Lecturer: Mohanad Muayad Alyas

Analytical Mechanics

2023-2024

Lec.13: Variation of gravity with height

2.11

Variation of gravity with height :-

Actually, the gravitational attraction of the earth on a body above the surface falls off as the inverse square of the distance (Newton's of gravity)

$$F = -\frac{GMm}{r^2} \quad \text{------} \quad (1)$$

When :-

G is the gravitational constant.

M is the mass of the earth.

m is the mass of the body.

r is the distance from the center of the earth to the body.

$$V(r) = -\frac{GMm}{r}$$

where $F = -\frac{\partial V}{\partial r}$

$$m\ddot{r} = -\frac{GMm}{r^2} \quad \text{------} \quad (2)$$

but $\ddot{r} = \dot{r}\frac{d\dot{r}}{dr}$

$$\ddot{r} = \frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt} = \frac{d\dot{r}}{dr}\dot{r}$$

eq. 1 becomes

$$m\dot{r}\frac{d\dot{r}}{dr} = -\frac{GMm}{r^2} \rightarrow \frac{1}{2}mv^2 = \frac{GMm}{r}$$

Integrate w.r.t r

$$\frac{1}{2} m \dot{r}^2 = \frac{GMm}{r} + E \quad \text{------------} \quad (3)$$

$$\frac{1}{2} m \dot{r}^2 - \frac{GMm}{r} = E$$

E is the constant of integration

Applying eq. 3) for the case of a projectile shot upward from the surface of the earth with intial speed v_0

$$\frac{1}{2} m v_0^2 - \frac{GMm}{r_e} = E$$

$r_e \longrightarrow$ the radius of the earth.

$$\frac{1}{2} m v^2 - \frac{GMm}{r_e + X} = \frac{1}{2} m v_0^2 - \frac{GMm}{r_e} = E$$

$$\frac{1}{2} v^2 = \frac{1}{2} v_0^2 + GM\left(\frac{1}{r_e + X} - \frac{1}{r_e}\right)$$

$$v^2 = v_0^2 + 2GM\left(\frac{1}{r_e + x} - \frac{1}{r_e}\right) \quad \text{------} \quad (3)$$

where

$$r_e + X = r$$

From eq. ①, g at the surface of the earth

$$F = mg = -\frac{GMm}{r_e^2}$$

$$g = \frac{-GM}{r_e^2}$$

$$v^2 = v_0^2 + 2GM\left(\frac{r_e - r_e - x}{r_e(r_e + x)}\right)$$

$$v^2 = v_0^2 - \frac{2GMx}{r_e^2}\left(\frac{1}{\frac{r_e(r_e + x)}{r_e^2}}\right)$$

$$v^2 = v_0^2 - 2gx\left(\frac{1}{1 + \frac{x}{r_e}}\right)$$

$$v^2 = v_0^2 - 2gx\left(1 + \frac{x}{r_e}\right)^{-1}$$

For small x, Where g is uniform $\frac{x}{r_e}$ is very small

$$v^2 = v_0^2 - 2gx$$