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Analytical Mechanics

2023-2024

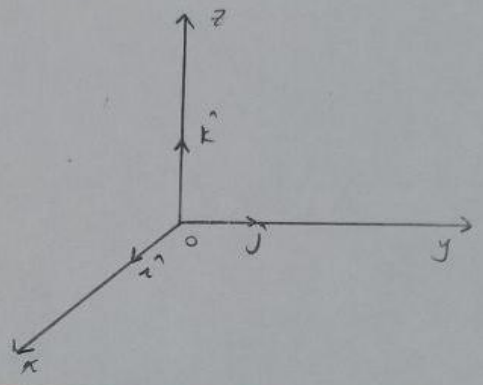
Lec.2: **Scalar and Vector product**

2.1 And Examples

5- Magnitude of a vector

$$|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

unit coordinate vector



6- The Scalar product : - (dot product)

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} = \hat{i} A_x + \hat{j} A_y + \hat{k} A_z$$

$$\vec{B} = \hat{i} B_x + \hat{j} B_y + \hat{k} B_z$$

$$\hat{i}\hat{i} = \hat{j}\hat{j} = \hat{k}\hat{k} = 1 \quad \text{و} \quad \hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$$

$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

2- Law of Cosines :-

For $\vec{C} = \vec{A} + \vec{B}$

$$\vec{C} \cdot \vec{C} = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B})$$
$$= \vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{B} + 2\vec{A} \cdot \vec{B}$$
$$C^2 = A^2 + B^2 + 2AB\cos\theta$$

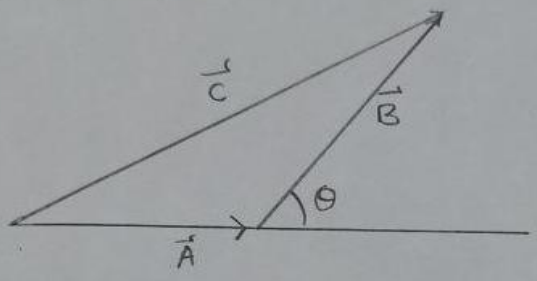


Fig 1.7 The Law of cosines

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8. The vector Product :-

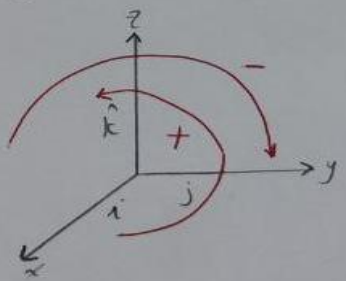
$$\vec{A} \times \vec{B} = [A_y B_z - A_z B_y , A_z B_x - A_x B_z , A_x B_y - A_y B_x]$$

Rules of cross product are

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$
$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$
$$n(\vec{A} \times \vec{B}) = (n\vec{A}) \times \vec{B} + \vec{A} \times (n\vec{B})$$

For example

$$\hat{i} \times \hat{j} = \hat{k} = -\hat{j} \times \hat{i}$$
$$\hat{j} \times \hat{k} = \hat{i} = -\hat{k} \times \hat{j}$$
$$\hat{k} \times \hat{i} = \hat{j} = -\hat{i} \times \hat{k}$$



1. Given the two vectors $\vec{A} = \hat{i} + \hat{j}$; and $\vec{B} = \hat{j} - \hat{k}$

Find

(a) $\vec{A} + \vec{B}$ and $|\vec{A} + \vec{B}|$

(b) $\vec{A} - \vec{B}$ and $|\vec{A} - \vec{B}|$

(c) $\vec{A} \cdot \vec{B}$

(d) $\vec{A} \times \vec{B}$ and $|\vec{A} \times \vec{B}|$

(e) $(\vec{A} + 2\vec{B}) \cdot (2\vec{A} - \vec{B})$

(f) $(\vec{A} + \vec{B}) \times (\vec{A} - \vec{B})$

Solution :-

a) $\vec{A} + \vec{B} = \hat{i} + \hat{j} + \hat{j} - \hat{k}$

$= \hat{i} + 2\hat{j} - \hat{k}$

$|\vec{A} + \vec{B}| = \sqrt{1 + 4 + 1} = \sqrt{6}$

b) $\vec{A} - \vec{B} = \hat{i} + \hat{j} - \hat{j} + \hat{k}$

$= \hat{i} + \hat{k}$

$|\vec{A} - \vec{B}| = \sqrt{1 + 1} = \sqrt{2}$

c) $\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j}) \cdot (\hat{j} - \hat{k})$

$= 1$