Lecturer: Mohanad Muayad Alyas

Analytical Mechanics

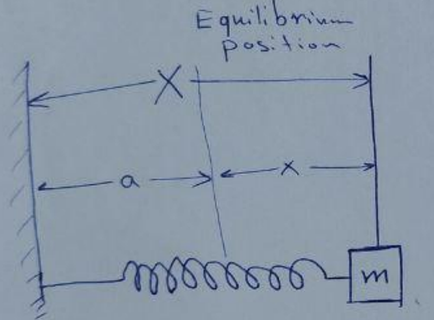
2023-2024

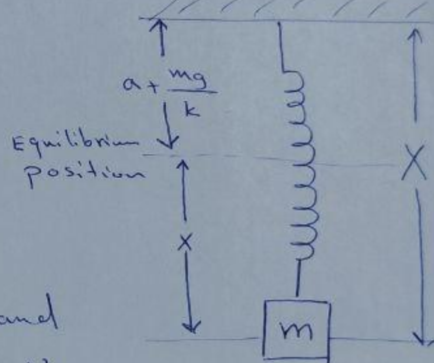
Lec.1: Linear restoring force: harmonic motion linear restoring force

2.12 Linear restoring force. Harmonic Motion linear restoring force

Is a force whose magnitude is proportional to the displacement of a particle from some equilibrium position and whose direction is always apposite to that of the displacement such a force is exerted by an elastic cord or by a spring obeying Hooke's Law

$$F = -k(X - a) = -kx$$





Where $\underline{X}$ is the total length, and

$\underline{a}$ is the unstretched (Zero load)

$\underline{x}$ is $x = X - a$ is the displacement of the spring from its equilibrium length

Fig. illustrating the linear harmonic oscillator by means of a block of mass m and a spring

a) Horizontal motion.

b) Vertical motion.

$$F = -kx \quad \text{———} \quad (1)$$

$$F = m\ddot{x} \quad \text{———} \quad (2)$$

$$m\ddot{x} + kx = 0 \quad \text{———} \quad (3)$$

Equation (3) canbe solved in anumberof ways.
It is on example of an important class of adifferential equations known as linear differential equations with constant coefficients.

$$m\frac{d^2}{dt^2}(Ae^{qt}) + k(Ae^{qt}) = 0$$

where $x = Ae^{qt}$

$$mq^2 + k = 0$$

$$q = \pm i\sqrt{\frac{k}{m}} = \pm i\omega_0$$

The general solution of equation (3)

$$x = A_+ e^{i\omega_0 t} + A_- e^{-i\omega_0 t}$$

since $e^{iu} = \cos u + i\sin u$, alternate forms of the solution are

$$x = a\sin\omega_0 t + b\cos\omega_0 t$$

$$x = A\cos(\omega_0 t + \theta_0)$$

prove that $x = A_+ e^{iw_\circ t} + A_- e^{-iw_\circ t}$ is equal to

$$x = A\cos(w_\circ t + \theta_\circ)$$

solution:-

$$x = A e^{iw_\circ t} + B e^{-iw_\circ t}$$

$$e^{iw_\circ t} = \cos w_\circ t + i\sin w_\circ t$$

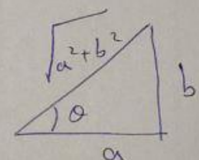
$$e^{-iw_\circ t} = \cos w_\circ t - i\sin w_\circ t$$

$$x = A(\cos w_\circ t + i\sin w_\circ t) + B(\cos w_\circ t - i\sin w_\circ t)$$

$$= (A+B)\cos w_\circ t + i(A-B)\sin w_\circ t$$

$$= a\cos w_\circ t + b\sin w_\circ t$$

$$x = \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\cos w_\circ t + \frac{b}{\sqrt{a^2+b^2}}\sin wt\right]$$



$$x = \sqrt{a^2+b^2}\left[\cos\theta_\circ \cos w_\circ t + \sin\theta_\circ \sin w_\circ t\right]$$

$$x = A'\cos(w_\circ t + \theta_\circ)$$