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Analytical Mechanics

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Lec.10: The force as a function of time: the concept of impulse

2.8 The force as a function of time. The concept of impulse.

If the force acting on a particle is known explicitly as as a function of time, then the eq. of motion

$$F(t) = m\frac{dv}{dt}$$

$$\int F(t)\,dt = mv(t) + c \quad \cdots\cdots (1)$$

$\int F(t)\,dt \longrightarrow$ impulse, it is equal to the momentum imparted to the particle by the force $F(t)$

The position of the particle as a function of time can be found by a second integration as follows

$$x = \int v(t)\,dt = \int \left[\int \frac{F(t')}{m}\,dt' \right] dt \quad \cdots (2)$$

Ex/ A block is initially at rest on a smooth horizontal surface. At time $t=0$, a constantly increasing horizontal force applied: $F = ct$.
Find the velocity and displacement as functions of time.

Solution:-

We have, for the differential eq. of motion

$$F(t) = m\frac{dv}{dt} \longrightarrow ct = m\frac{dv}{dt}$$

then $v = \frac{1}{m}\int_0^t ct\,dt = \frac{ct^2}{2m}$

and

$$x = \int_0^t \frac{ct^2}{2m}\,dt = \frac{ct^3}{6m}$$