Chapter Four

Simple Harmonic Motion

(4-1) Simple Harmonic Motion

Consider a physical system that consists of a block of mass **m** attached to the end of a spring, with the block free to move on a horizontal, frictionless surface (Figure 4-1). When the spring is neither stretched nor compressed, the block is at the position called the **equilibrium position** of the system. When the block is displaced to the right of equilibrium ($x > 0$), the force exerted by the spring acts to the left (Figure 4-1a). When the block is at its equilibrium position ($x = 0$), the force exerted by the spring is zero (Figure 4-1b). When the block is displaced to the left of equilibrium ($x < 0$), the force exerted by the spring acts to the right (Figure 4-1c).

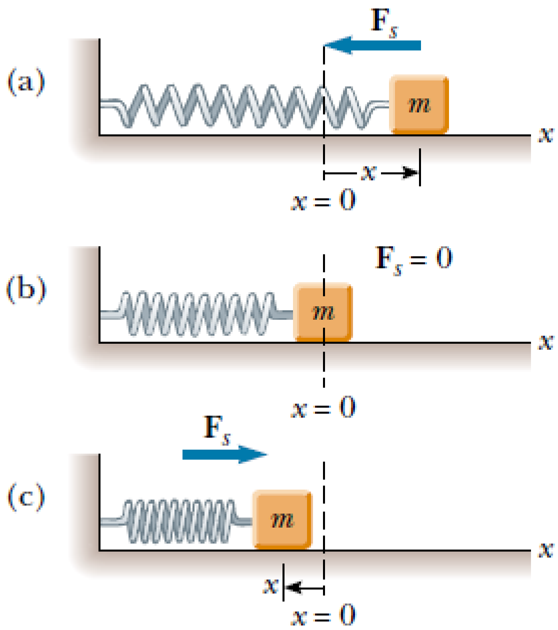

Figure (4-1)

We can understand the motion in Figure 4-1 qualitatively by first recalling that when the block is displaced a small distance **x** from equilibrium, the spring exerts on the block a force that is proportional to the displacement and given by Hooke's law

$$F_s = -kx \qquad \dots\dots\dots\dots\dots\dots\dots (4-1)$$

We call this a restoring force because it is always directed toward the equilibrium position and therefore **opposite** the displacement. That is, when the block is displaced to the right of in Figure 4-1, then the displacement is positive and the restoring force is directed to the left. When the block is displaced to the left of then the displacement is negative and the restoring force is directed to the right.

Applying Newton's second law to the motion of the block, together with Equation 4-1, we obtain

$$F_s = -kx = ma$$

$$a = -\frac{k}{m}x \qquad \dots\dots\dots\dots\dots\dots\dots (4-2)$$

That is, the acceleration is proportional to the displacement of the block, and its direction is opposite the direction of the displacement. Systems that behave in this way are said to exhibit **simple harmonic motion. An object moves with simple harmonic motion whenever its acceleration is proportional to its displacement from some equilibrium position and is oppositely directed.**

An experimental arrangement that exhibits simple harmonic motion is illustrated in Figure 4-2. A mass oscillating vertically on a spring has a pen attached to it. While the mass is oscillating, a sheet of paper is moved perpendicular to the direction of motion of the spring, and the pen traces out a wavelike pattern.

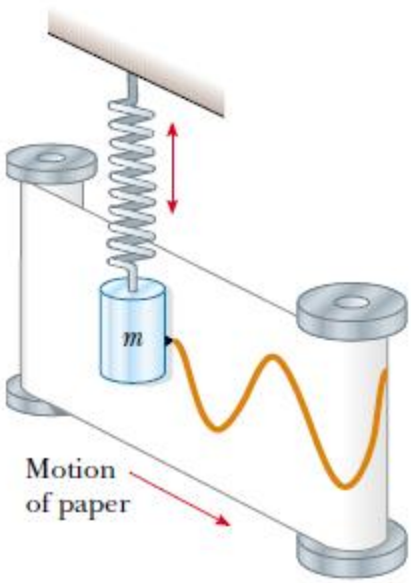

Figure (4-2)

The time it takes for a displaced object to make a complete oscillation back and forth about its equilibrium position is called the **period T**. The reciprocal of the period is the **frequency f**, which is the number of oscillations per second:

$$f = \frac{1}{T} \quad \dots\dots\dots\dots\dots\dots\dots (4-3)$$

The unit of frequency is the reciprocal second (s^{-1}), which is called a **hertz (Hz)**. For example, if the time for one complete oscillation is 0.25 s, the frequency is 4 Hz.

Figure 4-2 shows how we can experimentally obtain x versus t for a mass on a spring. The general equation for such a curve is

$$x = A\cos\,(\omega t + \phi) \quad \dots\dots\dots\dots\dots\dots\dots (4-4)$$

where A, ω, and ϕ are constants. To give physical significance to these constants, we have labeled a plot of x as a function of t in Figure 4-3a. This is just the pattern that is observed with the experimental apparatus shown in Figure 4-2. The **amplitude A** of the motion is the maximum displacement of the particle in either the positive or negative **x** direction. The constant **ω** is called the **angular frequency** of the motion and has units of radians per second. The constant angle **ϕ**, called the **phase constant** (or phase angle), is determined by the initial displacement and velocity of the particle. If the particle is at its maximum position x = A at t = 0 then and the curve of **x** versus **t** is as shown in Figure 4-3b. If the particle is at some other position at t = 0, the constants ϕ and A tell us what the position was at time t = 0. The quantity (ωt + ϕ) is called the **phase** of the motion and is useful in comparing the motions of two oscillators.