مع مادة الوسط (الالكترون مثلا).

M = كتلة الهدف.

T = طاقة الجسيم الساقط.

المعنى اذا كان مَعّلَم التصادم (**b**) = **صفر** فان معدل فقدان الطاقة للجسيم المشحون(**e**) تغطي بالعلامة ⊛ اعلاه بمعنى تصادم رأسي.

$$\left(-\frac{dT}{dx}\right) = w\, i \quad \text{--------} \quad ⑩$$

Where i = **number of ions per unit path of the heavy particle.**

W ⇒ is the energy needed to produce <u>one</u> ion pair and it is practically independent of the kinetic energy and the type of the incident particle.

Particle	Energy	W(eV)
Electron	5 KeV	35.0
α - particle	5.3 MeV	35.2
Proton	340 MeV	33.3

Mean Range $\bar{R}$:

The mean range is the average distance travelled by the incident particle before it **loses** all its kinetic energy.

$$\bar{R} = \int_0^{\bar{R}} dx = \int_{T_0}^{0} \frac{dx}{dT} \cdot dT = \int_0^{T_0} \left(-\frac{dT}{dx}\right)^{-1} dT \cong \int_0^{T_0} T\, dT \longleftarrow$$

يمكن كتابتها بهذا الشكل اذا كان $\left(-\frac{dT}{dx}\right)$ لا تعتمد على السرعة للجسيم المشحون

$$\bar{R} = \int_0^{T_0} \left[\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi z^2 nZ}{m_0 v^2} \ln \frac{2 m_0 v^2}{I_{ave.}} \right]^{-1} dT \text{ -------- } (11)$$

If the $\frac{2 m_0 v^2}{I_{ave.}}$ is independent on the speed v, then

$$\bar{R} \propto \int_0^{T_0} T\, dT \propto T_0^2 \text{ -------- } (12)$$

This is a very simple range relation.

The range of a single particle may be slightly larger or smaller than expression (11), because there are statistical variations in the amount of energy lost per unit path length $\left(-\frac{dT}{dx}\right)$ and in the total number of ion pairs formed.

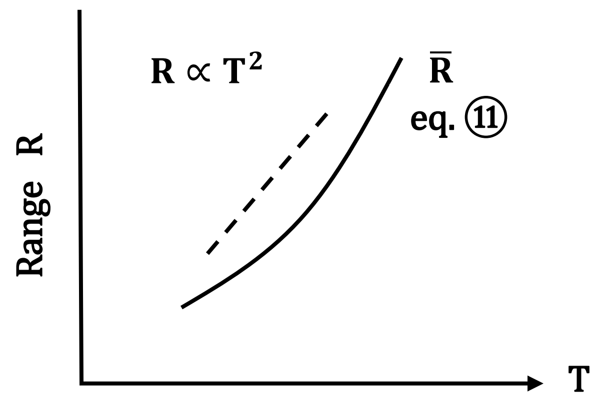


Fig. 2

This statistical nature is responsible for the **straggling phenomenon** which states that : identical particles of identical energies **do not** lose the same energy in passing through a target of given thickness.

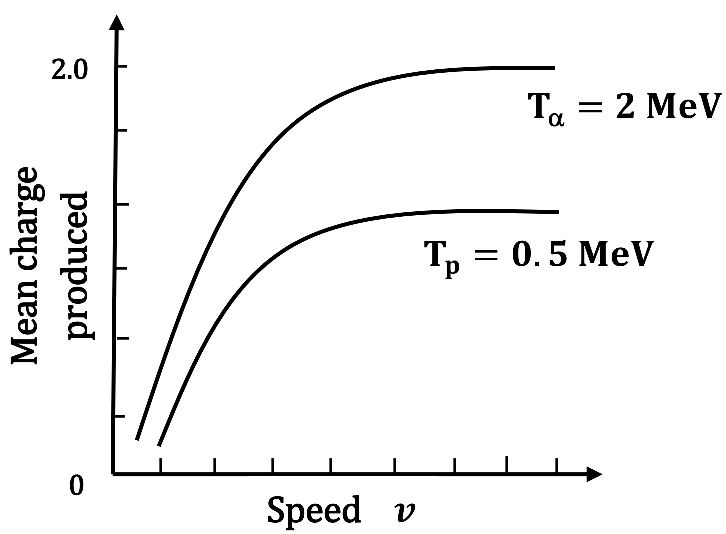


Fig. 3

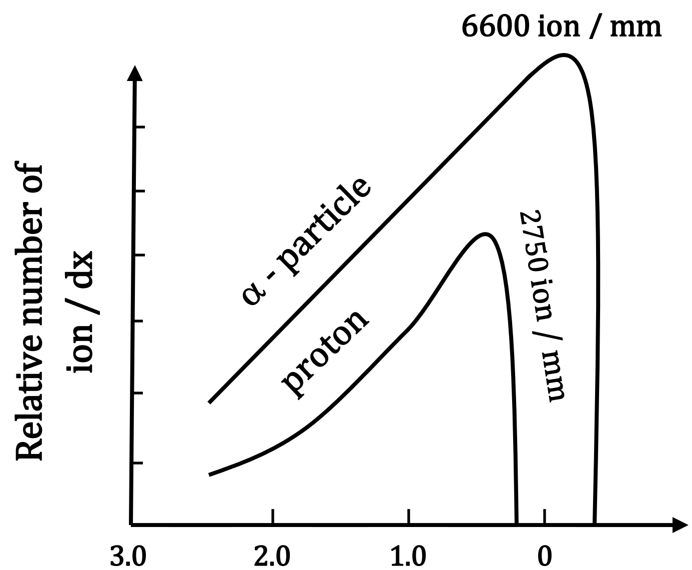


Fig. 4 Residual range in air

The residual range : is the distance left to travel until the particle comes to rest.

مسافة متبقية قصيرة يجب قطعها من قبل الجسيم حتى يصل الى السكون.

Bragg – Kleeman rule : the relationship between ranges in different materials can be obtained using a semi empirical relationship known as the **Bragg – Kleeman rule**.

$$\frac{R_1}{R_0} = \frac{\rho_0}{\rho_1}\sqrt{\frac{A_1}{A_0}} \text{ -------- } ⑬$$

Where **R** is the range , **ρ** the density and **A** the atomic weight. The subscripts **0** and **1** refer to the known and unknown ranges and materials respectively.

كما يمكن كتابة المعادلة ⑬ بالشكل الاتي

$$\frac{R_1}{R_2} = \frac{M_1}{M_2} = \frac{Z_2^2}{Z_1^2}$$

إذاً : R_1 , M_1 , Z_1 هو المدى و الكتلة و العدد الذري للجسيم الاول و الرقم ② للجسيم الثاني