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Analytical Mechanics

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Lec.2: Energy considerations in harmonic motion

Energy Considerations in Harmonic Motion :-

Let us calculate the work W done by an external force(F_a) in moving the particle from the equilibrium position ($x=0$) to some position x

القاة الإزاحة applied force $\quad F_a = -\underset{\text{restoring force}}{F} = +kx$

$$W = \int F_a \, dx = \int_0^x (kx)\, dx = \tfrac{1}{2} k x^2$$

The work W is stored in the spring as potential energy

$$V(x) = W = \tfrac{1}{2} k x^2$$

لما نشتق ذلك $\quad F = -\dfrac{dV}{dx} = -kx$

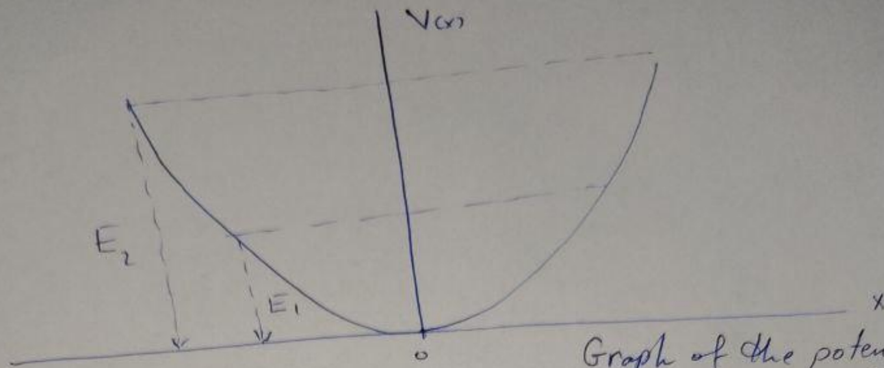
Total energy $\quad E = \tfrac{1}{2} m \dot{x}^2 + \tfrac{1}{2} k x^2$

$$\dot{x} = \left(\frac{2E}{m} - \frac{k}{m} x^2\right)^{1/2}$$ velocity as a function of displacement

Integrate to obtain t as a function of x

$$t = \int \frac{dx}{\sqrt{\frac{2E}{m} - \frac{k}{m} x^2}} = \sqrt{\frac{m}{k}} \cos^{-1}\left(\frac{x}{A}\right) + C$$

$A = \sqrt{\dfrac{2E}{k}}$, C is constant of integration.



Graph of the potential energy function of H.O

$$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2$$

When $x = 0$

$$E = \frac{1}{2} m v_{max}^2$$

when $\dot{x} = 0$

$$E = \frac{1}{2} k A^2$$

From the energy eq.

$$E = \frac{1}{2} m v_{max}^2 = \frac{1}{2} k A^2$$

or $v_{max} = \sqrt{\frac{k}{m}} A = \omega_0 A$