(2-10) Archimedes' Principle

Archimedes' principle states that a body immersed in a fluid is buoyed up by a force that is equal to the weight of the fluid displaced. This principle can be verified with the help of figure 2-9.

If we submerge a cylindrical body into a fluid, such as water, then the bottom of the body is at some depth $\mathbf{h_1}$ below the surface of the water and experiences a water pressure $\mathbf{P_1}$ given by

$$P_1 = \rho\ g\ h_1$$

where $\boldsymbol{\rho}$ is the density of the water. Because the force due to the pressure acts equally in all directions, there is an upward force on the bottom of the body. The force upward on the body is given by

$$F_1 = P_1 A \qquad \dots\dots\dots\dots\dots\dots\dots (2-22)$$

where **A** is the cross-sectional area of the cylinder. Similarly, the top of the body is at a depth $\mathbf{h_2}$ below the surface of the water, and experiences the water pressure $\boldsymbol{P_2}$ given by

$$P_2 = \rho\ g\ h_2$$

However, in this case the force due to the water pressure is acting downward on the body causing a force downward given by

$$F_2 = P_2 A \qquad \dots\dots\dots\dots\dots\dots\dots (2-23)$$

Because of the difference in pressure at the two depths, $\mathbf{h_1}$ and $\mathbf{h_2}$, there is a different force on the bottom of the body than on the top of the body. Since the bottom of the submerged body is at the greater depth, it experiences the greater force. Hence, there is a net force upward on the submerged body given by

$$Net\ force\ upward = F_1 - F_2$$

Replacing the forces $\mathbf{F_1}$ and $\mathbf{F_2}$ by their values in equations 2-22 and 2-23, this becomes

$$Net\ force\ upward = P_1 A - P_2 A$$

Replacing the pressures $\mathbf{P_1}$ and $\mathbf{P_2}$, this becomes

$$Net\ force\ upward = \rho\ g\ h_1 A - \rho\ g\ h_2 A$$

$$Net\ force\ upward = \rho\ g\ A(h_1 - h_2)$$

But, $\boldsymbol{A}\,(\boldsymbol{h_1} - \boldsymbol{h_2}) = \boldsymbol{V}$ the volume of the cylindrical body, and hence the volume of the water displaced. Above equation thus becomes

$$Net\ force\ upward = \rho\ g\ V$$

But, **ρ=m/V**, thus

$$Net\ force\ upward = \frac{m}{V}\ g\ V = m\ g$$

But **mg = w**, the weight of the water displaced. Hence,

Net force upward = Weight of water displaced

The net force upward on the body is called the **buoyant force (BF)**. When the buoyant force on the body is equal to the weight of the body, the body does not sink in the water but rather floats. Since the buoyant force is equal to the weight of the water displaced, **a body floats when the weight of the body is equal to the weight of the fluid displaced.**

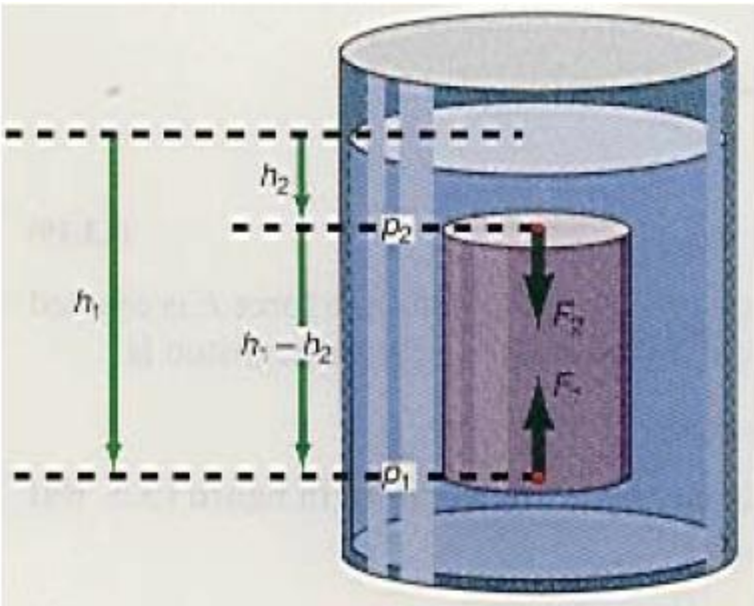

Figure 2-9

Example 21: A block of oak wood **5 cm** high, **5 cm** wide and **10 cm** long is placed into a tub of water. The density of the wood is **7.2×10^2 kg/m^3**. How far will the block of wood sink before it floats?

Answer:

Example 22: A block of iron **5 cm** high, **5 cm** wide and **10 cm** long is placed into a tub of water. The density of the iron is **7860 kg/m^3**. Will this
Iron body now float or will it sink?

Example 23: Find the buoyancy force on **2×10^{-4} m^3** of iron immersed in water?

Example 24: Find the buoyancy force **20 kg** of Iron immersed in water? the density of Iron **7860 kg/m^3**.

Example 25: Find the buoyancy force **75 N** of copper immersed in water? the density of copper **8960 kg/m^3**.

Example 26: If the average density of a human being is **980 kg/m^3** what fraction of a human body floats above water in a fresh water pond and what fraction floats above seawater in the ocean? The specific gravity of seawater is **1.025**.