Lecturer: Mohanad Muayad Alyas

Analytical Mechanics

2023-2024

Lec.14: Drill exercises

1. **A gun is fired straight up. Assuming that the air drag on the bullet varies quadratically with speed, show that the speed varies with height according to the equations:**

$$v^2 = Ae^{-2kx} - \frac{g}{k} \quad \textit{(upward motion)}$$

$$v^2 = \frac{g}{k} - Be^{2kx} \quad \textit{(downward motion)}$$

in which A and B are constants of integration, g is the acceleration of gravity, and $k = c_2/m$ where c_2 is the drag constant and m is the mass of the bullet. (Note: x is measured positive upward, and the gravitational force is assumed to be constant.)

sol.

Going up: $F_x = -mg - c_2v^2$

$$a = v\frac{dv}{dx} = -g - kv^2, \quad k = \frac{c^2}{m}$$

$$\int_{v_0}^{v} \frac{vdv}{-g - kv^2} = \int_0^x dx$$

$$-\frac{1}{2k}\ln\left(-g - kv^2\right)\Big|_{v_0}^{v} = x$$

$$\frac{g + kv^2}{g + kv_0^2} = e^{-2kx}$$

$$v^2 = \left(\frac{g}{k} + v_0^2\right)e^{-2kx} - \frac{g}{k}$$

Going down: $F_x = -mg + c_2v^2$

$$v\frac{dv}{dx} = -g + kv^2$$

$$\int_0^{v} \frac{vdv}{-g + kv^2} = \int_0^x dx$$

$$\frac{1}{2k}\ln\left(-g + kv^2\right)\Big|_0^{v} = x - x_0$$

$$1 - \frac{k}{g}v^2 = e^{2kx}e^{-2kx_0}$$

$$v^2 = \frac{g}{k} - \left(\frac{g}{k}e^{-2kx_0}\right)e^{2kx}$$

2. Given that the velocity of a particle in rectilinear motion varies with the displacement x according to the equation

$$\dot{x} = bx^{-3}$$

Where b is a positive constant, find the force acting on the particle as a function of x.

$$F = m\ddot{x} = m\dot{x}\frac{d\dot{x}}{dx}$$

$$\dot{x} = bx^{-3}$$

$$\frac{d\dot{x}}{dx} = -3bx^{-4}$$

$$F = m\left(bx^{-3}\right)\left(-3bx^{-4}\right)$$

$$F = -3mb^2x^{-7}$$

3. A metal block of mass m slides on a horizontal surface that has been lubricated with a heavy oil so that the block suffers a viscous resistance that varies as the 3/2 power of the speed:

$$F(v) = -cv^{3/2}$$

If the initial speed of the block is v_0 at x = 0, show that the block cannot travel farther than

$$2mv_0^{1/2}/c.$$

Sol.

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\cdot\frac{dv}{dx} = -\frac{c}{m}v^{\frac{3}{2}}$$

$$v^{-\frac{1}{2}}dv = -\frac{c}{m}dx$$

$$\int_{v_0}^{v} v^{-\frac{1}{2}}dv = \int_0^{x_{max}} -\frac{c}{m}dx$$

$$-2v_0^{\frac{1}{2}} = -\frac{c}{m}x_{max}$$

$$x_{max} = \frac{2mv_0^{\frac{1}{2}}}{c}$$