University of Mosul

College of Science

Department of Physics

Second stage

Lecture 12

Sound and wave Motion

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Lecture 12: Resonance

Preparation

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Resonance

When an external force of angular frequency p acts on an oscillator of normal angular frequency ω, the resonance Occurs: when the frequency of the exciting force p is equal to the natural frequency of the oscillator ω. For the sake of accuracy, this definition of resonance is completely inaccurate except under purely theoretical conditions because the oscillator always experiences frictional forces and so a factor must be taken decay into account. For a detailed study of resonance, we will analyze the special solution of equation (4) in a descriptive manner To reach the appropriate scientific definition of resonance. The particular solution that represents the steady state forced vibration he:

$$x = \frac{f_o sin(pt + \theta)}{\sqrt{(\omega^2 - p^2)^2 + (2rp)^2}} \qquad (23)$$

Whereas

$f_o/\{(\omega^2-p^2)^2+(2rp)^2\}^{1/2}$

It represents the forced vibration amplitude, and we will denote it with the letter A.

$$A = \frac{f_o}{\sqrt{(\omega^2 - p^2)^2 + (2rp)^2}} \qquad (24)$$

Therefore, equation (23) becomes as follows:

$$x = Asin(pt + \theta) \qquad (25)$$

The equation (25) indicates that the oscillator vibrates at the forced frequency p (the frequency of the external periodic force) rather than its natural frequency ω. The resulting motion is un-damped harmonic motion.

We also notice from equation (24) that the vibration amplitude A depends on both the natural non-damping frequency ω and the forced frequency of the applied force p, taking into account that fo and r are constants. If the forced frequency p is significantly different from the natural frequency ω, the resulting motion amplitude A is smaller, and as the frequency of the external influencing force p approaches the non-decaying natural frequency ω, the resulting motion amplitude A increases. The capacitance A reaches its peak when the forced frequency P equals the natural frequency ω, a phenomenon known as resonance. The forced frequency P, also known as the resonant frequency, corresponds to the peak in the capacitance of the forced oscillator and depends on the damping coefficient r, which measures the amount of frictional force acting on the oscillator. Substituting the value of ω = p in equation (24) when r = 0, the capacitance A becomes maximum in magnitude. This condition corresponds to complete absence of damping, meaning no frictional force exists. Although this condition is not practically achievable since there must always be frictional forces, the capacitance value becomes large but limited. To find the frequency at which the capacitance reaches its peak, we need to differentiate the capacitance A with respect to p, equate the result to zero, and then calculate the value of p, obtaining the maximum value of the capacitance A from equation (24) assuming that:

$$y = (\omega^2 - p^2)^2 + (2rp)^2$$

Substituting this result into equation (24), we get:

$$A = \frac{f_0}{\sqrt{y}} \qquad (26)$$

The maximum value of A occurs when the value of y is at its minimum. The value of y is at its minimum or grand when:

$$\frac{dy}{dp} = -4p(\omega^2 - p^2) + 8r^2 p = 0 \qquad (27)$$

$$p(p^2 - \omega^2 + 2r^2) = 0 \qquad (28)$$

From equation (28), we find that the p values are either p = 0 or $p = (w^2 - 2r^2)^{\frac{1}{2}}$, now we differentiate again, and we find that:

$$\frac{d^2y}{dp^2} = -4\omega^2 + 12p^2 + 8r^2 \qquad (29)$$

When we substitute p = 0 we get:

$$\frac{d^2y}{dp^2} = -4(\omega^2 - 2r^2) < 0 \qquad (30)$$

And when we make up $p = (w^2 - 2r^2)^{\frac{1}{2}}$ we get:

$$\frac{d^2y}{dp^2} = 8(\omega^2 - 2r^2) > 0 \qquad (31)$$

And on it $p = (w^2 - 2r^2)^{\frac{1}{2}}$ gives the value of the minimum limit of y. When the value of y is at its minimum limit, then The amplitude value A must be at its maximum value, and therefore the value of the coercive frequency p that leads to the maximum value The amplitude of vibration A is :

$$p = p_r = \sqrt{\omega^2 - 2r^2} \qquad (32)$$

By squaring both sides of equation (32), we get

$$p^2 = p_r^2 = (\omega^2 - 2r^2) \qquad (33)$$

This equation (33) gives the value of the resonant frequency p_r and it is noted that this frequency is always less than the value of the natural frequency ω It is noted that if the value of the decay coefficient ω > r and the value of r is less than 1, then the value of r^2 is getting smaller, so $\omega^2 >> r^2$ Then the condition for resonance to occur is ω = p acceptable with a high degree of accuracy. But if the value of r is large the value of $2r^2$ cannot be completely neglected in the equation(33).

Using equation (24), the graphical relationship can be drawn between the amplitude A and the coercive frequency p for different values of the decay coefficient r as shown in Figure (4)(1) shows that the capacitance becomes infinitely large when $r = 0$, i.e. in the case of no decay, and this happens when
p is completely equal to ω and this theoretical case is not practically achieved because the vibrator will be completely destroyed. If there is no decay The capacitance became infinity. If we assume, for the sake of argument, that the vibrator was not destroyed, then its huge capacity means that it inevitably exceeded limits Flexibility and therefore not subject to Hooke's law. It is for this reason that we mentioned at the beginning of the chapter that the exact definition For resonance the decay factor must be taken into account.

The three curves (2), (3) and (4) show the difference in the vibration amplitude with the different values of the decay coefficient r.

The lower the value of r, the resonance occurs near ω, and when the value of r increases, the position of the peak (peak) creeps to the left. . This indicates that the resonant frequency of the same oscillator varies with the value of r. It controls the value of the resonant frequency p Equation (33).

Thus, we see that resonance practically occurs when the forced frequency approaches the natural undamped frequency of the oscillator ω, and the oscillation's amplitude reaches its peak. In this case, the effectiveness of the excitation force on the oscillator reaches its maximum energy. It is said that the excitation force is in resonance with the oscillator. For practical purposes, resonance occurs at ω or near it. The curve (5) in figure (4), which corresponds to the damping factor ($r=0.707\omega$), ($\omega2=2r2$), represents the transition line between two states: above this line, when the value of r is less than 0.707 ω, the resulting curves have maxima, and below this line, when the value of r is greater than 0.707

ω, the resulting curves have no real maximum. Therefore, the curves that lie above the curve are in a state of resonance.

Curves (5) correspond to each of them a resonance state except for the case when r = 0 and the curves that fall under curve (5) do not No trace of resonance can be observed in it. This is expected because when offset $\omega^2=2r^2$ in equation (33), the resonant frequency The result is zero. Curve (6) in Figure (4) which corresponds to the decay coefficient ω = r lies below the transition line (5) Therefore, no peak is observed, which indicates that there is no significant resonance. From this curve, the vibration amplitude can be found Forced in case of excess decay. The decaying natural frequency ωo is less than the non-natural frequency The decay ω of the vibrator. It is noted that the resonant frequency p is not equal to both the un-decayed natural frequency ω and the natural decaying frequency ω_o. But it is smaller than both. If the frictional forces experienced by the oscillator small, the differences are small. So that the resonant frequency p can be considered equal to the natural frequency ω and the error is So small that it can be neglected.

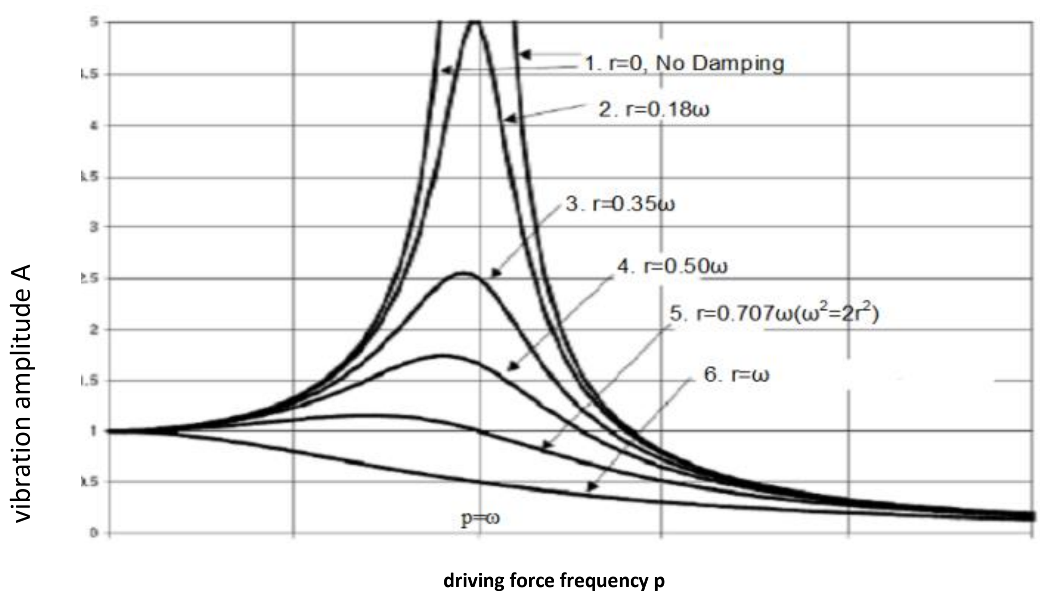

Figure (4) shows six curves for six degrees of decay:

1. Absence of decay.2. Minus decay.6. Criticality of decay.

Vibration amplitude at resonance:

It is clear from the previous figure (4) that the maximum value of the amplitude A at resonance is a function of the decay coefficient r. The exact value of the exact vibration amplitude in this case is by substituting the resonant frequency p given in Equation (33) into Equation (24), then we obtain the maximum value of the amplitude, Amax, at the resonant frequency.

$$A_{max} = \frac{f_o}{2r\sqrt{\omega^2 - r^2}} \qquad (34)$$

It is noted from this equation (34) that the capacitance in the resonant state clearly depends on the decay coefficient r The value of this coefficient is small, less than 1, the value of r^2 can then be neglected in comparison with ω^2.And so you become Equation (34) is as follows:

$$A_{max} = \frac{f_o}{2rw} \qquad (35)$$

But

$f_o=F_o/m$ و $2r=R/m$

Therefore it is

$$A_{max} = \frac{F_o}{R\omega} \qquad (36)$$

Equation (36) indicates that when the frictional resistance experienced by the oscillator is small, the amplitude of the oscillation At resonance it is inversely proportional to the resistance constant R. The capacitance becomes very large as the value of R approaches zero. Thus, we notice that when resonance occurs, the vibration amplitude increases without limits and is controlled only by the amount of resistance friction experienced by the oscillator. Thus, we expect that the greater the frictional resistance in an oscillator, the lower its intensity Resonant vibration caused by a certain external excitation.

Practical examples of resonance:

We have previously found that the response of any oscillator to a periodic external force depends on the relationship between the frequency of the applied force and the natural frequency of the oscillator. An oscillator may be very simple with a single natural frequency, or it may be complex with multiple natural frequencies. If a periodic force with a specific frequency affects the oscillator, it causes it to vibrate at the same frequency. The closer the frequency of this force is to one of the natural frequencies of the oscillator, resonance occurs, leading to intense vibration. It may result in dangerous resonant vibrations from small successive impulses, properly timed on the oscillator, whereas the same may not occur if large successive impulses are used and not properly timed. This illustrates that for resonance to be effective, two fundamental conditions must be met: first, the frequency of the vibrating force must be equal to the oscillator's frequency. Second, the phase of the applied force must be in sync with the oscillator's motion. The significance of forced vibration becomes apparent when resonance occurs, and despite the many practical benefits of the phenomenon of resonance, it is not without troublesome aspects that may lead to disasters. To avoid undesirable effects of resonance, knowledge of the natural frequency of the oscillator and appropriate measures to prevent resonance must be taken. Due to the importance of the phenomenon of resonance, we will mention some simple and different examples to see the extent of the impact of this phenomenon.

1- Air column resonance :

We hold a vibrating fork over a partially filled glass tube. This causes forced vibration in the column of air above the water surface. By adjusting the water level, we can make the natural frequency of the air column inside the tube equal to the frequency of the vibrating fork. When this happens, the effectiveness of the vibrating fork in supplying the air with vibrational energy reaches its maximum. As a result, we will hear a strong resonance in the air inside the tube in response to the sound produced by the vibrating fork. What actually happens in this case is that the sound wave emitted by the vibrating fork travels in the air column inside the tube, and when it reaches the end of the column, it reflects back to the position of the vibrating fork, where it is reflected again after being reinforced downwards. Thus, the vibrating sound fork will continuously work to reinforce the reflected sound wave, thereby sustaining the resonance. The resonance is not limited to a specific height of the air column, but it also occurs at different heights equal to multiples of the length of the air column in which the initial resonance occurred.

2- swing ring:

When a child makes periodic pushes on the swing to rock it, he also knows that he must push the swing. Over time periods, the amplitude of the vibration increases gradually, and when the frequency of the pulses that the child applies to the swing is repeated with the natural frequency of the swing, resonance occurs, and thus the amplitude of the swing is relatively large. We must remember that it is not only important that the two frequencies be equal for resonance to occur, but that the two vibrational movements must also be consistent in phase so that the efficiency of energy transfer from one oscillator to the other is at its highest levels.

3- Ringing forks:

We take two tuning forks of the same frequency and attach them to two identical boxes placed at a distance from each other Their open sides are opposite each other, as in Figure (5). When one of the forks, A, is struck with a rubber bat, it will

It begins to vibrate, and from there the vibration is transmitted to the box that carries it, and this in turn leads to the vibration of the air inside it Forcibly, if the length of the air column inside the box is equal to a quarter of the wavelength of the sound emitted by the fork The vibrating resonator, a strong resonance will occur, which will lead to the emission of strong sound waves from the mouth of the box. This is amazing The waves when they fall on the second fork B will cause it to vibrate. The vibration of the second fork without knocking or touching it is caused by the sound waves from A being A series of pulsations consisting of compressions and rarefactions, after traveling through the air, reaches B. Part of it hits the fork

B directly, causing vibration in it, and a part falling on the box B, causing the air inside it to vibrate, which leads to vibration The box is forced and this forced vibration is transmitted to fork B and its vibration increases. As the frequency of the incident waves equal to the frequency of fork B, this fork will vibrate at the same frequency as a result of resonance. This can be explained by silencing The first fork by holding it in the hand, and listening to the second fork that continues to emit a sound.

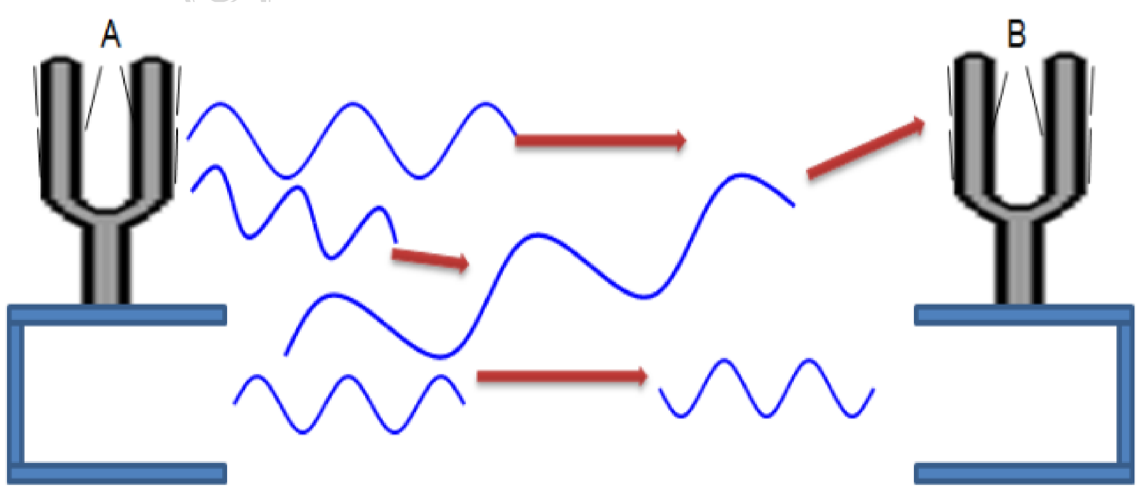

Figure 5: Fork B responds for the movement of the fork A

1- suspension bridges:

The suspension bridge is a complex vibrator that is easy to shake due to its relatively small decay factors. If a column of soldiers walks on a suspension bridge with regular steps, the successive foot strikes that strike the bridge at the same stage and at regular intervals exert a periodic force on the bridge. If the soldier's movement coincides with one of the natural frequencies of the bridge, a violent resonance will occur, and the amplitude of the vibration in this case may be large, which leads to a violent resonance that leads to the collapse of the bridge. This has already happened if the ringing caused many bridges to break. This is why soldiers, even if they are a small group, are ordered to abandon their military march when crossing the bridge to avoid disasters.

And there are examples Numerous on the diamonds caused by the resonance in the suspension bridges. The Burton Suspension Bridge over the River Irwell has been destroyed In the British city of Manchester, under the footsteps of a military column consisting of only sixty men. However, the biggest tragedy occurred in 1850, when a battalion of French infantry, consisting of about five hundred men, destroyed a bridge Angier commentator. The soldiers fell into a deep ravine, and 226 of them were killed. Only a few years ago, a suspension bridge collapsed in the Qanater El Khayriyah Gardens in Egypt, due to some people swinging on top of it with a frequency that matched its natural frequency, completely destroying it.

Perhaps one of the most famous resonant vibrations is what happened to the Tacoma Bridge in Washington State. The American Bridge, which ranks third in the world rankings in terms of length. This vibration led to its collapse, and it was only opened four months ago. The cause of the collapse was not understood at that time (1940). Because the vibrations caused by the wind in suspension bridges were not the focus of designers. The Tacoma Bridge was exposed to constant winds that caused it to oscillate strongly, and when the frequency of the oscillatory force caused by the regular winds coincided with one of the natural frequencies of this bridge, this led to an increase in the amplitude of its vibration and it collapsed. The Tacoma Bridge disaster has attracted a great deal of attention to avoid a recurrence. After extensive studies and research, the bridge was rebuilt and many bridges were redesigned, so that they would be stable from an aerodynamic point of view.

Example: A mass of 2 kg attached to a spring is given an external force of

$F = 3\ N\cos(2\pi t)$ if constant Find the force of the spring 20 N / m:

1. vibration time.

2. Movement capacity assuming there is no movement decay.