University of Mosul

College of Science

Department of Physics

Second stage

Lecture 10

Sound and wave Motion

2024-2025

Lecture 10: summary of Damping

Preparation

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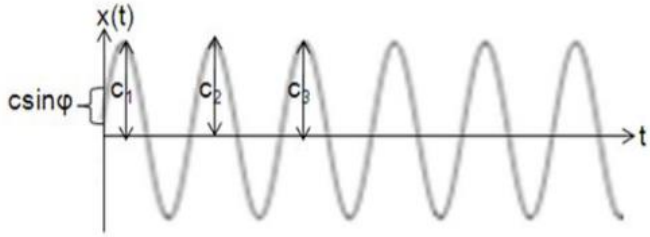
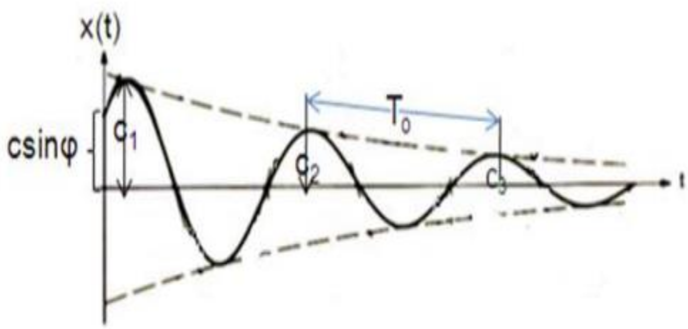
A summary of the results obtained in the four cases that represent the special cases of the general solution to an equation Decreased harmonic motion as in Figure (8):

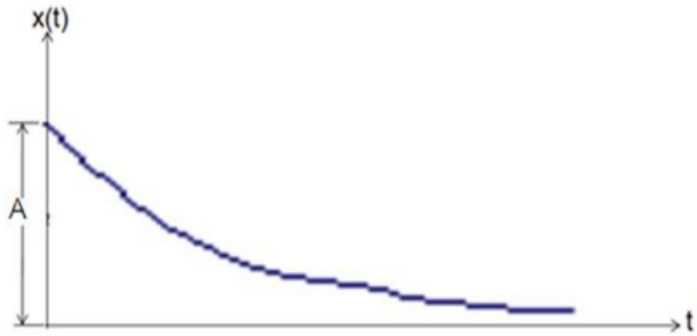
Motion Equation (14) Vibratory motion, amplitude of motion c is the periodic time constant T_0 is the natural frequency constant f.



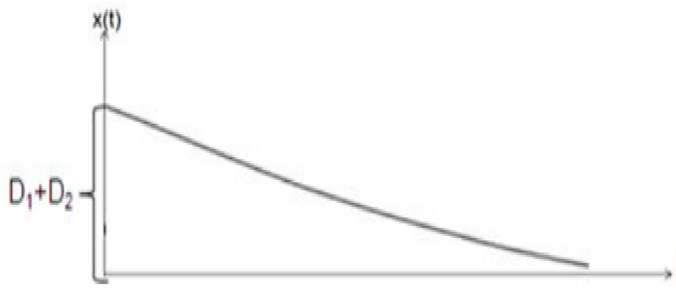
Motion Equation (17) Motion is oscillatory amplitude Ce^{-rt} decreases with time. Motion T<To periodic time indecision $f > f_0$.



Equation of Motion (21) Motion is non-vibratory The particle returns to its equilibrium position in the shortest time maybe.



Motion Equation (22) Non-vibrational motion of a particle It returns to its equilibrium position slowly over a period of time The time it takes in critical condition.



Example: An oscillator consists of a particle and a helical spring. As it vibrates along the sinusoidal axis, it is subjected to two forces. The first is the restoring force, which is proportional to the instantaneous displacement x, and the second is the damping force, which is proportional to the instantaneous velocity v. If the restoring force is numerically equal to 40x dyne and the damping force is equal to 200 dyne when its instantaneous velocity is 10 S/cm, assuming that the mass of the particle is 5 g, and that it starts its motion from rest at point A, 20 cm from the equilibrium position. Find the following:

1- The differential equation for the movement of the oscillator and the conditions that describe its movement.

2. The instantaneous position of the particle at any time.

3. Amplitude, periodic time, and frequency of fading oscillations.

4. Draw the movement graphically.

5. Natural frequency and natural periodic time of vibration.

solution

1. Since the restoring force is equal to

$Fk = -kx = -40x$

The resistance constant, R, is equal to

$R=F_R/v=200dyne/10cm/S=20dyne/cm/S$

Therefore, the quench force or resistance F_R and its magnitude is

$F_R=-Rv=-20(\frac{dx}{dt})$

Applying Newton's second law of motion, it turns out that

$\sum F=m(\frac{d^2x}{dt^2})$

$m(\frac{d^2x}{dt^2})= -R(\frac{dx}{dt})-kx$

$5(\frac{d^2x}{dt^2})=-20(\frac{dx}{dt})-40x$ ………………………………….1

That is, by dividing both sides of equation (1) by 5 and arranging its limits, we get:

$(\frac{d^2x}{dt^2})+4(\frac{dx}{dt})+8x=0$ ……………………………………..2

This is the differential equation of motion of the oscillator.

2-To find the instantaneous position x of the particle at any time t, the solution to the equation of motion (2) must be found. We have The initial conditions for motion are.

X=20 at the moment in time t=0

V=0 at the moment in time t=0

We compare Equation (2) with the standard equation for fading harmonic motion, and we find that

$\frac{d^2x}{dt^2}+2r\,(\frac{dx}{dt})+\omega^2\,x=0$

$\omega^2=8$, $2r=4$, $r^2=4$

This indicates that the value $\omega^2>r^2$ That is, the oscillatory motion is diminished and the appropriate solution is

$x = e^{-rt}(A\cos W_0\,t + B\sin W_0\,t)$ 3

We substitute the appropriate values

$\omega_o=(\omega^2-r^2)^{1/2}=(8-4)^{1/2}=2$

r=2 , In equation (3), it becomes:

$x=e^{-2t}(A\cos 2t+B\sin 2t)$.................................4

To find the values of A,B, we substitute the initial conditions into equation (4), so when we substitute the first initial condition, we find that The capacitance A is equal to :

20=1(A+0) → A=20cm

To compensate for the second condition, we must differentiate equation (4) with respect to time, so we get:

$V= \;\; =-2e^{-2t}(A\cos 2t+B\sin 2t) + e^{-2t}(-2A\cos 2t+B\sin 2t)$

0=-2(A+0)+(0+2B)=-2(20)+(2B)=-40+2B

40=2B B=20cm

Thus, we find that the instantaneous position x of the particle at any time t is

$x=e^{-2t}(20\cos 2t+20\sin 2t)$

And the previous solution can be expressed in another way, so we have the relation

$A\cos 2t + B\sin 2t = C\cos(2t-\varphi)$

Whereas

$C=(A^2+B^2)^{1/2}=(20^2+20^2)^{1/2}=20(2)^{1/2}$cm

and that

$\tan\varphi=A/B=20/20=1$

$\varphi=\arctan(1)=\pi/4$

Therefore, the general solution becomes as follows:

$x=20(2)^{1/2}e^{-2t}\cos(2t-\pi/4)$ ……………………………………5

This equation enables us to find the position of the particle x at any point in time t.

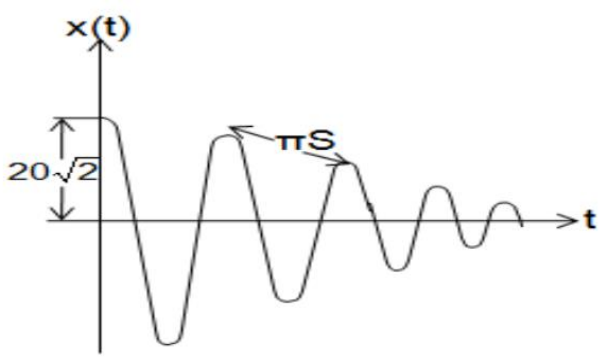
3-From equation (5), we find that the amplitude, periodic time and frequency of the decaying motion is equal to:

$C=20(2)^{1/2}e^{-2t}$ cm

$T_o=2\pi/\omega_o=2\pi/2=\pi S$

$f_o=\omega_o/2\pi=2/2\pi=1/\pi Hz$

4 -The movement of the particle can be represented graphically through equation (5) by plotting the instantaneous displacement x with time t We find from this figure that the amplitude of vibration decreases gradually with time.



5-To find the natural frequency and the periodic time in the absence of damping (or resistance), we substitute t = 0 into the equation (3) Putting

$\omega = \omega_o$, we get. $x=e-^{2t}(A\cos 2t+B\sin 2t)$ whereas $\omega^2=k/m$

Among them we find that

$\omega^2=k/m=40/5=8$ $\quad$ $,\omega=2(2)^{1/2}$

Among them, we find the natural periodic time T and the natural frequency f as follows:

$\omega=2\pi/T$

$T=2\pi/\omega=2\pi/2(2)1/2=\pi/(2)1/2S$ $\quad$, $\quad$ $f=1/T=(2)1/2/\pi Hz$

Question: It is the value of the decay constant r with respect to the value of ω that determines the nature of the motion of the oscillator, explain that.

H.W.

1. When the value of r is greater than the value of ω, no vibration occurs, and if the oscillator is removed from its equilibrium position and left free, It slowly returns to its equilibrium position.

2. If the value of r = ω, then the movement is critical, meaning that this value separates two behaviors, which are either vibratory behavior or non-vibrational behavior. In the case of r = ω, if the oscillator is removed from its equilibrium position and left free, it returns to a position Balance it in the shortest possible time (without vibration).

3. When the value of r is less than the value of ω, then the oscillator can vibrate, but its vibration is weak, i.e. Its capacity is gradually reduced. And the speed of capacitance decrease depends on the value of r, and in an extreme case when r is non-existent (0=r). The oscillator continues to oscillate and its amplitude remains constant

Tenth lecture