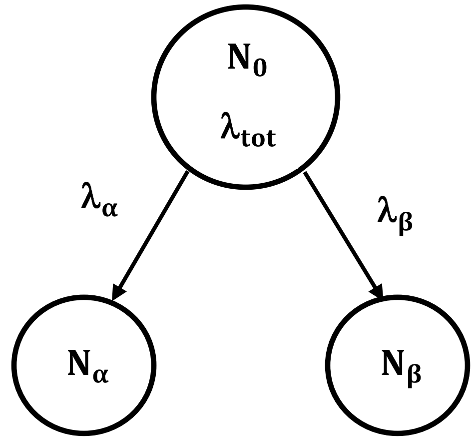
Multi Process Decay



If a single nuclide can decay by more than one process, for example by **alpha** and **β - decay** the probability of decay increased.

$$-dN_0 = dN_\alpha + dN_\beta$$

$$-dN_0 = \lambda_\alpha N_0\, dt + \lambda_\beta N_0\, dt = (\lambda_\alpha + \lambda_\beta) N_0\, dt$$

this equation can be solved to obtain

$$N = N_0\, e^{-(\lambda_\alpha+\lambda_\beta)t} \quad \text{or} \quad N = N_0\, e^{-\lambda_{tot}\, t}$$

Where $\lambda_{tot} = \lambda_\alpha + \lambda_\beta + \cdots$

$\frac{\lambda_\alpha}{\lambda_{tot}} \Rightarrow$ **is the branching ratio for α - decay.**

$\frac{\lambda_\beta}{\lambda_{tot}} \Rightarrow$ **is the branching ratio for β - decay.**

The experimental **half – life** of the parent is **:**

$$\left(T_{1/2}\right)_{parent} = \frac{0.693}{\lambda_{tot}} \quad \text{and} \quad T_\alpha = \frac{0.693}{\lambda_\alpha} \quad \text{and} \quad T_\beta = \frac{0.693}{\lambda_\beta}$$

The total activity of the parent is **:**

$$\left(\frac{dN}{dt}\right)_{tot} = \frac{dN_\alpha}{dt} + \frac{dN_\beta}{dt} = \lambda_\alpha N_0\, e^{-\lambda_{tot}\, t} + \lambda_\beta N_0\, e^{-\lambda_{tot}\, t}$$

$$= (\lambda_\alpha + \lambda_\beta) N_0\, e^{-\lambda_{tot}\, t} = \lambda_{tot} N_0\, e^{-\lambda_{tot}\, t}$$

$$\therefore \left(\frac{dN}{dt}\right)_{tot} = \lambda_{tot} N$$

The activity of **α - branch is :** $\frac{dN_\alpha}{dt} = \lambda_\alpha N$

The activity of **β - branch is :** $\frac{dN_\beta}{dt} = \lambda_\beta N$

And the activity $\left(\frac{dN}{dt}\right)_{tot} = \lambda_\alpha N + \lambda_\beta N = (\lambda_\alpha + \lambda_\beta)N$

$$\left(\frac{dN}{dt}\right)_{tot} = \lambda_{tot} N$$

Width of Decaying States

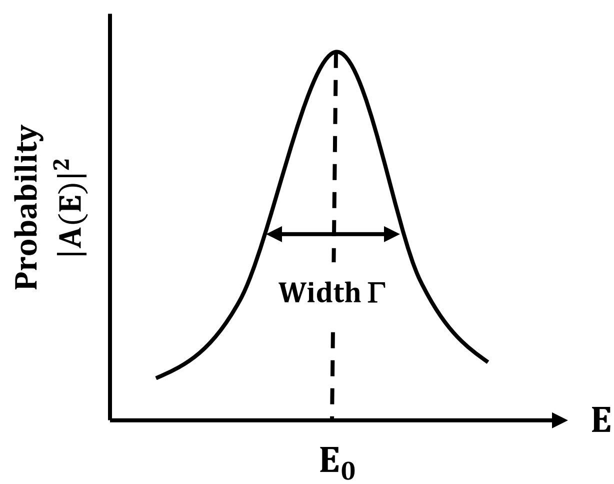
Each excited state has a finite life time (greater than $\mathbf{10^{-10}s}$) and this case an uncertainty in its energy , so the energy will have a **spread** which called the **Width** $\boldsymbol{\Gamma}$**.** The probability of finding a decaying state with a given energy **E** is proportional to absolute square of the amplitude **A(E):**

$$|A(E)|^2 = \frac{1}{4\pi^2}\frac{1}{(E - E_0)^2 + (\pi/2)^2}$$

The **width** of the state is given by $\Gamma = \hbar\lambda = \frac{\hbar}{\tau}$ where $\tau \Rightarrow$ **is the mean life time of the state.**

$$\hbar = h/2\pi = \frac{6.62 \times 10^{-34}\ J.s}{2\pi} = 1.054 \times 10^{-34}\ \text{Joule . sec.}$$

$$\therefore \Gamma(\text{in eV}) = \frac{1.054\times10^{-34}\ \text{J.sec}}{1.6\times10^{-19}\ \text{J} \times\tau(\text{sec})} = \frac{0.66\times10^{-15}}{\tau(\text{sec})} = \frac{0.46\times10^{-15}}{T_{1/2}\ (\text{sec})}$$



Probability to observe the energy of an unstable state with width Γ

Units of Radioactivity

The activity of radioactive material , originally , measured by unit called the **Curie** (**Ci**) and is defined as the amount of radioactive material that gives $\mathbf{3.7 \times 10^{10}\ dis/sec}$ or ($\mathbf{3.7 \times 10^{10}}$ **Bequerel** ($\mathbf{1\ Bq = 1\ dis/sec}$)**.** Therefore , the **Curie** measures only the quantity (**grams** or **kilograms**) of radioactive material and not measure rate of decay ($\mathbf{dN/dt}$).

The activity of ($\mathbf{1\ gm}$) of $^{226}_{88}\mathbf{Ra}$ ($\mathbf{T_{1/2} = 1620\ years}$) is equal to **1 Ci.**

$$\mathbf{Activity = \lambda N} \quad , \quad \text{where } \lambda = \frac{0.693}{T_{1/2}}$$

$$\lambda = \frac{0.693}{1620 \times 365 \times 24 \times 3600} = 1.36 \times 10^{-11}\ \text{sec}^{-1}$$

and

$$N = \frac{W(g) \times N_A}{A} = \frac{1(gm) \times 6.025 \times 10^{23}\ \text{atom/mole}}{226\ \text{gm/mole}} = 0.0266 \times 10^{23}\ \text{atom}$$

$$\therefore\ \text{Activity} = 1.36 \times 10^{-11}\ (\text{sec}^{-1}) \times 0.0266 \times 10^{23} \cong 3.7 \times 10^{10}\ \text{dis/sec}$$

$$1\ \text{Curie} = 3.7 \times 10^{10}\ \text{dis/sec} = 3.7 \times 10^{10}\ \text{Bq}$$

Specific Activity :

Note that the **Curie** , although it is used as a unit of quantity , but it does not mentioned anything about the relation between the mass of radioactive material and the activity , this relation is called the **Specific Activity :**

$$S.A. = \frac{\text{Activity}}{\text{Weight}} = \left(\frac{\lambda N}{W}\right) = \frac{\lambda\, N_A}{A} = 6.025 \times 10^{23} \left(\frac{\lambda}{A}\right) \text{dis/sec/g}$$

$$= \frac{6.025 \times 10^{23}}{3.7 \times 10^{10}} \left(\frac{\lambda}{A}\right) = 1.63 \times 10^{13} \left(\frac{\lambda}{A}\right)\ \text{Curie/g}$$

$$= \frac{1.63 \times 10^{13}}{A} \times \frac{0.693}{T_{1/2}} = 1.13 \times 10^{13} \left(\frac{1}{A \times T_{1/2}}\right)\ \text{Curie/g}$$

Exposure Dose and the Röntgen Unit (R)

One common property of nuclear radiation is its ability to **ionize** (**knock electrons from**) atoms with which **interacts**. For this reason nuclear radiation is often called **ionizing radiation**. The passage of **x – ray** and **γ - ray** photons through air will ionize the atoms in the air through a variety of processes (**photoelectric** , **Compton scattering** , **electron – positron pair production**) each one creates a **free electron** , often of reasonably high energy. These electrons can themselves produce **ionization** (**secondary ionization**) and additional electrons. The total electric charge **Q** on the ions produced in a given mass **m** of air is called the **Exposure X** , which equals to $\mathbf{X = \frac{Q}{m}}$ and is measured in the **SI** units of $\frac{\mathbf{Coulmb}}{\mathbf{Kilogram}} = \frac{\mathbf{C}}{\mathbf{Kg}}$.

Röntgen : the Röntgen is a unit of **Exposure Dose** , which is mostly used in work with **X – ray** and **γ - rays**.

The **Röntgen** is defined as the **amount** of radiation which release in $\mathbf{1\ cm^3}$ of air at $\mathbf{0^{\circ}C}$ and **760 mm** pressure (i.e. in **0.001293 g** of **dry air**) one electrostatic unit of charge ($\mathbf{e = 4.8 \times 10^{-10}\ esu}$) of either sign (**either positive** or **negative**) thus

$$\mathbf{1\ Röntgen = 1\ R = \frac{1\ esu}{0.0012939} = \frac{4.6 \times 10^{-10}\ esu}{1.6 \times 10^{-19}\ C \times 0.001293 \times 10^{-3}} = 2.58 \times 10^{-4}\ C/kg}$$

Exposure **1 R** means that $\mathbf{2.58 \times 10^{-4}\ \frac{C}{Kg} / 1.6 \times 10^{-19}\ C = 1.61 \times 10^{15}}$ **ions** are formed per **kg** of air or $\mathbf{2.08 \times 10^{9}\ ions/cm^3}$.

On the average about **34 eV** are needed to form one ion in air and thus an exposure of **1 R** results in an energy absorption by the air of $\mathbf{7.08 \times 10^{19}\ eV/cm^3}$ or $\mathbf{0.113\ erg/cm^3}$ or $\mathbf{88\ erg/g}$

Absorption Dose (Rad)

The **Radiation Absorbed Dose** , **Rad** , is a basic unit to measure the absorbed energy per unit mass of tissue , and equal to an energy absorption of $\mathbf{100\ erg/g}$ of material.

$$\mathbf{1\ Rad = 100\ \ erg/g = 10^{-5}\ J/g = 10^{-2}\ J/kg}$$

Thus $\mathbf{1\ R\ddot{o}ntgen = 0.88\ Rad\ (in\ air)}$

$\therefore$ $\mathbf{1\ Rad = \frac{1}{0.88}\ R}$

The **Rad** unit is used for an **external irradiation** by **γ - ray,** neutrons , charged particles as well as for **internally deposition** of radioactive isotopes **:**

Gray (Gy): The **SI** unit for absorbed dose (**Rad**) is the **gray** which is equal to the absorption of $\mathbf{1\ \ Joule/kg}$ of material , so

$$\mathbf{1\ Gy = 100\ Rad}$$

Rem (Röntgen Equivalent Man)

In a unit of measure of the **Dose – Equivalent** (**DE**). The **DE** is the product of the dose (**D**) and a number that expresses the relative biological effectiveness (**RBE**) , which is called the **Quality Factor** (**QF**). The (**RBE**) is the ratio of the dose of a certain radiation to the dose of **X – rays** that produces the same biological effect. For radiation protection of human beings , it is necessary to have some measures of the biological effects of different kinds of radiation , because some radiations may deposit their energy over a very long path , so that relatively little energy is deposited over any small interval (the size of a typical human cell as an example) ; β and γ **rays** are examples of such radiations. Other types of radiations , **α - particles** lose energy **more rapidly** and deposit essentially **all** of their energy over a very short path length. The probability of **cell damage** from **1 Rad** of **α - radiation** is much greater than that from **1 Rad** of **γ - radiation** , the differences known as (**RBE**).

Since the **RBE** is relatively difficult quantity to measure , its customary to work with instead of the **quality factor** (**QF**) , which is calculated for a given type and (**energy**) of radiation according to the energy deposited per unit path length , as shown in the following table **:**

Quality Factor for Absorbed Radiation

Radiation	QF
X – ray , β , γ	1
Low – energy p , n (~ KeV)	2 – 5
High – energy p , n (~ MeV)	5 – 10
α – particle , heavy ions	20

The effect of a certain radiation on a biological system then depends on the **absorbed dose** (D) and on the **quality factor** (QF) of the radiation. The **dose equivalent** (DE) is obtained by multiplying these quantities together :

$\mathbf{DE = D \times QF}$ the **DE** is measured in units of **rem** , when dose **D** is in **Rads**

$DE\ (rem) = D\ (Rad) \times QF$

When the **SI** unit of **Gray** is used for **D** , then the **DE** is in **Sievert** (**Sv**) unit.

$DE\ (Sv) = D\ (Gy) \times QF$

Therefore **1 Sv = 100 rem**

The international commission on **Radiation Protection** (**ICRP**) has recommended limiting annual whole – body absorbed dose to **0.5 rem** per year for the general public and **5 rem / year** for those who work with radiation. Unfortunately , the physiological effects of radiation exposure are difficult to calculate and measure , and so we must keep the exposure as low as possible.

Table **Quantities and Units for Measuring Radiation**

Quantity	Measure of	Traditional unit	SI unit
Activity (A)	Decay rate	Curie (Ci)	Becquerel (Bq)
Exposure	Ionization in air	Röntgen	C/kg
Absorbed Dose	Energy absorption	Rad	Gray (Gy)
Dose - Equivalent	Biological effectiveness	rem	Sievert (Sv)