Alpha Decay Systematics

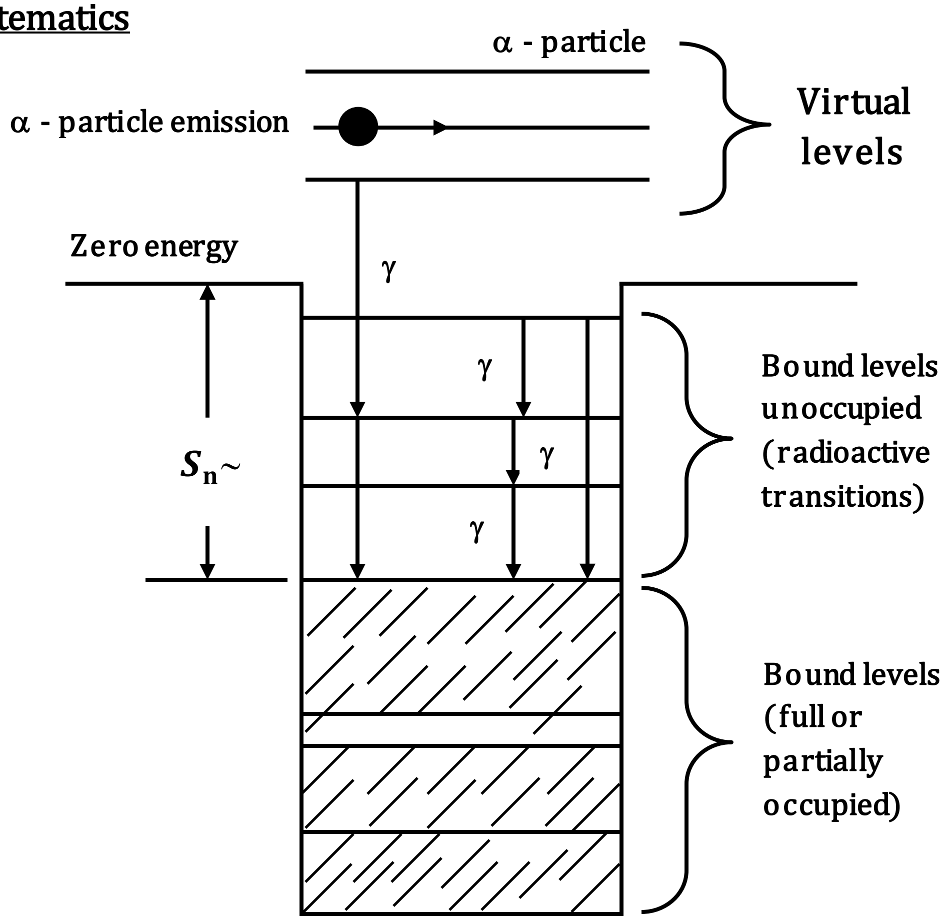


Fig. A. Nuclear Levels

Alpha decay is the emission of α – **particle** from a virtual nuclear levels

$$^{A}_{Z}X_{N} \xrightarrow{\alpha\text{ - emission}} {}^{A-4}_{Z-2}Y_{N-2} + {}^{4}_{2}He_{2} + Q_{\alpha}$$

This decay can be happened by **most** nuclides with $A > 190$ and (**many with** $150 < A < 190$) are energetically can decay by **α - emission.**

In **α - decay** , several systematics trends are apparent **:**

① α – **emission** with large disintegration (**decay**) energy (Q_α) had short half – lives and conversely ,as in the **α - decay** of ^{232}Th ($\sim 1.4 \times 10^{10}$ **y** , $Q_\alpha = 4.08$ **MeV**) and ^{218}Th (1.0×10^{-7} **sec** , $Q_\alpha = 9.85$ **MeV**) , this means that a change by a factor **2** in energy ($9.85/4.08$) corresponds a factor of 10^{24} (10^{10} **year**/10^{-7} **sec**) in half – life , the theoretical explanation of this **Geiger – Nuttall** rule in **1928** was one of the first triumphs of quantum mechanics.

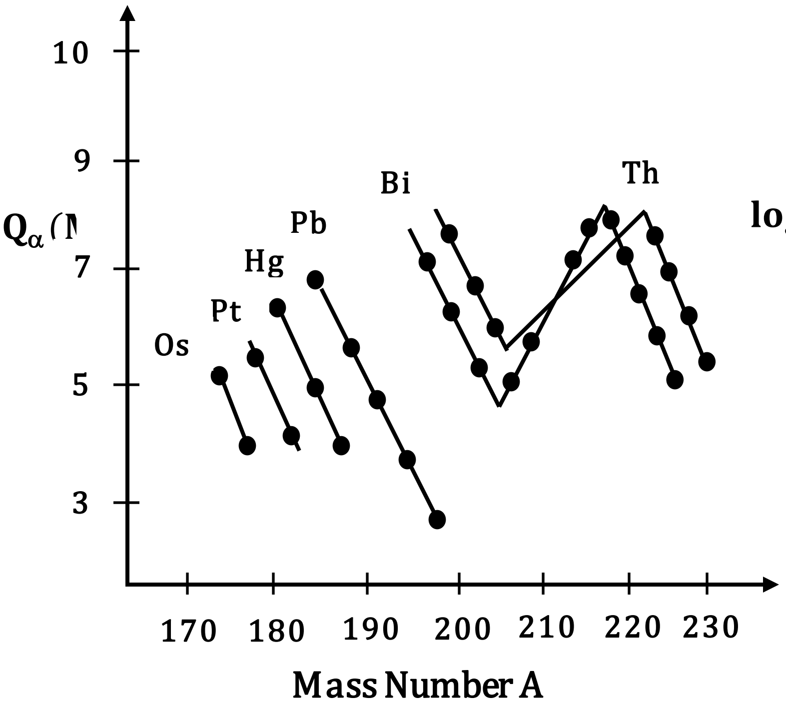


Fig (B). The effect of closed shells

at $A = 212$, $N = 126$

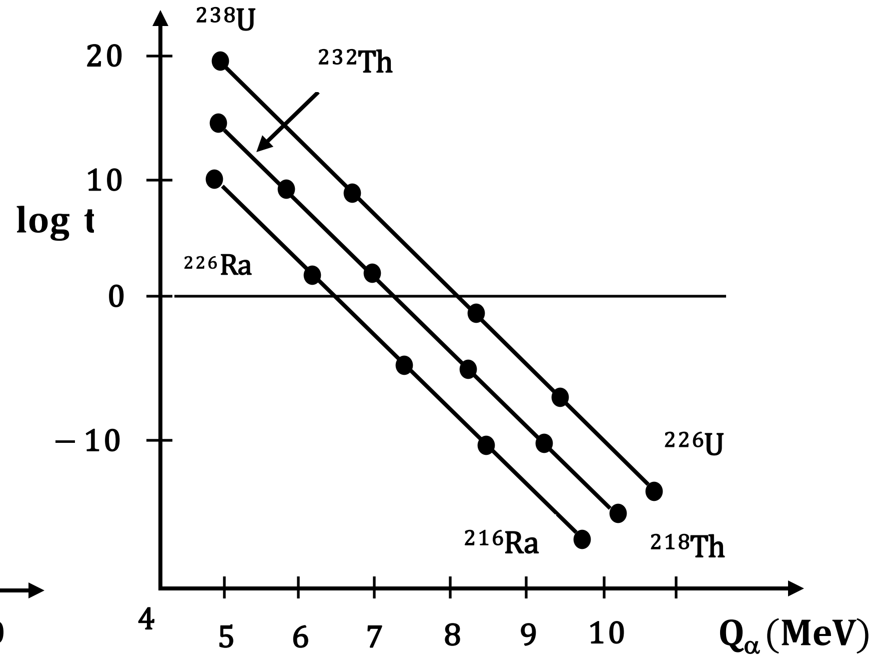


Fig (C). The inverse relationship between α - decay half - life and decay energy Q_α (Geiger - Nutall rule)

For ground state decay between **Even – Even** nuclides, the following **Geiger – Nutall rule** is valid :

$$\log t_{1/2} = a + \frac{b}{\sqrt{Q_\alpha}}$$

Which means that when the decay energy (Q_α) **increases**, the half – life **decreases**, i.e. the decay constant increases.

a and **b** are functions of **Z** and they are equal to **:**

$$a \approx -1.61\, Z_D^{2/3} - 21.4$$

$$a \approx 1.61\, Z_D$$

Z_D is the atomic number of daughter nucleus.

② The dependence of the decay energy (Q_α) on **A** or (**Z** or **N**) is regular **except** near magic numbers (**Fig. B**).

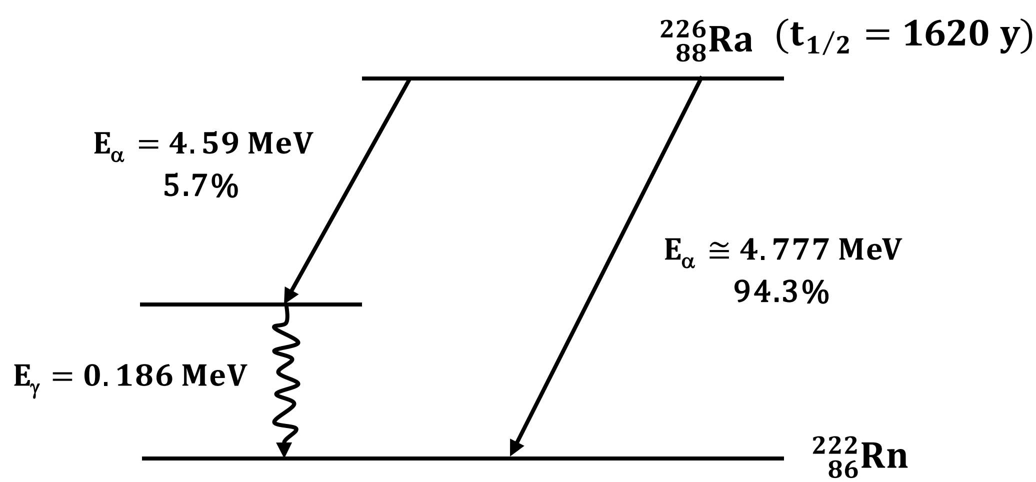
③ For nuclides with a given **Z**, the half – life is a smooth function of the decay energy (Q_α) , especially for **even – even** nuclei (**Fig. C**).

④ Adding neutrons to a nucleus, especially for $A > 212$, <u>reduces</u> the disintegration energy (Q_α) as shown in (**Fig. B**), which accordance to **Geiger – Nutall rule**, **<u>increases</u>** the half – life ($\log t_{1/2}$) as shown in (**Fig. C**).

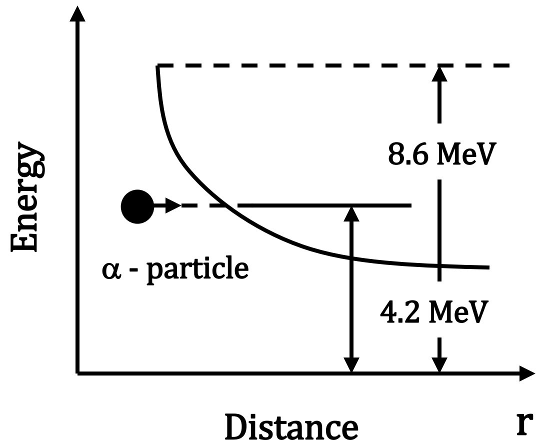
⑤ The decay constant **<u>decreases</u>** exponentially with Q_α and near $A = 150$, the decay energy Q_α is practically **zero**.

⑥ The energy spectrum of α **- particles** give information about the level scheme of the parent or daughter nuclei.

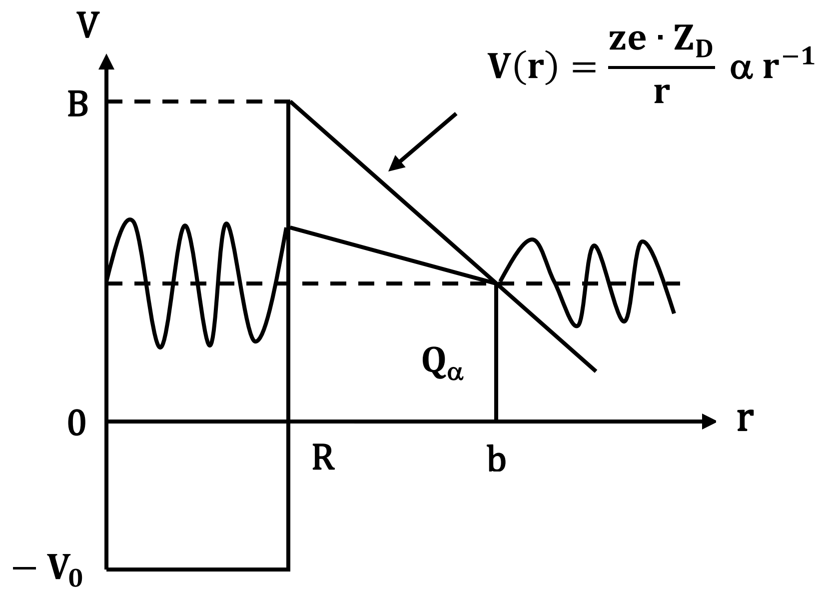
$$E_\gamma = 4.777 - 4.591 = 0.186 \text{ MeV}$$



The Theory of α - decay :



Coulomb barrier 8.6 MeV
for $^{238}_{92}U$ and $^{234}_{90}Th$



α - particle Decay Mechanism

As we know that nuclei consist of protons , neutrons and there is no **α - particles** inside the nucleus. According to **Gamow et. al.** theory **α - particles** are preformed inside the parent nucleus just before **α - decay** . In this theory **α - particles** are assumed to move in spherical region determined by the daughter (Z_D) nucleus , which is known as **one – body** model . From the **Fig.** it is clear that there are **three regions** of interest.

① **The spherical region** ($r < R$) inside the nucleus , where the potential well is of depth ($- V_0$) , V_0 is of positive value . classically **α - particles** can move in this region with kinetic energy $Q + V_0$, but it cannot escape from it . Quantum mechanically , however , there is a chance of **"Leakage"** or **"Tunneling"** through such barrier.

② **The annular shell region** ($R < r < b$) forms a potential barrier because here the potential energy is more than the total available energy **Q** . Classically **α - particle** cannot enter this region from either direction.

③ The region ($r > b$) is classically permitted region outside the barrier.

From the <u>classical</u> point of view, an **α - particle** in the spherical potential well ($r < R$) would sharply reverse its motion every time it tries to pass beyond $r = R$
From the <u>semi classical</u> point of view, the probability of decay per unit time (λ_α) is equal to the number of collisions per second which **α - particle** makes with the wall of the confining potential well multiplied by the probability of **penetration** the potential barrier.

$$\lambda_\alpha \approx fP = \left(\frac{\upsilon_{in}}{2R}\right) P \quad \text{----------- (1)} \longrightarrow \frac{\sqrt{2Q/m}}{2R}$$

Where λ_α is the decay constant, **f = the frequency of collisions.**
υ_{in} **= the velocity of α - particle inside the nucleus.**
P = probability of penetration.
$(\upsilon_{in}/2R)$ **= number of collisions /sec.**

In ^{238}**U**, for example the leakage probability is so <u>small</u> that the **α - particle**, on the average must make $\sim \mathbf{10^{38}}$ tries before it escape
($\sim \mathbf{10^{21}}$ **tires/sec for** $\sim \mathbf{10^{9}}$ **year**). The quantum mechanical treatment gives about the same result for this problem. since $\mathbf{f} = \left(\frac{\upsilon_{in}}{2R}\right)$ and the energy of **α - particle** emitted ranged from **4 – 5 MeV**, then $\mathbf{f \approx 6 \times 10^{21}}$ **tries/sec** for **Q ≈ 5 MeV.**

The coulomb barrier in the **Fig.** has height **B** at **r = R**

Where $$\mathbf{B} = \frac{1}{4\pi\epsilon_0}\frac{ze \cdot \acute{Z}_D e}{R} = \left(\frac{e^2}{4\pi\epsilon_0}\right)\frac{2\,\acute{Z}_D}{R} \quad \text{----------- (2)}$$

$\left(\frac{e^2}{4\pi\epsilon_0}\right)$ **= 1.44 MeV. fm.**, **ze = 2e** is the **charge of α - particle** and $\acute{Z}_D e$ is the **charge of daughter nucleus.** The height of the barrier thus varies from **(B – Q)** to $\frac{1}{2}$**(B – Q)** and $\mathbf{R/b = Q/B}$.

For a typical heavy nucleus (**Z = 90**, **R = 7.5 fm**) the barrier height **B** is about **34 MeV.** The radius **b** at which **α - particle "leaves"** the barrier is found from the equality of particle's energy and the potential energy.

$$\mathbf{b} = \frac{1}{4\pi\epsilon_0}\frac{z\,Z_D e^2}{Q} = \left(\frac{e^2}{4\pi\epsilon_0}\right)\frac{z\,Z_D}{Q} \quad \text{----------- (3)}$$

The penetration probability of the barrier is **:**

$$\mathbf{P = e^{-2G}} \quad \text{-----------} \quad ④$$

Where **G** is the **Gamow factor**

$$\mathbf{G = \sqrt{\frac{2m}{\hbar^2}} \int_R^b [V(r) - Q]^{1/2}\, dr} \quad \text{-----------} \quad ⑤$$

From equations ② and ③ **:**

- **∗** λ_α decreases as **Z** increases , i.e. the height of the barrier (the thickness **b** increases , **eq.** ③).

- **∗** λ_α increases as **R** increases , i.e. the barrier height **B** and the thickness **b** decreases.

From **eq.** ⑤

$$\mathbf{G = \sqrt{\frac{2m}{\hbar^2 Q_\alpha}} \cdot \frac{2Z_d\, e^2}{4\pi \in_0} \cdot \left[\frac{\pi}{2} - 2\sqrt{\frac{Q}{m}}\right]}$$

و بعدها نجد قيمة التردد (**f**)

و بتعويض قيمة **G** (**معامل كامو**) في معادلة ④ نجد احتمالية الاختراق (**P**)

بعدما وجدنا فيه **f** , **P**

نطبق المعادلة ① لنجد λ_α ليتسنى لنا معرفة العمر النصفي.

مثال محلول ص **170 - 171**