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Analytical Mechanics

2023-2024

Lec.11: The harmonic oscillator in two and three dimensions

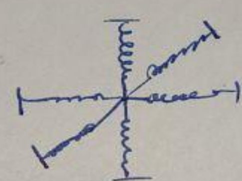
4.9 The Harmonic oscillator in two and three dimensions

A particle subject to a restoring force directed always to a fixed point

The general eq. of motion

$$\vec{F} = -k\vec{r}$$

$$m\frac{d^2\vec{r}}{dt^2} = -k\vec{r} \quad ----(1)$$



Model of a 3D H.O.

The two dimensional oscillator

Two components

$$m\ddot{x} = -kx$$

$$m\ddot{y} = -ky$$

The solution can be written as

$$x = A\cos(\omega t+\alpha) \quad , \quad y = B\cos(\omega t+\beta) \quad ---(2)$$

Where $\omega = \sqrt{\frac{k}{m}}$

A, B, α and β are constant

- To find the eq. of path, eliminate time t between the two eq.s to do so rewrite

$$y = B\cos(\omega t + \alpha + \Delta)$$

$$\Delta = \beta - \alpha$$

Then

$$y = B\left[\cos(\omega t + \alpha)\cos\Delta - \sin(\omega t + \alpha)\sin\Delta\right] \quad \text{--- (3)}$$

but from eq. (2)

$$\cos(\omega t + \alpha) = \frac{x}{A}$$

$$\sin(\omega t + \alpha) = \sqrt{1 - \frac{x^2}{A^2}}$$

eq. (3) becomes

$$\frac{y}{B} = \frac{x}{A}\cos\Delta - \sqrt{1 - \frac{x^2}{A^2}}\,\sin\Delta \quad \text{------ (4)}$$

prove

square eq4, we get

$$\frac{x^2}{A^2} - xy\frac{2\cos\Delta}{AB} + \frac{y^2}{B^2} = \sin^2\Delta \quad \text{------ (5)}$$

which is a quadratic eq.

The general quadratic eq. is

$$ax^2 + bxy + cy^2 + dx + ey = f$$

represents an ellipse, a parabolla, or hyperbola
قطع ناقص قطع مكافئ قطع زائد

depending on whether the discriminant
المقدار المميز

$b^2 - 4ac$

is $-ve$, zero or positive

$$b^2 - 4ac = \left(-\frac{2\cos\Delta}{AB}\right)^2 - 4\,\frac{1}{A^2}\,\frac{1}{B^2}$$

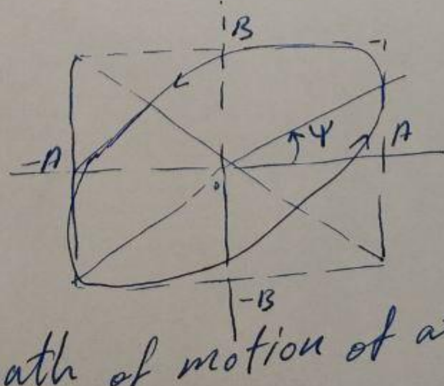
$$= \frac{4}{A^2B^2}\cos^2\Delta - \frac{4}{A^2B^2}$$

$$= \frac{4}{A^2B^2}(\cos^2\Delta - 1)$$

$$= -\frac{4}{A^2B^2}\sin^2\Delta$$

$$= -\left(\frac{2}{AB}\sin\Delta\right)^2$$

The discrinant is negative, so Path is an ellipse as shown in fig.



Elliplical path of motion of a two-dimensional H.O