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Analytical Mechanics

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Lec.9: The concepts of kinetic and potential energy

2.7 The concepts of Kinetic and potential energy :-

If force is independent of velocity or time then the differential equation of motion for rectilinear motion is simply

$$F(x) = m\ddot{x}$$

$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{dx}{dt} \cdot \frac{d\dot{x}}{dx} = v\frac{dv}{dx}$$

So the differential equation of motion may be written

$$F(x) = mv\frac{dv}{dx} = \frac{m}{2}\frac{dv^2}{dx} = \frac{d}{dx}\left(\frac{1}{2}mv^2\right) = \frac{dT}{dx} \quad \text{----} (1)$$

where $T = \frac{1}{2}mv^2 \Rightarrow$ Kinetic energy of the particle

Equation (1) can be expressed

$$F(x) = \frac{dT}{dx}$$

$$\int F(x)\,dx = \int dT = \frac{1}{2}m\dot{x}^2 + \text{const.}$$

$\int F(x)\,dx \Rightarrow$ the work done on the particle by the impressed force $F(x)$

Let us define a function $V(x)$ such that

$$-\frac{dV}{dx} = F(x)$$

$V(x) \Rightarrow$ potential energy function

$$T = \int F(x)\,dx = -\int \frac{dV}{dx}\,dx = -V(x) + \text{constant}$$

then $T + V = \frac{1}{2} m v^2 + V(x) = \text{constant} = E$ ----- (2

$E \Longrightarrow$ Total energy

In words :-

The sum of kinetic and potential energy remains in this case is said to be conservative.

No conservative forces, that is, those for which no potential energy function exists, are usually of a dissipational nature, such as friction.

The motion of a particle can be obtained by solving the energy equation (2

$$v = \frac{dx}{dt} = \pm\sqrt{\frac{2}{m}\left[E - V(x)\right]}$$

$$t = \int \frac{\pm\, dx}{\sqrt{\frac{2}{m}\left[E - V(x)\right]}}$$

Ex/ The motion of a freely falling body discussed previously under the case of a constant force is a special case of conservative motion. If we choose the x direction to be positive upward, then the gravitational force is equal to $-mg$ and the potential energy function is $V = mgx + c$.

Here c is an arbitrary constant whose value depends merely on the choice of the reference level for V.

~~Forxezo~~

Solution: -

$$\left.\begin{aligned} F &= -mg \\ F &= -\frac{dV}{dx} \end{aligned}\right\} -mg = -\frac{dV}{dx}$$

$$\int dV = \int mg\,dx \Longrightarrow V = mgx + c$$

For $c = 0$, the total energy is just

$$E = \frac{1}{2} m \dot{x}^2 + mgx$$

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m \dot{x}^2 + mgx$$

if $\dot{x} = 0$

$$\frac{1}{2} m v_0^2 = mgx \longrightarrow \boxed{h = X_{max} = \frac{v_0^2}{2g}}$$

$$E = \frac{1}{2} m v_0^2 = \frac{1}{2} m \dot{x}^2 + mgx$$

$$v_0^2 - 2gx = \dot{x}^2 \longrightarrow \dot{x} = \frac{dx}{dt} = \sqrt{v_0^2 - 2gx}$$

$$\int_0^t dt = \int_0^x \frac{dx}{\sqrt{v_0^2 - 2gx}}$$

$$t = \int_0^x (v_0^2 - 2gx)^{-1/2}\, dx$$

$$t = -\frac{1}{2g} \int_0^x (v_0^2 - 2gx)^{-1/2} (-2g)\, dx$$

$$t = -\frac{1}{\cancel{2}g} (v_0^2 - 2gx)^{1/2}\, \cancel{2} \Big|_0^x$$

$$t = -\frac{1}{g} (v_0^2 - 2gx)^{1/2} \Big|_0^x$$

$$t = -\frac{1}{g} (v_0^2 - 2gx)^{1/2} + \frac{1}{g} (v_0^2)^{1/2}$$

$$t = \frac{v_0}{g} - \frac{1}{g} (v_0^2 - 2gx)^{1/2}$$

Verfy above eq.

$$x = \frac{1}{2} a t^2 + v_0 t + x_0 \quad \text{------} \; ①$$

$$F = ma = -mg$$

$$a = -g$$

$$g = -a$$

Sub. in eq. ①, we get

$$t = \frac{v_0}{-a} - \frac{1}{-a}\left(v_0^2 - 2(-a)x\right)^{1/2}$$

$$t + \frac{v_0}{a} = \frac{1}{a}\left(v_0^2 + 2ax\right)^{1/2}$$

$$x = \frac{1}{2} a t^2 + v_0 t + 0$$