University of Mosul

College of Science

Department of Physics

Second stage

Lecture 11

Sound and wave Motion

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Lecture 11: **Applications of S.H.M**

Preparation

M. Maysam Shihab Ahmed

Chapter FIVE

forced vibration

introduction

So far, our study has been limited to studying free decaying and non-decaying vibration, as we found that when a non-decaying oscillator is removed from its equilibrium position and left free, it will vibrate at a frequency that depends on the elastic constants. And inertia, this frequency is called free frequency or natural frequency. This is what helps the free vibration continue, is the energy stored in the oscillator at the beginning of the vibrating movement, but because of the inevitable frictional resistance that is always present, no matter how small, the amplitude of the vibration will change and gradually diminish with time until the vibrator stops vibrating.

In order to keep the vibration going, the oscillator must be continually supplied with energy to overcome the impact of the vibration Frictional resistance. "And if the means is to supply the vibrating energy in the form of a periodic external force, then the vibrator is said to be It is in a state of forced shaking or forced shaking." Perhaps the simplest and most familiar example of forced vibration is movement The swing, the rocking swing, if left alone, will stop rocking after a while. This is due to the continuous loss of energy as a result of friction, but if given in successive small batches and at intervals Suitable time they will continue to vibrate as a result of continuous compensation for lost energy. In fact, if it is better The timing of the impulses so that they are effective in the same direction as the movement and not opposite it, the amplitude of the vibration is large. And there There are many practical examples of forced vibration, including the vibration of the bridge under the footsteps of a military column at the crossing. And the vibration of the engine structure as a result of the periodic strikes of the pistons inside the combustion cylinders, and the vibration of musical instruments, String, pneumatic and thin-film types when excited mechanically or electrically. The problem of forced vibration is a general question in physics. The solution to this problem is not limited to vibrational and wave movements, but extends to many other fields in acoustics, alternating current circuits, and atomic physic.

The equation of motion for a decaying oscillator under the influence of a periodic external force We will consider here the oscillator consisting of a particle of mass m attached to the end of a spiral spring of fixed elasticity k and its end fixed

the other with provisions as in Figure (1).

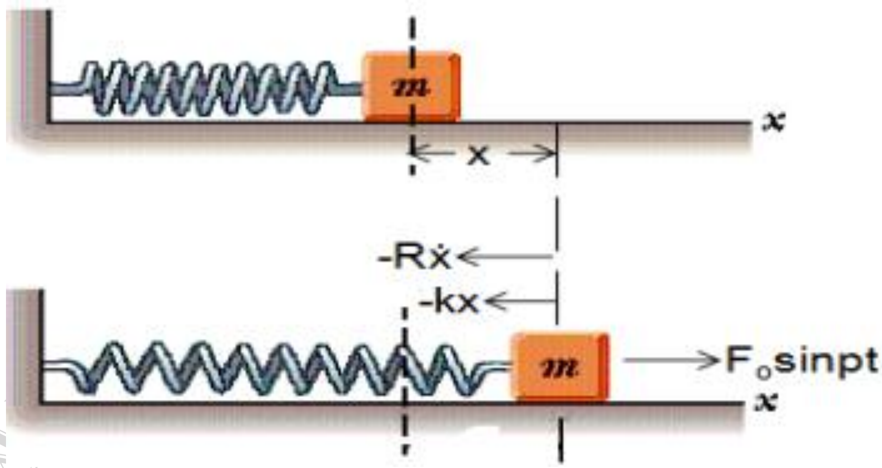

Figure (1) A decaying oscillator under the influence of a periodic external force

Suppose that the particle vibrates in air (or any viscous medium) at a low speed such that the frictional resistance experienced by the particle is directly proportional to its velocity. We assume that the particle is subjected to a periodic external force F_p of $F_o$sinpt where p is the angular frequency of this force. This frequency is completely independent of the natural angular frequency ω or frequency angular decay ω_o. This rotating force feeds the vibrator with energy to compensate for the energy it loses during the movement. The vibrating particle in this case is simultaneously subject to three different forces, which is the restoring force F_k=-kx, the frictional force $F_R = -R\dot{x}$,the periodic external force F_p =$F_o$sinpt, and the resultant of this force acting in an extension The x-axis is.

$$\sum F = F_p + F_R + F_k \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Now we apply Newton's second law of motion and substitute the values of the forces acting on the vibrating particle into equation (1), It turns out that:

$$m\left(\frac{d^2x}{dt^2}\right) = F_o sinpt - R\left(\frac{dx}{dt}\right) - kx \qquad (2)$$

Divide both sides of equation (2) by m and arrange it to become:

$$\left(\frac{d^2x}{dt^2}\right) + \frac{R}{m}\left(\frac{dx}{dt}\right) + \left(\frac{k}{m}\right)x = \left(\frac{F_o}{m}\right)sinpt \qquad (3)$$

And we assume that $(f_o = F_o/m)$ And as it was $(\omega^2 = k/m)$, $(2r = R/m)$ So equation (3) becomes as follows:

$$\left(\frac{d^2x}{dt^2}\right) + 2r\left(\frac{dx}{dt}\right) + \omega^2 x = f_o sinpt \qquad (4)$$

Solve the Forced Motion Equation (Special Solution) To solve this equation, we must remember that an external force applied at an angular frequency p will force the particle to vibrate at this Frequency, so each term of this equation on the left side must include a function that changes harmonic with Time with an angular frequency p For this purpose, we will impose the experimental solution by considering the instantaneous displacement x changes harmonic with time According to the following equation:

$$x = Asinpt + Bcospt \qquad (5)$$

To test the validity of this solution we find,($\frac{d^2x}{dt^2}$,($\frac{dx}{dt}$)) to equation (5) and substitute it into equation (4):

$$\frac{dx}{dt} = pAcospt - pBsinpt$$

$$\frac{d^2x}{dt^2} = -p^2Asinpt - p^2Bcospt$$

$$-p^2Asinpt - p^2Bcospt + 2rpAcospt - 2rpBsinpt + \omega^2Asinpt + \omega^2Bcospt = f_o sinpt$$

We arrange this equation to become:

$$(-p^2A - 2rpB + \omega^2A)sinpt + (-p^2B + 2rpA + \omega^2B)cospt = f_o sinpt \qquad (6)$$

If the assumed solution is correct, then the right side of this equation must equal the left side at any instant of time t. This means that the coefficient of sin(pt) on the left side must equal the coefficient of sin(pt) on the right side at any instant of time t, and also to equal the coefficients of cos(pt) on both sides. For the coefficients sin(pt) and cos(pt), we find that:

$$-p^2A - 2rpB + \omega^2A = f_o$$

$$-p^2A - 2rpB + \omega^2A = 0$$

Solving these two simultaneous equations, we get the values of A and B as follows:

$$A = \frac{(\omega^2 - p^2)f_o}{(\omega^2 - p^2)^2 + (2rp)^2} \qquad (7)$$

$$B = \frac{-2rpf_o}{(\omega^2 - p^2)^2 + (2rp)^2} \qquad (8)$$

Substituting A and B into the solution to equation (5), we find that:

$$x = \left[\frac{(\omega^2 - p^2)f_o}{(\omega^2 - p^2)^2} + (2rp)^2\right] sinpt - \left[\frac{2rpf_o}{(\omega^2 - p^2)^2} + (2rp)^2\right] cospt \qquad (9)$$

In order to express this solution in a simpler mathematical way and in a way that facilitates the physical interpretation of the behavior of the vibrating particle, The directional method is preferred for solution reduction. For this purpose we have the instantaneous displacement equation x which is :

$$x = Asinpt + Bcospt$$

And we have the equation for the periodic external force, F_o, which is

$$F = F_o sinpt$$

From the comparison of the two equations, we notice that the force vector Fo is in the same direction as the displacement component A because both are positive and multiplied by sinpt, so it is said that both are in the same phase. This is expressed in Figure (2).

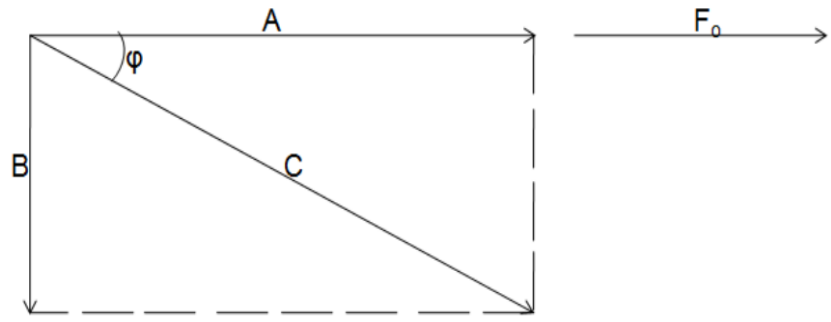

Figure (2) shows the phase angle between the driving force vector of the oscillator and the displacement vector.

As for the displacement component B, it is multiplied by the cospt, so it is definitely out of phase at an angle 90^0 on F_o. But o The B sign is negative in equation (8) so it must be lagging behind the F_o phase by an angle 90^0 from Figure (2). The resultant C and the phase angle θ can be found.

where

$$C = \sqrt{(A^2 + B^2)} \qquad (10)$$

$$\tan\theta = \frac{B}{A} \qquad (11)$$

Thus, equation (5) takes the following form:

$$x = C sin(pt + \theta) \qquad (12)$$

We substitute the values of A and B from equations (7) and (8) into equations (10) and (11), and we get:

$$C = \frac{f_o}{\sqrt{(\omega^2 - p^2)^2 + (2rp)^2}} \quad (13)$$

$$\tan\theta = \frac{-2rp}{(\omega^2 - p^2)} = 2rp/(p2 + \omega2) \quad (14)$$

$$x = \frac{f_o sin(pt + \theta)}{\sqrt{(\omega^2 - p^2)^2 + (2rp)^2}} \quad (15)$$

The last equation (15) represents a valid solution to the equation of motion (4) as long as it agrees with it without decreasing, and this solution It represents a steady state solution because it continues in one pattern and does not change over time. It represents the particle's response to vibration force at an angular frequency p (the frequency of the vibrating external force) without regard to the initial conditions of motion or to the natural frequency of the oscillator. Therefore, this solution does not represent a general case for the equation of motion because it does not contain constants Optional determined by the initial conditions for the motion of the oscillator and this solution is called the special solution. complementary solutions (transient solutions) There is another correct solution to the equation of motion (4). It is the solution resulting from the free movement of the oscillator without the influence of force external to it.

That is, he solved equation (4) when the right-hand side equals zero. We have already found that the general solution for such a case is. We have found that in the case of frictional resistance, this is what is expected in all practical cases. The oscillator does not continue to move or vibrate freely, but rather stops after a period of time. Equation (5) has three different solutions when the right side is equal to zero and each solution depends on the relative values of the resistance coefficient r and the natural frequency of the oscillator ω. These solutions are mathematically called complementary solutions, and these solutions are:

1. When ω>r the complement solution is:

$$x = Ce^{-rt} sin(\omega o t + \theta) \qquad (16)$$

This solution represents the state of vibration incomplete decay.

2. and when r = ω the complement solution is:

$$x = e^{-rt}(A + B/t) \qquad (17)$$

This solution represents the critical motion condition.

3. and when ω< r the complement solution is:

$$x = e^{-rt}\left(D_1 e^{+\sqrt{(r^2-\omega^2)}t} + D_2 e^{-\sqrt{(r^2-\omega^2)}t}\right) \qquad (19)$$

This solution represents the case of excess motion decay.

In all of these solutions, it is noted that the value of the displacement x approaches zero with the passage of time t. So it's like this Solutions are described physically as transient solutions because they represent temporary states and last only for a short period of time.

general solutions:

Since the equation of motion (5) is a linear equation, therefore its general solution is the sum of the special and complementary solutions. On this basis, there are three complete general solutions to equation (5), and each solution depends on the relative values of r and ω.

1. When ω>r, the complete solution is:

$$x = \frac{Ce^{-rt}sin(\omega_o t + \theta) + f_o sin(pt + \theta)}{\sqrt{(\omega^2 - p^2)^2 + (2rp)^2}} \qquad (20)$$

2. When $r = \omega$, the complete general solution is:

$$x = \frac{e^{-rt}(\frac{A + B}{t}) + f_o sin(pt + \theta)}{\sqrt{(\omega^2 - p^2)^2 + (2rp)^2}} \qquad (21)$$

3. When $\omega<r$, the complete general solution is:

$$x = e^{-rt}\left(D_1 e^{+\sqrt{(r^2-\omega^2)}t} + D_2 e^{-\sqrt{(r^2-\omega^2)}t}\right) + \frac{f_o sin(pt + \theta)}{\sqrt{(\omega^2 - p^2)^2 + (2rp)^2}} \qquad (22)$$

It is noted that each of these general solutions consists of two terms, the first term is mathematically called the complementary solution and represents Physically the transient part of the general solution, and the second term is called mathematically the special solution and physically represents the stable part from the general solution.

The complete general solution corresponding to this case can be represented graphically as shown in Figure (3). Where part (a) corresponds to the term The first is from the solution, and it represents the state of free waning vibration, and it is noted that it fades after a short period of time This part of the solution is a transient state. And part (b) of the figure corresponds to the second term of the solution and it is noted that it continues vibrating with the same amplitude and does not change with time as long as the periodic force applied to the oscillator is continuous effect, so this part is called the steady-state solution. And part (c) represents the sum of the two transient and stable solutions, It is noted that the effect of the transient part appears for a short period from the start of the vibration, then it disappears completely and the part remains The stable of the solution is dominant in its effect on the vibrator .

In the study of forced vibration, the transient part of the general solution is usually neglected after a short period of time has passed from initiation vibration, and attention is then focused on a steady-state solution that completely controls the behavior of the oscillator.

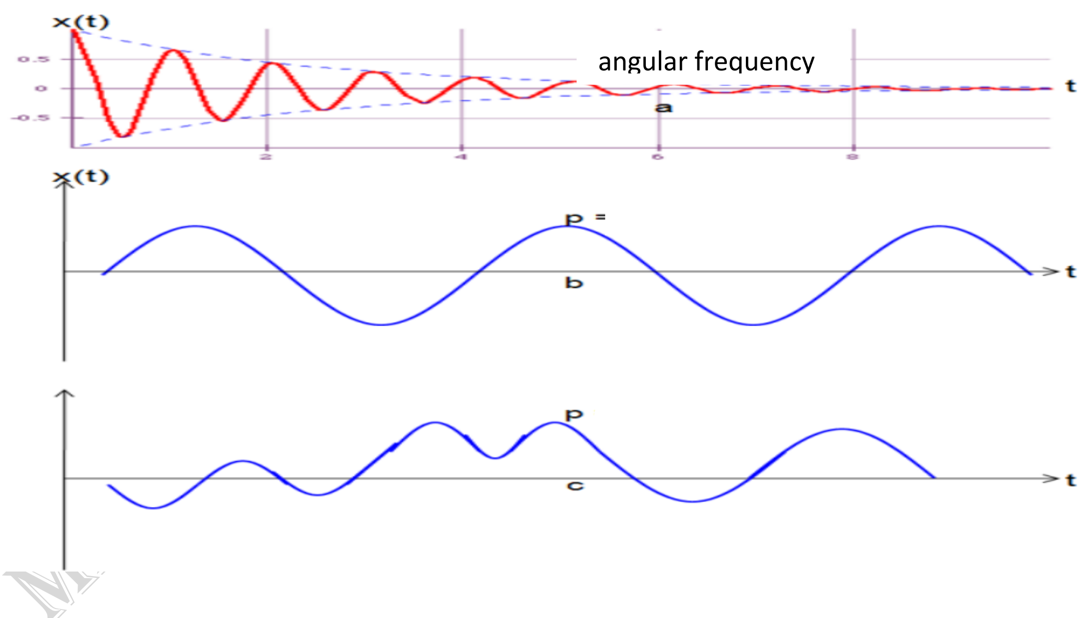

Figure (3) Solutions of the equation of forced vibrational motion (a) representing the transient solution (b) the steady-state solution (c) the solution The complete result from the superposition of the two solutions (a) and (b).