Lecturer: Mohanad Muayad Alyas

Analytical Mechanics

2023-2024

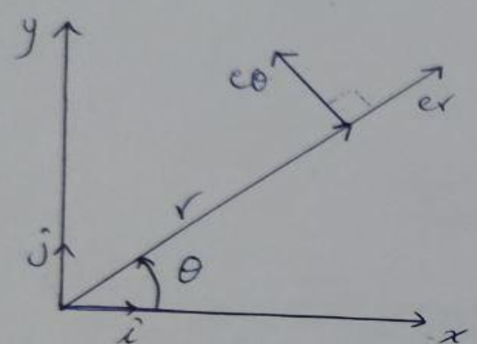
Lec.6: velocity and acceleration in plane polar coordinates

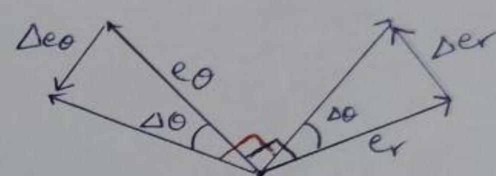
Velocity and Acceleration in plane Polar coordinates :-

It is often convenient to employ polar coordinates (r, θ)

$\vec{r} = r\,\hat{e}_r$

$\vec{V} = \frac{d\vec{r}}{dt} = \dot{r}\,\hat{e}_r + r\frac{de_r}{dt}$ ------- ①





unit vectors for plane coordinates

$\Delta e_r \simeq \hat{e}_\theta\,\Delta\theta$

$\frac{de_r}{dt} = \hat{e}_\theta\,\frac{d\theta}{dt}$ ------- ②

$\Delta e_\theta = -\hat{e}_r\,\Delta\theta$

$\frac{de_\theta}{dt} = -\hat{e}_r\,\frac{d\theta}{dt}$ ------ ③

sub. eq. ② in eq. ①, we get

$\vec{V} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,e_\theta$

$$\vec{a} = \frac{d\vec{V}}{dt} = \ddot{r}\, e_r + \dot{r}\, \frac{de_r}{dt} + (\dot{r}\dot{\theta} + r\,\ddot{\theta})\, e_\theta + r\dot{\theta}\, \frac{de_\theta}{dt} \quad \ldots\ldots (4)$$

sub. eq. (2) and eq. (3) in eq. 4, weget

$$a = (\ddot{r} - r\dot{\theta}^2)\, e_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\, e_\theta$$

Thus the magnitude of the radial component of the acceleration vector is

$$a_r = \ddot{r} - r\dot{\theta}^2$$

and that of the transverse component is

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

Example:- A particle moves on a spiral path such that the position in polar coordinates is given by

$$r = bt^2 \quad ; \quad \theta = ct$$

where b and c are constant

$$\vec{V} = e_r \frac{d}{dt}(bt^2) + e_\theta\, (bt^2)\frac{d}{dt}(ct)$$

$$\vec{V} = (2bt)\, e_r + (bct^2)\, e_\theta$$

$$\vec{a} = e_r(2b - bt^2c^2) + e_\theta\, [0 + 2(2bt)c]$$

$$\vec{a} = b(2 - t^2c^2)\, e_r + 4bct\, e_\theta$$