University of Mosul

College of Science

Department of Physics

Second stage

Lecture 3

Sound and wave Motion

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Lecture 3: Simple Harmonic Motion

Preparation

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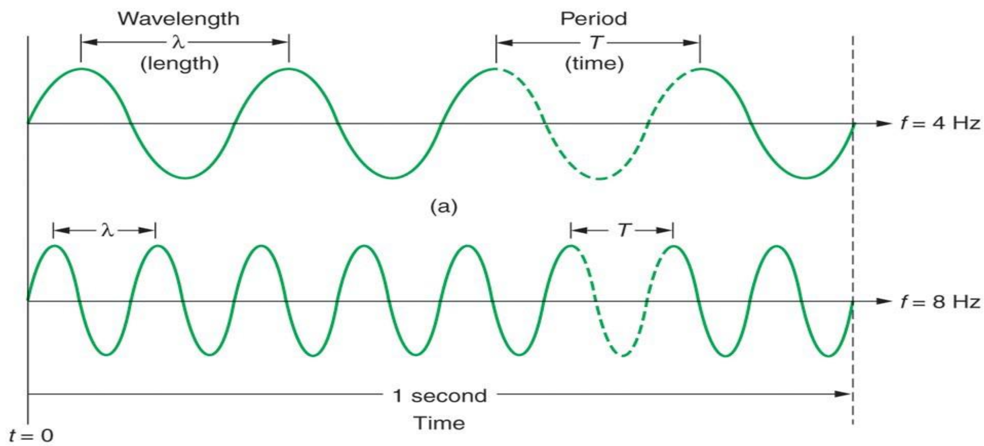
Chapter two

Concepts in Simple Harmonic Motion:

1- **Complete oscillation**: The body movement in which a path is traversed back and forth.



2- **The period of the oscillation** ,T, which is the time required for a complete oscillation.

3- **Frequency** (f) The number of vibrations caused by a body's vibration during one second, and its unit is Hz

4- **Displacement** (x) is the distance of the body from its resting position at any moment.

5- **The amplitude** (A) is the greatest value of the displacement.

6- **Periodic motion:** It is the continuous back and forth movement of a particle around a fixed point called the position of equilibrium or stability.

7-**Equilibrium position**: is the point at which the resultant of the forces acting on the particle is zero, representing a state of rest when it ceases (stop) to vibrate.

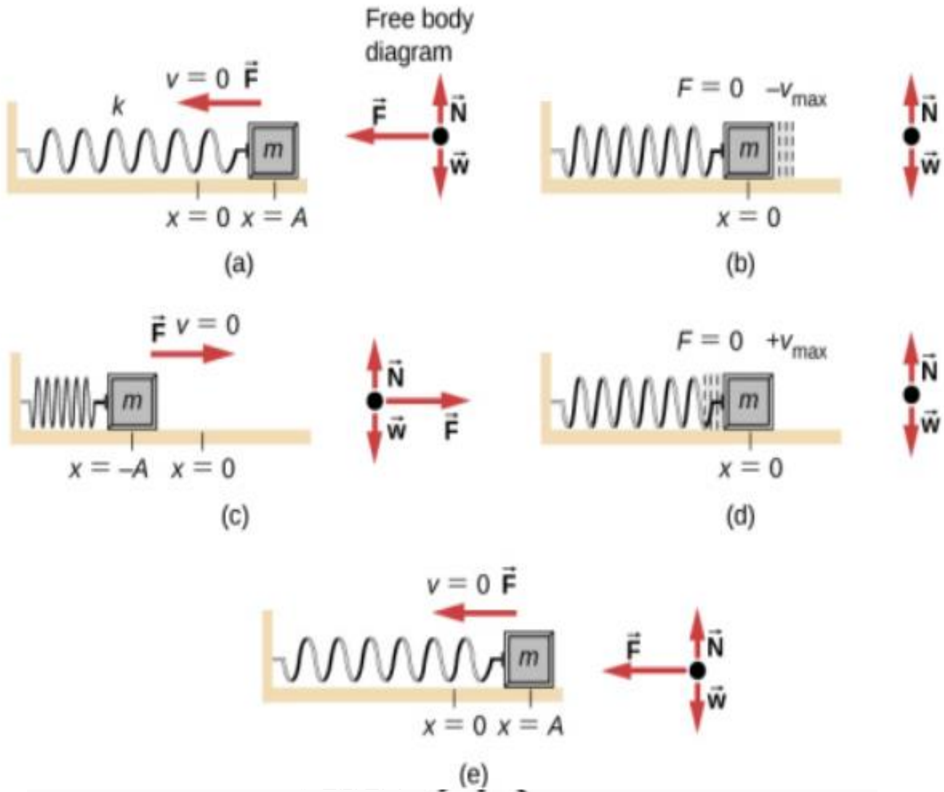
8-**Simple linear Harmonic Motion** (SlHM): is the movement of a particle along a straight line with an acceleration that is directly proportional to its displacement from a fixed point representing the equilibrium position. The direction of the acceleration is always towards that reference point. Examples of SHM include the motion of a simple pendulum.

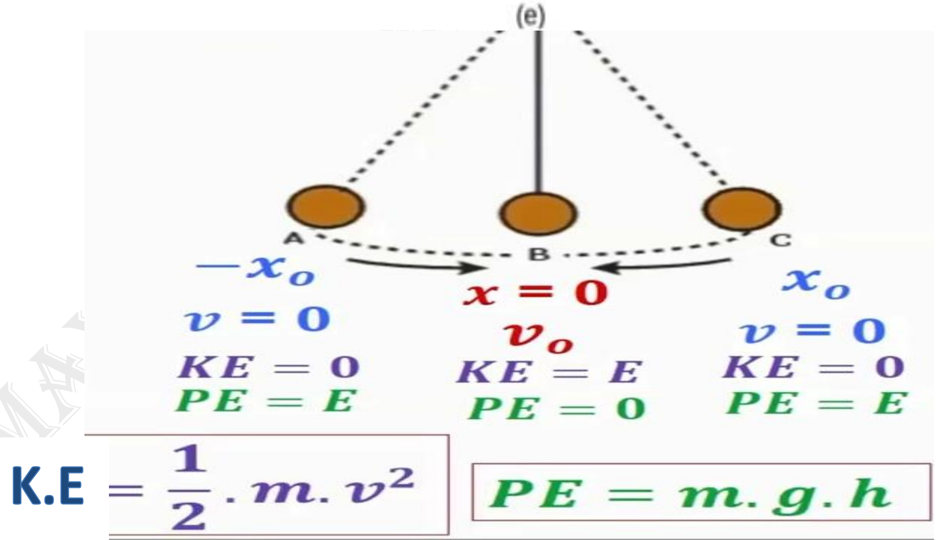
(Its equilibrium and direction are always towards that reference point).

Simple harmonic motion equation

This is the movement in which the effective force F is directly proportional to the displacement relative to a fixed point and always directed towards that point within the limits of elasticity.

To illustrate, we took a thin strip of steel, attached it to one end of a rigid (solid) body, and made it move with a small impulse (push) in a straight line. The restoring forces resulting from the effect of the steel strip attempt to return the body to its stable position. The body moves towards the center with increasing speed, but the rate of increase is not constant, as the acceleration force decreases as the body approaches the center. When the body reaches the center, the restoring force decreases to a minimum (zero). However, the body overshoots the equilibrium position and continues its movement towards the other side due to the acquired velocity. If there is no energy loss due to friction, the motion will continue uninterrupted to find the resulting relationship of this movement as illustrated in the figure.





When a body oscillates at a distance X from its equilibrium position, where F represents the resultant force acting on the body and equals the restoring force -Kx, according to Hooke's law, expressed as F= -kx. This is derived from Newton's second law.

$$F = -Kx = ma = mv(dv /dX)$$

where K: is the constant of proportionality

m :is the mass of the moving body

a: is acceleration and v is velocity

$mv\ (dv/\ dx) + Kx = 0 \quad \longrightarrow \quad m\int vd\ v + k\int x\ dx = 0$

$1/\ 2\ mv^{2} + 1/\ 2\ kx^{2} = c_1$

C_1 =(   ) H.w

The first term represents kinetic energy, the second term represents potential energy, and c_1 represents total energy for the body.

The velocity at the site (o) is the greatest possible, V_{max}. The total energy is all kinetic, meaning that:

$E_{total} = Ek = 1/\ 2\ mv^{2}_{max}$

where the v sign depends on the direction of motion

$v_{max} = \pm\sqrt{2E/m}$

At the end of the path, all energy is potential and kinetic is zero

$E = p\ E = 1/\ 2\ Kx^{2}_{max}$

$$x_{max} = \pm\sqrt{\frac{2E}{K}}$$

The velocity V at any displacement X is equal to

$v = v = \sqrt{2E - kx^2/m}$

When substituting the c_1 into the equation of v (AT ANY POIN), it is obtained

$$dx/\,dt = v = \sqrt{\frac{K}{m}}\sqrt{A^2 - x^2}$$

$$\int dx/\sqrt{A^2 - x^2} = \sqrt{\frac{K}{m}}\int dt$$

$$sin^{-1}\frac{x}{A} = \sqrt{\frac{K}{m}}\,t + C_2$$

If x_0 is the value of X at the moment t = 0, then

$$C_2 = sin^{-1}\frac{X_\circ}{A}$$

Therefore, C_2 is the angle and we denote it by the symbol θ_0

$$sin\theta_o = \frac{X_\circ}{A}$$

$$\therefore sin^{-1}\frac{x}{A} = \sqrt{\frac{K}{m}}\,t + \theta_o$$

$$x = Asin\left(\sqrt{\frac{K}{m}}\,t + \theta_o\right)$$

$\theta_\circ$ is an initial phase angle.

$\left(\sqrt{\frac{K}{m}}\,t + \theta_o\right)$ is the phase angle of motion

The motion of a simple harmonic wave can also be represented as a cosine function if the initial phase angle is equal to π/2. Furthermore, because the sine or cosine function oscillates between the values 1 and -1, the object's displacement, denoted by x, varies between -A and A.

In addition, it should be noted that the displacement x has the same value at time t and time (T+ t), while the phase angle ($\sqrt{K/m}\ t + \theta_\circ$) increases by 2π during time. This indicates a gradual change in phase angle.

$$\left(\sqrt{\frac{K}{m}}(t+T)+\theta_\circ\right)=\left(\sqrt{\frac{K}{m}}\,t+\theta_\circ\right)+2\pi$$

$$\sqrt{\frac{K}{m}}\,T=2\pi \quad \Rightarrow \quad T=\sqrt{\frac{m}{K}}\,2\pi$$

T depends only on the mass m and the proportionality constant k, so the T It is constant whether the body capacity is small or large.

The frequency f is → $w = 2\pi f$

$$\omega=\sqrt{\frac{K}{m}}$$

at any time

$$\therefore x = A\sin(wt+\theta_o)$$

$$\frac{dx}{dt}=v=w\sqrt{A^2-x^2}=wA\cos(wt+\theta_o)$$

$$\frac{dx^2}{dt^2}=\frac{dv}{dt}=a=-w^2A\sin(wt+\theta o)$$

$$a=-w^2x$$

$\ddot{x}+w^2x=0$ → Equation

The last equation is the general form of the equation of simple harmonic motion and its solution is:

$x = A\sin(wt+\theta o)$ → Solution

Acceleration in simple harmonic motion is directly proportional to the displacement and opposite to it, and that the acceleration reaches its maximum value when → $X_{max} = \pm A$.

Subsequently, the velocity becomes non-existent, as indicated by the value of:

$a_{max} = \pm w^2 A$.

At the moment when the body passes through the equilibrium position ($X = 0$), its velocity reaches its maximum value. Furthermore, during this instance, there is no acceleration, signifying that the maximum velocity is $v_{max} = \pm WA$.

It should be noted that the maximum value of the sine or cosine function is ±1.