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Analytical Mechanics

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Lec.3: Examples of the cross and scalar products

3.1 Component of a vector work

3.2 Moment of a force

3.3 Geometric interpretation of the cross peoduct

7. Some Examples of the Scalar Product :-

1. Component of a vector Work :-

Work ΔW done by the force is given by the product of the component of force $\vec{F}$ in the direction of the displacement $\vec{\Delta s}$, multiplied by the magnitude Δs of the displacement,

$$\Delta W = (F \cdot \cos\theta)\, \Delta s$$

Where

θ is the angle between $\vec{F}$ & $\vec{\Delta s}$

then

$$\Delta W = \vec{F} \cdot \vec{\Delta s}$$

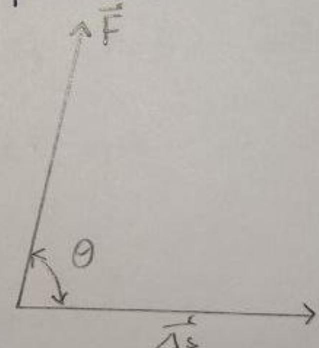


Fig. 1.6 A force under going a displacement

10- An example of the cross product: Moment of a force:-

Let a force $\vec{F}$ act at a point $p(x,y,z)$, as shown in Fig 1.9 and Let the vector $\vec{op}$ be designed by $\vec{r}$, that is

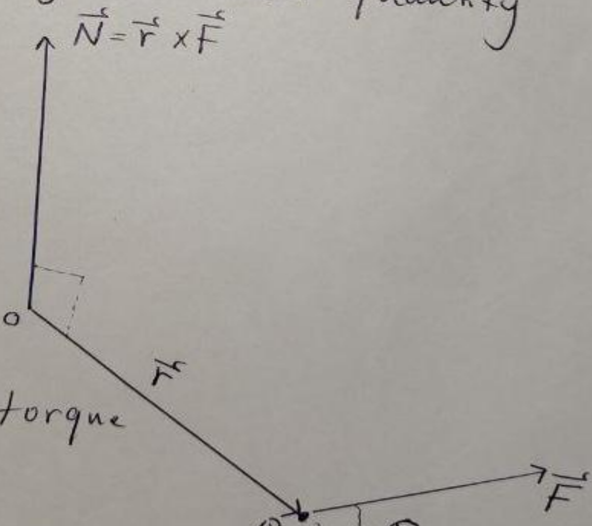
$$\vec{op} = \vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

The moment $\vec{N}$ or the Torque about a given point o is defined as

$$\vec{N} = \vec{r} \times \vec{F}$$

unit of $\vec{N}$ is nt.m vector quantity

unit of W is nt.m joule scalar quantity



The magnitude of the torque is

$$|\vec{N}| = |\vec{r} \times \vec{F}| = rF\sin\theta$$

For example

$$\hat{\imath} \times \hat{\jmath} = [1, 0, 0] \times [0, 1, 0]$$
$$= [0-0 , 0-0 , 1-0]$$
$$= [0, 0, 1] = \hat{k}$$

$$\begin{array}{ccc} i & j & k \\ 2 & & 1 \end{array}$$
$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

او تقديم وتأخير بالأعمدة لإيجاد $\hat{\jmath}$

prove

$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$
$$\hat{k} \times \hat{\imath} = \hat{\jmath}$$

9- Geometric Interpretation of the cross product :-

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \hat{\imath} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} + \hat{\jmath} \begin{vmatrix} A_z & A_x \\ B_z & B_x \end{vmatrix} + \hat{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

اشتقاقه طويل

$$|\vec{A} \times \vec{B}| = AB \sin\theta$$

1. Given the two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$; $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$

Find $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$

Solution:- $\vec{A} \cdot \vec{B} = (2)(1) + (1)(-1) + (-1)(2)$

$= 2 - 1 - 2 = -1$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}(2-1) + \hat{j}(-1-4) + \hat{k}(-2-1)$$

$$= \hat{i} - 5\hat{j} - 3\hat{k}$$

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2. Find the angle between $\vec{A}$ and $\vec{B}$

Solution:-

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{A B}$$

$$\cos\theta = \frac{-1}{\left[2^2 + 1^2 + (-1)^2\right]^{1/2} \left[1^2 + (-1)^2 + 2^2\right]^{1/2}}$$

$$\cos\theta = \frac{-1}{\sqrt{6}\,\sqrt{6}} = -\frac{1}{6}$$

Hence

$$\theta = \cos^{-1}\left(-\frac{1}{6}\right) = 99.6^\circ$$

- The condition for rotational equilibrium that the vector sum of all the moments is zero

$$\sum_i (r_i \times F_i) = \sum_i N_i = 0$$

Note :- $\vec{A} = A\hat{n}$ or $\vec{A} = |\vec{A}|\hat{n}$

Example :- Find a unit vector normal to the plane containing the two vectors $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$

solution :-

$$\because \vec{A} \times \vec{B} = |\vec{A} \times \vec{B}|\,\hat{n}$$

$$\therefore \hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{\begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix}}{|\vec{A} \times \vec{B}|}$$

$$\vec{A} \times \vec{B} = \hat{i}(2-1) - \hat{j}(4+1) + \hat{k}(-2-1)$$

$$= \hat{i} - 5\hat{j} - 3\hat{k}$$

$$\hat{n} = \frac{\hat{i} - 5\hat{j} - 3\hat{k}}{\pm\sqrt{1^2 + 5^2 + 3^2}} = \pm\left(\frac{\hat{i}}{\sqrt{35}} - \frac{5\hat{j}}{\sqrt{35}} - \frac{3\hat{k}}{\sqrt{35}}\right)$$

There could be two unit vectors