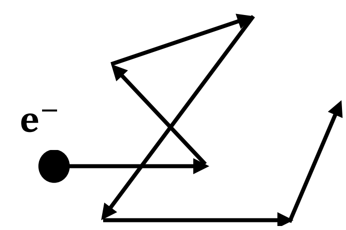
Energy Loss by Electrons

The fast electrons lose their energy by ionization and excitation in gases , liquids and solids <u>almost</u> by the same way as heavy charged particles do . electrons (**positive** or **negative** e^{+} , e^{-} , β^{+} , β^{-}) , therefore , interact through coulomb scattering (**coulomb force**) from atomic electrons . the energy loss formula **eq.** ⑦ and **eq.** ⑨ are practically the same . there are , however , some important differences :

① Electrons practically which emitted in **β – decay** travel at relativistic speeds , because their mass is small , $m_e = \frac{1}{1836} m_p$.

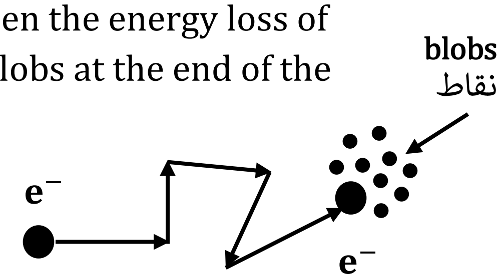
② Because the incident electron and the electron in the stopping material have the same mass , then there is much more scattering of the incident electron , i.e. the electron defects through larger angles than the heavier particles do , therefore the path length of electron in the stopping material will be larger and (**erratic**)than the straight line of the heavy particle.



erratic path

③ In head – on – collision of one electron with another , a large fraction of the initial energy may be transferred to the struck electron , because they have the same mass.

④ As the charge of the <u>incident electron</u> never change , then the energy loss of electrons by ionization is in the **eV** range , this cause the blobs at the end of the electron path.



⑤ Because the electron may suffer rapid change in the direction and the magnitude of its velocity , it is subject to large accelerations , and accelerated charged particles <u>must</u> radiate Electromagnetic energy , which is called **Bremsstrahlung** (bra radiation) . therefore , the energy loss by radiation for electrons is important at much lower energies then the protons , as shown in the **Fig. 5**

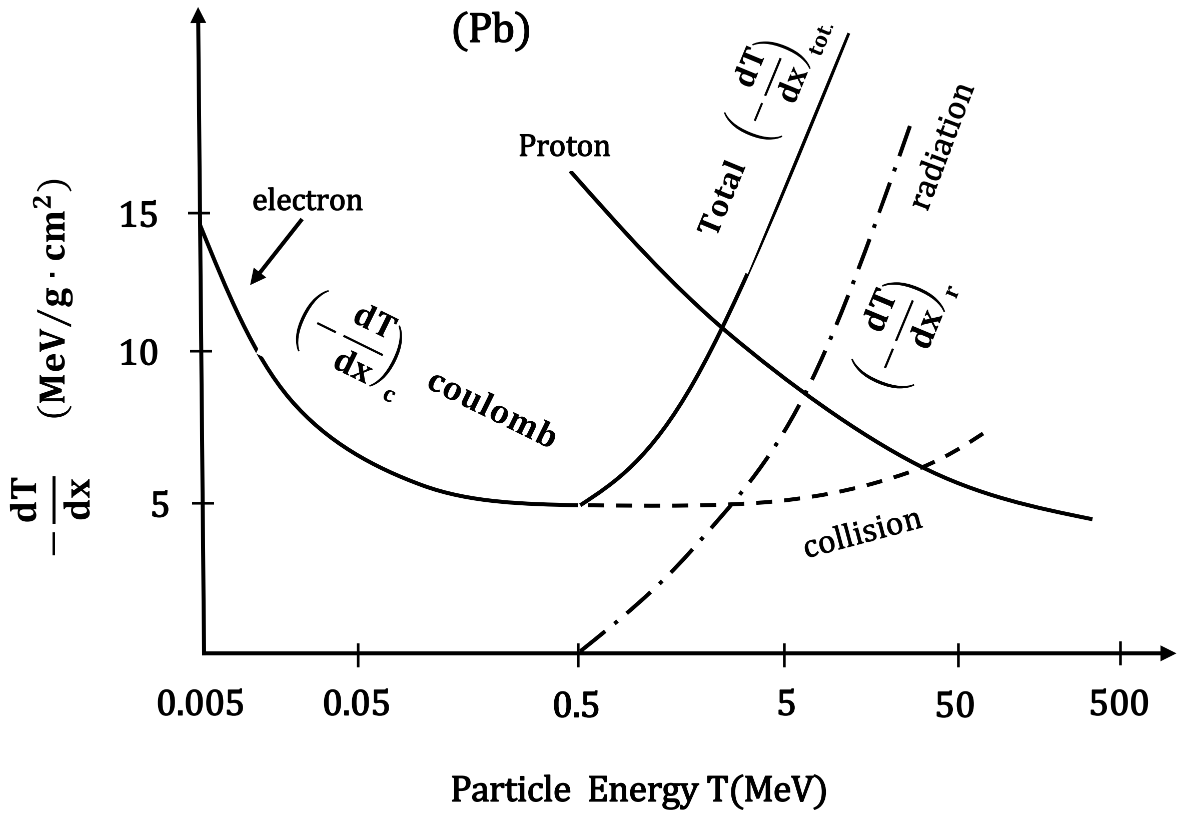


Fig. 5 Energy Loss of Electrons and Protons in Lead (Pb)

Note : from the figure we see that the radiative term is significant (important) only at high energy and in heavy materials (**z** ≫). The total energy loss :

$$\left(-\frac{dT}{dx}\right)_{tot.} = \left(-\frac{dT}{dx}\right)_c + \left(-\frac{dT}{dx}\right)_r$$

$$\frac{(-\,dT/dx)_r}{(-\,dT/dx)_c} \approx \frac{T.+m_e c^2}{m_e c^2} \cdot \frac{Z}{100}$$

Z ⇒ is the atomic number of the stopping material.

T ⇒ kinetic energy of e^-.

The experimental arrangement and the absorption curves for charged particles emitted by a radioactive sources are shown below :

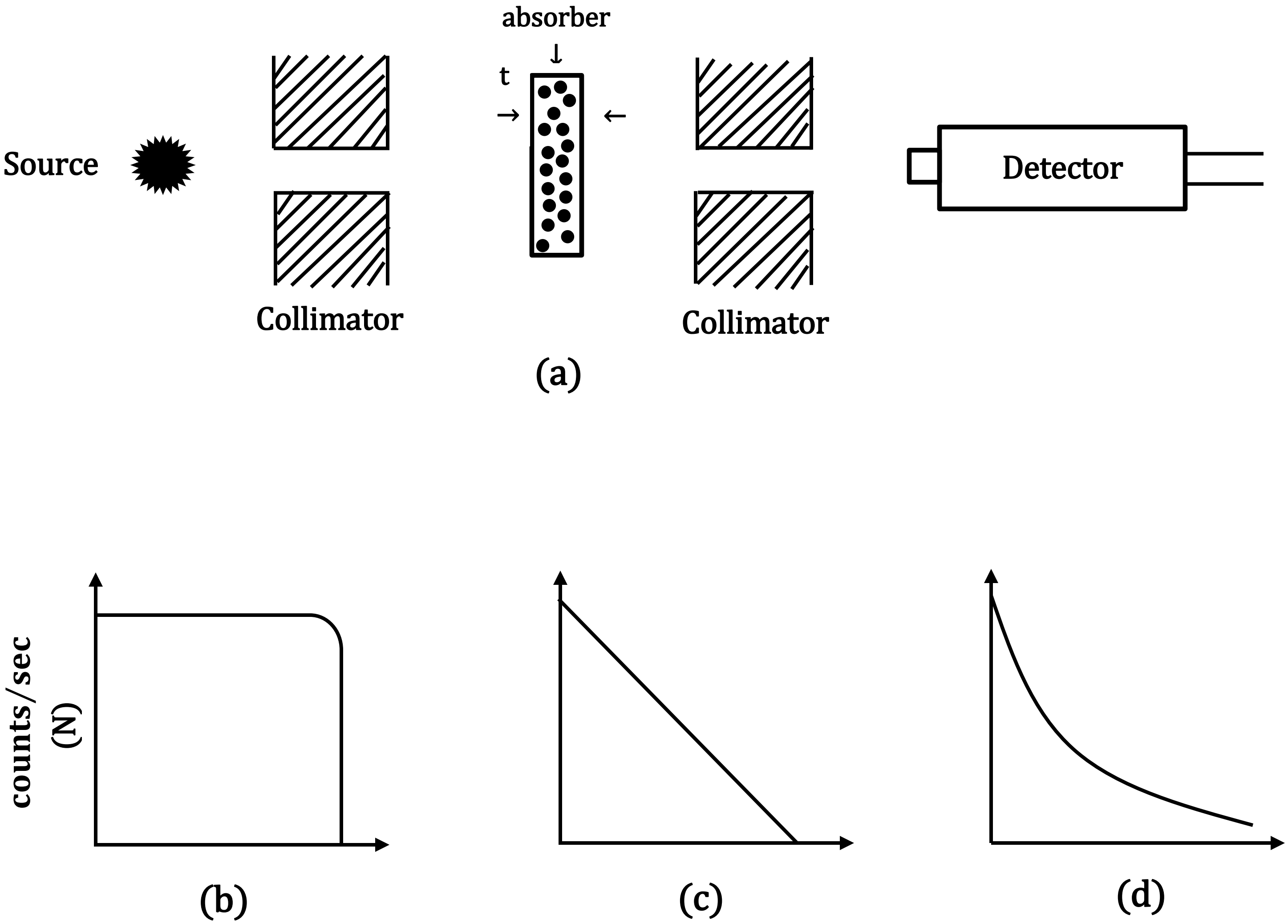


Fig. 6 (a) Experimental arrangement. (b) Absorption curve for heavy particles.
(c) Absorption curve for monoenergetic electrons e^-.
(d) Absorption curve for β – decay.

Interaction of Neutrons with Matter

Neutron Slowing

The interaction of neutrons with matter is very different from that of charged particles or **γ – rays** . it was formed by **Fermi (1934)** that the **radioactivity** of targets bombarded by neutrons is **increased** when the **fast neutrons** slowed – down by hydrogenous material placed in front of the target , which is called **moderator** such as H_2O , D_2O , **wax** and in general the light materials . when fast neutrons , their energy of the order of **2 MeV** , are introduced in a medium (**moderator**) , a number of collisions with nuclei takes place (**occurs**) . the neutrons are defected in direction on **each** collision , they lose energy , and they tend to go away from their origin. Each neutron has its own history (**way**) and it is impractical to trace all of them so we took the average behavior of all of them. Since the neutron is an uncharged particle , it is unaffected by the **coulomb force** (**Coulomb barrier**) so neutrons of even very low energy (**eV** or **less**)can penetrate the nucleus and initiate nuclear reactions . Neutrons passing through matter have negligible interactions with the atomic electrons , therefore they don't not produce primary ionization events in detections materials . the first experimental observation of the neutron occurred in **1930** , when **Bothe** and **Becker** bombarded **beryllium** with **α - particles** (**from radiative decay** , e.g. ^{226}**Ra**) and obtained a very **penetrating** but **nonionizing** radiation , named by **Chadwick** in **1932** by neutrons.

$$ {}^{4}_{2}\mathrm{He} + {}^{9}_{4}\mathrm{Be} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n} \qquad Q = 5.7\ \mathrm{MeV} $$

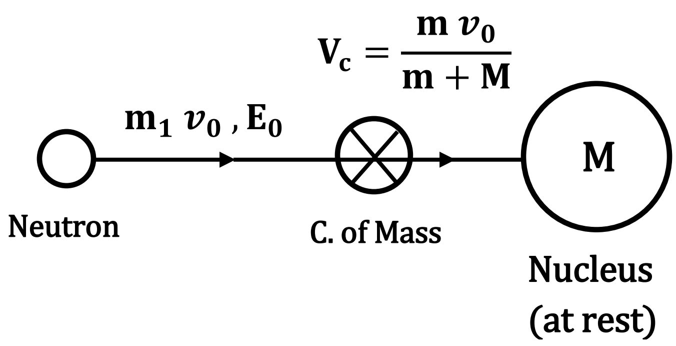
The amount of energy lose (**of neutrons**) in a **single** collision can be determined by solving the equations of conservation of energy and momentum. The other method , which is simpler by using two reference systems of coordinates.

① The **laboratory system** , **L – system** , in which the target nucleus is assumed to be at rest before the collision.

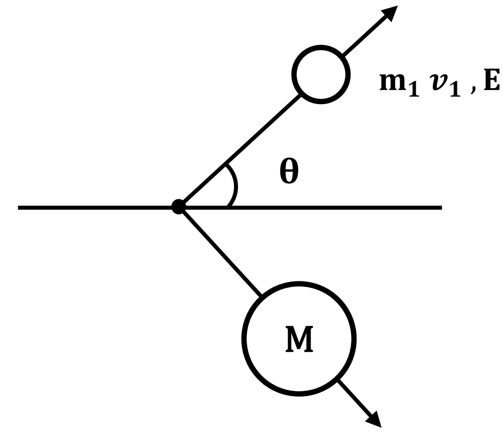
② The **Center – of – Mass** , **C – system** , in which the center of mass of neutron and the nucleus is considered to be at rest and both the neutron and the nucleus approach it.

In an elastic collision , the **struck** nucleus is not excited and the momentum and the kinetic energy are conserved , the two systems are **sketched** before and after collision **:**

L – System :

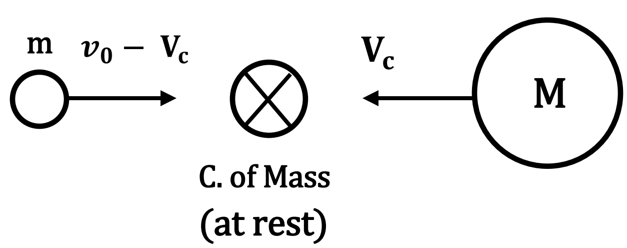


L – system before collision

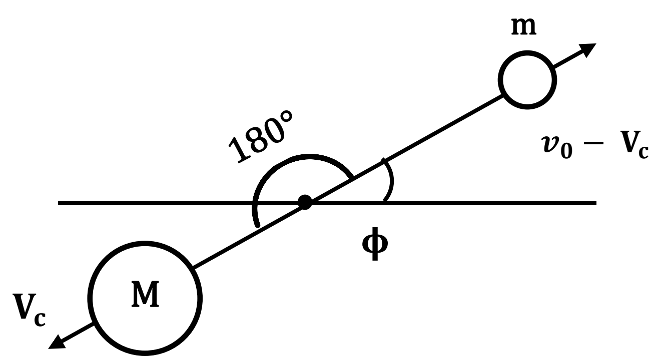


L – system after collision

C – System :



C – system before collision



C – system after collision

Fig. 7 L – system and C – system coordinates

After Collision : in **L – system** the neutron moves with velocity v and energy **E** at angle **θ** and the nucleus moves at an **particular** angle . In **C – system** the neutron moves at angle **ϕ .** Since the total momentum **must be** conserved , its value **must be zero** (**before collision also equal zero**) and the **nucleus must** move off at angle **180 ° + ϕ .**

In glancing collision :

اقل فقدان للطاقة

i.e. $\phi \approx 0°$, i.e. $E = E_0$ -------- (1)

$v = v_0 \frac{M}{M+m} + v_0 \frac{m}{M+m} = v_0$, i.e. $v = v_0$ -------- (1a)

The amount of lost energy by the neutron is **negligible** and $E = E_0$.

In Head – on – collision : $\phi = 180°$, the speed of the neutron is given by :

$$v = v_0 \frac{M}{M+m} - v_0 \frac{m}{M+m} = v_0 \left(\frac{M-m}{M+m}\right)$$

$$\frac{v}{v_0} = \left(\frac{M-m}{M+m}\right) \quad ; \quad \frac{v^2}{v_0^2} = \left(\frac{M-m}{M+m}\right)^2 \quad ; \quad \frac{\frac{1}{2} m v^2}{\frac{1}{2} m v_0^2} = \left(\frac{M-m}{M+m}\right)^2$$

or $\frac{E}{E_0} = \left(\frac{M-m}{M+m}\right)^2$ -------- ②

since $M \approx A$ where $M \Rightarrow$ the mass of nucleus.

$m_n \approx 1$ $A \Rightarrow$ the mass number of the target.

$m_n \Rightarrow$ the mass of neutron.

$\therefore \frac{E}{E_0} \cong \left(\frac{A-1}{A+1}\right)^2 = \alpha$ where $\alpha = \left(\frac{A-1}{A+1}\right)^2$ -------- ③ اعظم فقدان في طاقة النيترون المستطير

$\therefore E_{min} = \alpha E_0$ -------- ④

The neutron loses most energy in a **head – on – collision** , when the moderator (**target**) is graphite **M = 12** , then :

$$\frac{E}{E_0} = \left(\frac{A-1}{A+1}\right)^2 = \left(\frac{12-1}{12+1}\right)^2 = 0.72$$

$E_{min} = 0.72\ E$

The neutron lose up to **28%** of its energy in a collision with a carbon nucleus , i.e.

$$\left(\frac{E_0 - E_{min}}{E_0}\right) \times 100\% = \left(\frac{1-0.72}{1}\right) \times 100\% = 28\%$$

If the moderator is Hydrogen (**A = 1**) then $\frac{E}{E_0} = \left(\frac{1-1}{1+1}\right)^2 = 0$ this means that the neutron loses almost all its energy ($E \approx 0$) this is why the H_2O is used as a moderator .

For Intermediate Value of ϕ

The neutron speed (v) after collision can be found as a function of **ϕ** .

$$v^2 = v_0^2 \left(\frac{M}{M+m}\right)^2 + v_0^2 \left(\frac{m}{M+m}\right)^2 + 2\, v_0^2 \left(\frac{M}{M+m}\right)\left(\frac{m}{M+m}\right) \cos\phi \quad \text{--------} \ ⑤$$

$$\frac{E}{E_0} = \frac{v^2}{v_0^2} = \frac{M^2+m^2+2Mm\cos\phi}{(M+m)^2} \quad \text{--------} \ ⑥$$

This is a general equation which can be used to determine the energy of the neutron after any type of collision **:**

① In glancing collision , where $\phi \approx 0$, $\cos\phi = \cos 0 = 1$, from **eq.** ⑥ we have **:**

$$\therefore \ \frac{E}{E_0} = \frac{M^2 + m^2 + 2Mm}{(M+m)^2} = \frac{(M+m)^2}{(M+m)^2} = 1$$

$\therefore \ E = E_0$, as in **eq.** ①

② In **head – on – collision** , where $\phi = 180°$, $\cos\phi = -1$, from **eq.** ⑥ we have :

$$\frac{E}{E_0} = \frac{M^2 + m^2 - 2Mm}{(M+m)^2} = \frac{(M-m)^2}{(M+m)^2}$$, as in **eq.** ②

* How to determine the velocity of the center of mass V_c **:**

From the conservation law of momentum in **L – system** we have **:**

Momenta before collision = Momenta after collision

$$m\, v_0 + M\, V_{target} = (m_0 + M)\, V_c$$

Since the velocity (V_{traget}) of the target is equal to **zero** , because the target nucleus is at rest in the **L – system** then

$$m\, v_0 = (m_0 + M)\, V_c$$

$$\therefore \ V_c = \left(\frac{m}{m+M}\right) v_0 \quad \text{--------} \ ⑦$$

In **C – system** the velocity of scattered neutron ($v_0 - V_c$) is given by **:**

$$(v_0 - V_c) = v_0 - \frac{m}{m+M}\, v_0 = v_0\left[1 - \frac{m}{m+M}\right] = \left(\frac{M}{m+M}\right) v_0 \quad \text{--------} \ ⑧$$

The vector diagram for the velocity of the neutron (v) after the collision in **L – system** and **C – system**.

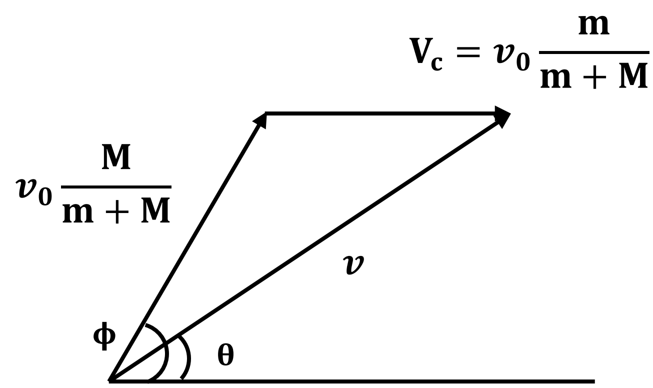


Fig. 8

The relation between the angles in **L – system** (θ) and in the **C – system** (ϕ) is given by **:**

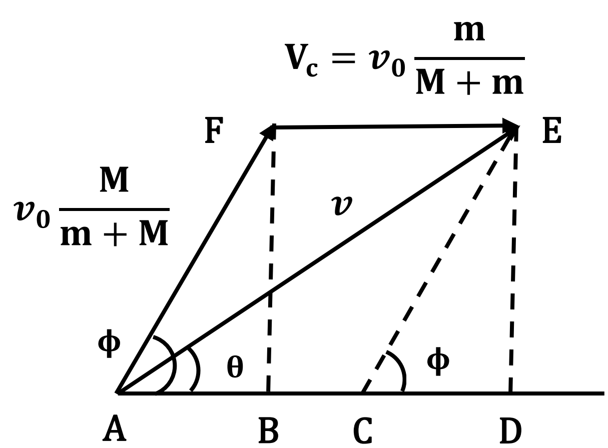


Fig. 9

<u>H.W</u>

$$\cot\theta = \frac{\cos\phi + 1/A}{\sin\phi} \quad \text{--------} \quad (9)$$

Where $A = \frac{M}{m}$, can be <u>**derived**</u> as follows **:**

$$\sin\theta = \frac{E}{v} = \frac{BF}{v}$$

$$\sin\phi = \frac{BF}{v_0 \frac{M}{m+M}} \quad , \quad \text{therefore} \quad BF = v_0 \frac{M}{m+M} \sin\phi$$

$$\therefore \ \sin\theta = \left(v_0 \frac{M}{m+M} \sin\phi\right)/v$$

$$\cos\theta = \frac{AD}{v} = \frac{AB+BD}{v}$$ where $AB = v_0 \frac{M}{m+M} \cos\phi$ and $BD = v_0 \frac{m}{m+M}$

$$\therefore \cos\theta = \frac{v_0\frac{M}{m+M} \cos\phi + v_0\frac{m}{m+M}}{v}$$

$$\frac{\cos\theta}{\sin\theta} = \frac{v_0\frac{M}{m+M} \cos\phi + v_0\frac{m}{m+M}}{\cancel{v}} \times \frac{\cancel{v}}{v_0\frac{M}{m+M} \sin\phi}$$

$$\cot\theta = \frac{v_0\left(\frac{M}{m+M}\right)\left[\cos\phi + \frac{m}{M}\right]}{v_0\frac{M}{m+M} \sin\phi}$$

Therefore

$$\cot\theta = \frac{\cos\theta + \frac{m}{M}}{\sin\phi}$$

or

$$\cot\theta = \frac{\cos\phi + 1/A}{\sin\phi}$$

Where $A = \frac{M}{m}$