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Analytical Mechanics

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Lec.1: **Fundamental concepts vectors:**

1.1 Scalar and vector quantities
1.2 Formal definitions and rules
1.3 Magnitude of a vector

1- Fundamental concepts vectors :

Time and space are two fundamental concepts in mechanical studies. In this course we are going to consider them indepentent from each other.

Time: An absolute measurement of an ordered squence of events.

space:: The physical space is described by 3 dimensional mathematical space of Euclidean geometry.

In order to define the position of a body in space, It is necessary to have coordinate system,. The basic one is a set of 3 mutually penpendicular axis

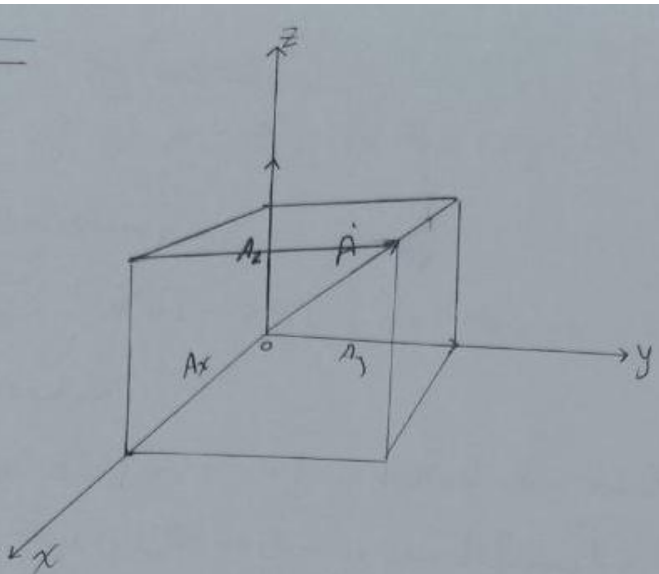
[Cartesian Coordinate system]

2- Scalar and vector Quantities ::

A scalar :- A physical quantity that is completely specified by a single magnitude such as (density, volume and temperature).

vectors :- Quantities require a direction and a magnitude for their complete specification. such as (velocity, displacement, force and acceleration).

3 Notation



A vector is represented diagramatically a directed line.

$$\vec{A} = [A_x, A_y, A_z]$$

$$\vec{A} = \hat{i} A_x + \hat{j} A_y + \hat{k} A_z$$

4- Formal Definitions and Rules

1. Equality of vectors

$$\vec{A} = \vec{B}$$

or

$$[A_x, A_y, A_z] = [B_x, B_y, B_z]$$

is equivalent to the three equations

$$A_x = B_x \quad , \quad A_y = B_y \quad ; \quad A_z = B_z$$

2. Vector Addition

The addition of two vectors is defined by the equation.

$$\vec{A} + \vec{B} = [A_x, A_y, A_z] + [B_x, B_y, B_z] =$$

$$[A_x + B_x, A_y + B_y, A_z + B_z]$$

3- Multiplication by a scalar :

$$c\vec{A} = c[A_x, A_y, A_z] = [cA_x, cA_y, cA_z] = \vec{A}c$$

4- Vector Subtraction :

$$\vec{A} - \vec{B} = \vec{A} + (-1)B = [A_x - B_x, A_y - B_y, A_z - B_z]$$

5- The Null vector

The vector $0 = [0, 0, 0]$ is called the null vector. The direction of the null vector is undefined.

6- The Commulative Law of Addition

This Law holds for vectors : that is

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

since $A_x + B_x = B_x + A_x$

7- The Associative Law

$$\begin{aligned}\vec{A} + (\vec{B} + \vec{C}) &= [A_x + (B_x + C_x), A_y + (B_y + C_y), [A_z + (B_z + C_z)]\\ &= [(A_x + B_x) + C_x, (A_y + B_y) + C_y, (A_z + B_z) + C_z]\\ &= (A + B) + C\end{aligned}$$

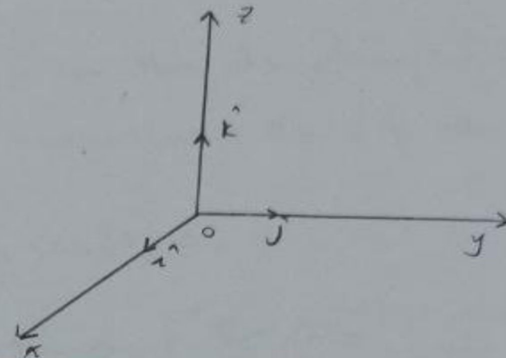
8- The Distributive Law

$$\begin{aligned}c(A + B) &= c[A_x + B_x, A_y + B_y, A_z + B_z]\\ &= [c(A_x + B_x), c(A_y + B_y], c(A_z + B_z)]\\ &= [cA_x + cB_x, cA_y + cB_y, cA_z + cB_z]\\ &= cA + cB\end{aligned}$$

5- Magnitude of a vector

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

unit coordinate vector



6- The Scalar product :- (dot product)

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} = \hat{i} A_x + \hat{j} A_y + \hat{k} A_z$$

$$\vec{B} = \hat{i} B_x + \hat{j} B_y + \hat{k} B_z$$

$$\hat{i}\hat{i} = \hat{j}\hat{j} = \hat{k}\hat{k} = 1$$ و $$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = \hat{j} \cdot \hat{k} = 0$$

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$