Chapter Three

Fluid in motion

(3-1) The Equation of Continuity

Up to now, we have studied only fluids at rest. Let us now study fluids in motion, the subject matter of **hydrodynamics**. The study of fluids in motion is relatively complicated, but the analysis can be simplified by making a few assumptions. Let us assume that the fluid is incompressible and flows freely without any turbulence or friction between the various parts of the fluid itself and any boundary containing the fluid, such as the walls of a pipe. A fluid in which friction can be neglected is called a **nonviscous fluid***.* A fluid, flowing steadily without turbulence, is usually referred to as being in **streamline flow***.* The rather complicated analysis is further simplified by the use of two great conservation principles: the conservation of mass, and the conservation of energy. The law of **conservation of mass** results in a mathematical equation usually called the **equation of continuity**. The law of **conservation of energy** is the basis of **Bernoulli's theorem***,* the subject matter of section 3-2.

Let us consider an incompressible fluid flowing in the pipe of figure 3-1. At a particular instant of time the small mass of fluid **Δm**, shown in the left-hand portion of the pipe will be considered. This mass is given by a slight modification of equation 2-1, as

$$\Delta m = \rho\, \Delta V \qquad \dots\dots\dots\dots\dots\dots\dots (3-1)$$

Because the pipe is cylindrical, the small portion of volume of fluid is given by the product of the cross-sectional area $\mathbf{A_1}$times the length of the pipe $\mathbf{\Delta x_1}$ containing the mass **Δm**, that is,

$$\Delta V = A_1 \Delta x_1 \qquad \dots\dots\dots\dots\dots\dots\dots (3-2)$$

The length $\mathbf{\Delta x_1}$ of the fluid in the pipe is related to the velocity $\boldsymbol{v_1}$ of the fluid in the left-hand pipe. Because the fluid in $\mathbf{\Delta x_1}$ moves a distance $\mathbf{\Delta x_1}$ in time $\boldsymbol{\Delta t}$, $\mathbf{\Delta x_1 = v_1 \Delta t}$. Thus,

$$\Delta x_1 = v_1 \Delta t \qquad \dots\dots\dots\dots\dots\dots\dots (3-3)$$

Substituting equation 3-3 into equation 3-2, we get for the volume of fluid,

$$\Delta V = A_1 v_1 \Delta t \qquad \dots\dots\dots\dots\dots\dots\dots\dots (3-4)$$

Substituting equation 3-4 into equation 3-1 yields the mass of the fluid as

$$\Delta m = \rho A_1 v_1 \Delta t$$

$$OR\ \frac{\Delta m}{\Delta t} = \rho A_1 v_1 \qquad \dots\dots\dots\dots\dots\dots\dots (3-5)$$

When this fluid reaches the narrow constricted portion of the pipe to the right in figure 3-1, the same amount of mass **Δm** is given by

$$\Delta m = \rho\ \Delta V \qquad \dots\dots\dots\dots\dots\dots\dots (3-6)$$

But since $\boldsymbol{\rho}$ is a constant, the same mass **Δm** must occupy the same volume **ΔV**. However, the right-hand pipe is constricted to the narrow cross-sectional area $\mathbf{A_2}$. Thus, the length of the pipe holding this same volume must increase to a larger value $\mathbf{\Delta x_2}$, as shown in figure 3-1. Hence, the volume of fluid is given by

$$\Delta V = A_2 \Delta x_2 \qquad \dots\dots\dots\dots\dots\dots\dots (3-7)$$

The length of pipe $\mathbf{\Delta x_2}$ occupied by the fluid is related to the velocity of the fluid by

$$\Delta x_2 = v_2 \Delta t \qquad \dots\dots\dots\dots\dots\dots\dots (3-8)$$

Substituting equation 3-8 back into equation 3-7, we get for the volume of fluid,

$$\Delta V = A_2 v_2 \Delta t \qquad \dots\dots\dots\dots\dots\dots\dots\dots (3-9)$$

It is immediately obvious that since $\mathbf{A_2}$ has decreased, $\mathbf{v_2}$ must have increased for the same volume of fluid to flow. Substituting equation 3-9 back into equation 3-6, the mass of the fluid flowing in the right-hand portion of the pipe becomes

$$\Delta m = \rho A_2 v_2 \Delta t$$

$$OR \quad \frac{\Delta m}{\Delta t} = \rho A_2 v_2 \qquad \dots\dots\dots\dots\dots\dots\dots (3-10)$$

Appling the law of conservation of mass

Mass flowing into the pipe = mass flowing out of the pipe

$$\left(\frac{\Delta m}{\Delta t}\right)_{into} = \left(\frac{\Delta m}{\Delta t}\right)_{out}$$

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$A_1 v_1 = A_2 v_2 \qquad \dots\dots\dots\dots\dots\dots\dots (3-11)$$

Equation 3-11 is a special form of the **equation of continuity for incompressible fluids** (i.e., liquids).
Applying equation 3-11 to figure 3-1, we see that the velocity of the fluid $\mathbf{v_2}$ in the narrow pipe to the right is given by

$$v_2 = \frac{A_1}{A_2}\, v_1$$

Because the cross-sectional area A_1 is greater than the cross-sectional area $\mathbf{A_2}$, the ratio $\mathbf{A_1/A_2}$ is greater than one and thus the velocity v_2 must be greater than v_1.

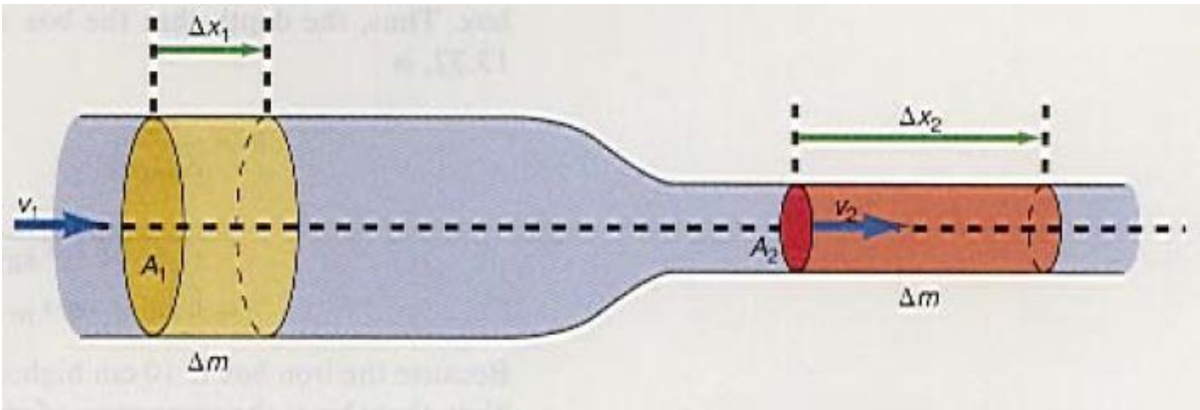

Figure (3-1)

Example 1: A pipe cross-sectional area of the left-hand was $\mathbf{7.85 \times 10^{-3}\ m^2}$ and the velocity was **0.322 m/sec**. If the diameter of the right-hand of the pipe is **4** cm.

a) Find the velocity of the fluid in the right-hand pipe?

b) What is the rate of mass flow for the right-hand side of the pipe?

Example 2: A garden hose fills a **32 L** wastebasket in **120 sec**. The opening at the end of the hose has a radius of **1 cm**.

a) How fast is the water travelling as it leaves the hose?

b) How fast does the water travel if **half** the exit area is obstructed by placing a finger over the opening?

Example 3: A garden hose having an internal diameter of **0.75 in** is connected to a lawn sprinkler that consists merely of an enclosure with **24 holes**, each **0.05 in** diameter. If the water in the hose has a speed of **3 ft/sec**, at what speed does it leave the sprinkler holes?