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Analytical Mechanics

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Lec.11: Velocity – dependent force

2.9 Velocity - Dependent Force

In the case of fluid resistance, it is found that, for Low velocities

resistance $\propto$ velocity

For higher velocity

resistance $\propto$ square of velocity

If there are no other forces acting, the differential eq. of motion can be expressed

$$F(v) = m\frac{dv}{dt}$$

to get t as a function of v

$$t = \int \frac{m\,dv}{F(v)} = t(v)$$

نهمل

we can omit the constant of integration, since its value depends only on the choice of time origin

Ex/ Suppose a block is projected with initial velocity v_0 on a smooth horizontal plane, but there is air resistance proportional to v, that is $F(v) = -cv$, where c is a constant of proportionality.
(The x-axis is along the direction of motion)

Solution :- The differential eq. of motion is

$$-cv = m\frac{dv}{dt}$$

$$t = \int_{v_0}^{v} -\frac{m\,dv}{cv} = -\frac{m}{c}\ln\frac{v}{v_0}$$

$$\ln\frac{v}{v_0} = -\frac{ct}{m} \longrightarrow \frac{v}{v_0} = e^{-ct/m}$$

Thus the velocity decreases exponentially with time

$$x = \int_0^t v_0\, e^{-\frac{ct}{m}}\,dt = -\frac{mv_0}{c}\, e^{-\frac{ct}{m}}\Big|_0^t$$

$$x = \frac{mv_0}{c}\left(1 - e^{-\frac{ct}{m}}\right)$$