1-3 Poisson's ratio

When a rod or bar is subjected to a tension stress, it not only elongates in the direction of the stress but its transverse dimension decreases. If

w_o= is the original transverse dimension

Δw= is the change in transverse dimension

L= is the original length

ΔL= is the change in length

Then,

$$Transverse\ strain = \frac{\Delta w}{w_o} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1-8)$$

$$Longitudinal\ strain = \frac{\Delta L}{L} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1-9)$$

The ratio of the transverse to longitudinal strain is called Poisson's ratio and represented by (σ)

$$\sigma = -\frac{transverse\ strain}{longitudinal\ strain} = -\frac{\Delta w/w_o}{\Delta L/L} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1-10)$$

The minus sign means that any increase in length always results in a decrease in transverse dimension and vice versa.

Example 1: A steel wire **1 m** long a diameter **1 mm** has **10 kg** mass hung from it. Give Young's modulus for steel **$21x10^{10}$ N/m^2**

a) How much will the wire stretch?

b) What is the stress on the wire?

c) What is the strain on the wire?

Example 2: A steel rod of **120 cm** long and cross-section area of **1 cm²**. The rod is stretched from both ends by two equal forces of **100 N** each. If Poisson's ratio equal **0.3** and Young's modulus **2x10^{11} N/m²** find: a) normal stress, b) longitudinal strain, c) increase in its length, and d) transverse strain

Example 3: A steel cable of diameter **3 cm** support a load of **2 KN**. What is the fractional length increase of the cable compared to the length when there is no load, if Y=**2x10^{11} N/m²**

1-4 Shear Modulus

Another type of deformation occurs when an object is subjected to a force tangential to one of its faces while the opposite face is held fixed by another force (Figure 1-5a). The stress in this case is called a shear stress. If the object is originally a rectangular block, a shear stress results in a shape whose cross-section is a parallelogram.

A book pushed sideways, as shown in Figure 1-5b, is an example of an object subjected to a shear stress. To a first approximation (for small distortions), no change in volume occurs with this deformation.

We define the **shear stress** as (F/A), the ratio of the tangential force to the area A of the face being sheared. The **shear strain** is defined as the ratio $(\Delta x/h)$, where Δx is the horizontal distance that the sheared face moves and **h** is the height of the object. In terms of these quantities, the **shear modulus (μ)** is (modulus of rigidity)

$$\mu = \frac{shearing\ stress}{shearing\ strain} = \frac{F/A}{\Delta x/h} = \frac{F \times h}{A \times \Delta x} \qquad \dots\dots\dots\dots\dots (1-11)$$

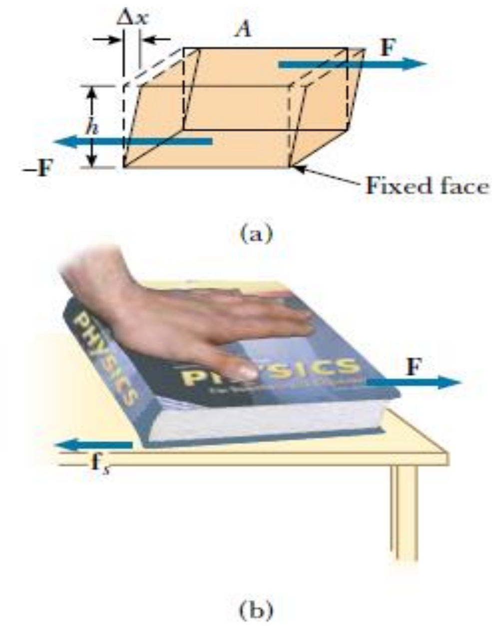

Figure 1-5 Shear Modulus

Example 4: A sheet of copper **0.750 m** long, **1 m** high, and **0.5 cm** thick is acted on by a tangential force of **50,000 N**, as shown in figure 1-6. The value of μ for copper is 4.20×10^{10} N/m^2. Find (a) the shearing stress, (b) the shearing strain, and (c) the linear displacement Δx.

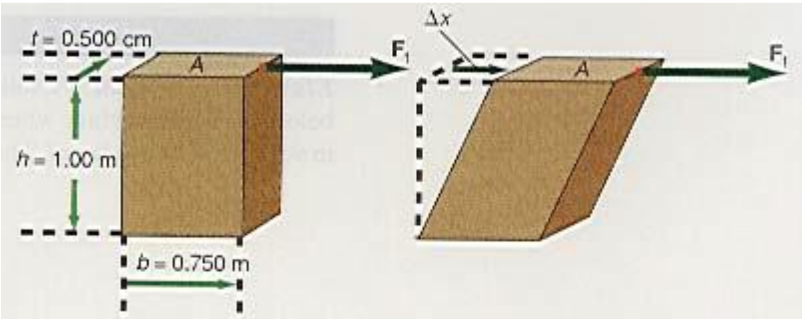

Figure 1-6

Example 5: A sheet of paper of thickness **0.2 mm** is cut with scissors that have blades of length **10 cm** and width **0.2 cm**. While cutting, the scissors blades each exert a force of **3 N** on the paper, the length of each blade that makes contact with the paper is approximately **0.5 mm.** What is the shear stress on the paper?

Example 6: A hole punch has a diameter of **8 mm** and presses onto **10 sheets** of paper with a force of **6.7 KN**. If each sheet of paper is of thickness **0.2 mm**, find the shear stress

1-5 Bulk Modulus

Bulk modulus characterizes the response of a substance to uniform squeezing or to a reduction in pressure when the object is placed in a partial vacuum. Suppose that the external forces acting on an object are at right angles to all its faces, as shown in Figure 1-7, and that they are distributed uniformly over all the faces. Such a uniform distribution of forces occurs when an object is immersed in a fluid. An object subject to this type of deformation undergoes a change in volume but no change in shape. The **volume stress** is defined as the ratio of the magnitude of the normal force F to the area A. The quantity $P = F/A$ is called the **pressure**. If the pressure on an object changes by an amount $\Delta P = \Delta F/A$, then the object will experience a volume change ΔV. The **volume strain** is equal to the change in volume ΔV divided by the initial volume V. Thus, from Equation 1-6, we can characterize a volume ("bulk") compression in terms of the **bulk modulus (B)**, which is defined as

$$B = \frac{volume\ stress}{volume\ strain} = -\frac{\Delta F/A}{\Delta V/V} = -\frac{\Delta P}{\Delta V/V} \qquad \dots\dots\dots\dots\dots (1-12)$$

A negative sign is inserted in this defining equation so that B is a positive number. This maneuver is necessary because an increase in pressure (positive ΔP) causes a decrease in volume (negative ΔV) and vice versa. The reciprocal of the bulk modulus is called the **compressibility (K)** of the material.

$$K = -\frac{1}{B} = -\frac{\Delta V}{PV} \qquad \dots\dots\dots\dots\dots\dots\dots (1-13)$$

$$\Delta V = -KVP \qquad \dots\dots\dots\dots\dots\dots\dots. (1-14)$$

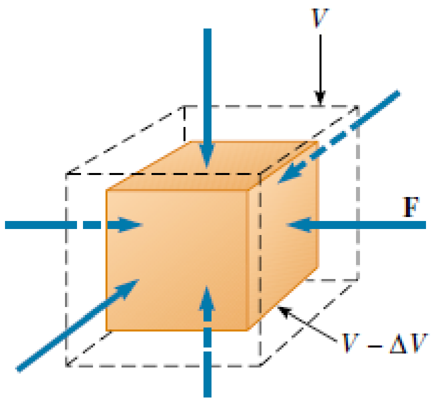

Figure 1-6 Bulk Modulus

Example 7: A solid copper sphere of 0.5 m^3 volume is placed **100 ft** below the ocean surface where the pressure is **3×10^5 N/m²**. What is the change in volume of the sphere? The bulk modulus for copper is **14×10^{10} N/m²**.

Example 8: A marble statue of volume **1.5 m^3** is being transported by ship from Athens to Cyprus. The statue topples into the ocean when an earthquake caused tidal wave sinks the ship, the statue ends up on the ocean floor, **1 km** below the surface. Find the change in volume of the statue in **cm^3** due to the pressure of the water. The density of seawater is **1025 kg/m^3**. The bulk modulus for marble is **70×10^9 Pa**