University of Mosul
College of science
Department of Physics
Third Stage
Lecture 3

Laser

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Lecture 3: Population Inversion in Three Levels

Preparation
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Three Level Laser with the Intermediate Level as the Upper Laser Level: (solid state laser)

For the traditional three-level arrangement i, u, l, with populations N_i, N_u, and N_1 and $E_i > E_u > E_l$, the laser transition occurs from level u to level l. The gain medium is in thermal equilibrium before we begin pumping from level l to level i at a rate Γ_{li}, where Γ denote an externally applied pumping.

γ_{iu} ⟶ decay from level i to level u

γ_{i1} ⟶ decay from level i to level l

γ_{ul} ⟶ decay from level u to level l

} radtiative and Collisional

A_{ul} ⟶ radiative decay from level u to level l

We assume no thermalizing excitation from l to level u and i, science the energies of u and i are high that such processes are very small. Thus

$N_u\gamma_{ul} = N_l\gamma_{lu}$ ⟶ at thermal equilibrium

From Boltzmann relationship

$$\frac{N_u}{N_l} = e^{-\Delta E_{ul}/KT} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ 40$$

$$\frac{\gamma_u}{\gamma_l} = e^{-(E_u - E_l)/KT} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 41$$

Now, we will consider the rate equations for the population densities of levels in i, u, l by considering the flux entering and leaving each level per unit time:

$$\frac{dN_l}{dt} = -\Gamma_{li}N_l + \gamma_{ul}N_u + \gamma_{il}N_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 42$$

$$\frac{dN_u}{dt} = -\gamma_{ul}N_u + \gamma_{iu}N_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 43$$

$$\frac{dN_i}{dt} = \Gamma_{li}N_l - (\gamma_{il} + \gamma_{iu})N_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 44$$

Steady state solutions of these equations, ⟶ i.e, equated them to zero. We assumed that no significant external pumping occurs from level l to level u, which is typically true for this type of laser.

Since

$N = N_l + N_u + N_i$ ……………….45

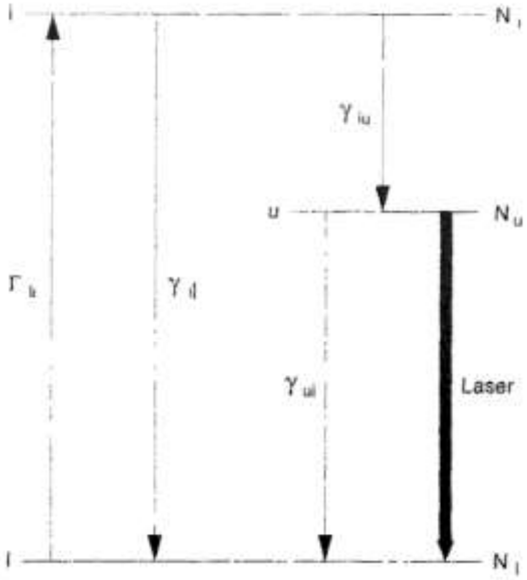

Fig. (6): Excitation and decay processes for three- level laser

Thus, we get

$N_u=(\gamma_{iu}/\gamma_{ul})N_i$ ……………..46

$N_l=((\gamma_{il}+\gamma_{iu})/\Gamma_{li})N_i$ ……..……..47

$$\frac{N_u}{N_l} = \frac{\gamma_{iu}\Gamma_{li}}{\gamma_{ul}(\gamma_{iu}+\gamma_{il})} > 1$$ ..…………..48

$$\Gamma_{li} > \frac{\gamma_{ul}(\gamma_{iu}+\gamma_{il})}{\gamma_{iu}} > 1$$ …...………49

$$\Gamma_{li} > \frac{\gamma_{ul}\gamma_{iu}\ (1+\frac{\gamma_{il}}{\gamma_{iu}})}{\gamma_{iu}}$$ ……………...50

$$\Gamma_{li} > \gamma_{ul}(1+\frac{\gamma il}{\gamma iu})$$ … …………51

$\frac{\gamma il}{\gamma iu} \longrightarrow$ be as small as possible in order to minimize the- requirements on the pumping rate Γ_{li}.

In fact, this is the case for most solid state laser since collisional de population occurs much more frequently between nearby energy levels than between levels separated by a large energy. Thus, if also $\gamma_{ul} \cong A_{ul}$ then the inversion is produced if

$$\Gamma_{li} > A_{ul} > 1/\tau_u \qquad \ldots\ldots\ldots\ldots\ldots 52$$

Example: Calculate the pumping flux necessary to reach a population inversion in ruby Laser, if the lifetime of the upper laser level is 3 $x10^{-3}$ s and the no. of atoms per unit volume in the ground state N_o=1.6 $x10^{25}$ / m^3.

Solution: $\tau_u = 3x10^{-3}$ s , $N_o = 1.6 \times 10^{25}$ / m^3

$$\Gamma_{li} > A_{ul} = 1/\tau_u \qquad \ldots\ldots\ldots\ldots\ldots 53$$

$$\Gamma_{li} = \frac{1}{3x10^{-3}} = 3.33x10^2\ s^{-1}$$

The pumping Flux (photons per cubic meter per second) is just $N_o\ \Gamma_{li}$, thus

$N_o\ \Gamma_{li} > 3.33x10^2 \times 1.6x10^{26} / m^3s$

$\Gamma_{li} > 5.33x10^{27}/m^3s$

which is the pumping flux necessary to reach a population inversion threshold. For significant laser output to occurs the inversion must be higher than that.

Three Level Laser with the Upper Laser Level as the Highest Level

Consider the energy level diagram shown in Fig. (7) with the highest level as the upper laser level, the intermediate level as the lower laser level l and the lowest level (ground) as level o.

Γ_{ou} & Γ_{ol} ⟶ pumping process provides flux to both – levels u and l from the ground state o

Before pumping, $N = N_o + N_u + N_l \approx N_o$ this is true for atomic system (gas laser), since ΔE_{lo}, and ΔE_{uo} are generally much greater than KT for the system.

$N_o\Gamma_{ou}$ ⟶ pumping flux from level o to level u.

$N_o\ \Gamma_{ol}$ ⟶ pumping flux from level o to level l.

γ_{ul}, γ_{uo} and γ_{lo} ⟶ are decay rates from levels u and l.

γ_{ou} , γ_{ol} and γ_{lu} ⟶ are thermal excitation and are neglected .

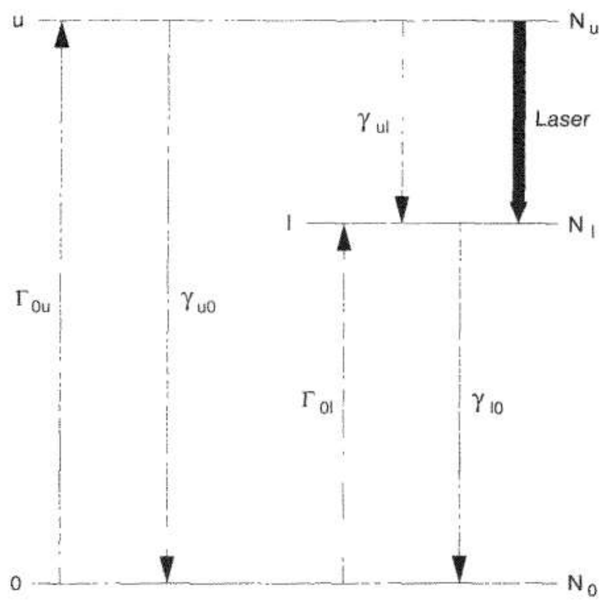

Figure (7): Energy- levels diagram for three- level system.

The rate equations are:

$$\frac{dN_u}{dt} = N_o\Gamma_{ou} - N_u(\gamma_{ul} + \gamma_{uo}) = 0 \longrightarrow \text{at steady – state} \quad \text{.......... 54}$$

$$\frac{dN_l}{dt} = N_o\Gamma_{ol} + N_u\gamma_{ul} - N_l\gamma_{lo} = 0 \longrightarrow \text{at steady – state} \quad \text{..........55}$$

$$N_u = \frac{N_o\Gamma_{ou}}{\gamma_{ul} + \gamma_{uo}} \quad \text{............. 56}$$

$$N_l = (N_o\Gamma_{ol} + N_u\gamma_{ul}) \; / \gamma_{lo} \quad \text{..............57}$$

∴

$$N_l = \frac{N_o[\Gamma_{ol} + \Gamma_{ou}\gamma_{ul} / (\gamma_{ul} + \gamma_{uo})]}{\gamma_{lo}} \quad \text{..............58}$$

$$\frac{N_u}{N_l} = \frac{\Gamma_{ou}}{(\gamma_{ul} + \gamma_{uo})} \frac{\gamma_{lo}}{(\Gamma_{ol} + \Gamma_{ou}\gamma_{ul}/(\gamma_{ul} + \gamma_{uo}))} \quad \text{..............59}$$

$$\frac{N_u}{N_l} = \frac{\Gamma_{ou}\gamma_{lo}}{\Gamma_{ol}\gamma_{uo} + \Gamma_{ol}\gamma_{ul} + \Gamma_{ou}\gamma_{ul}} \quad \text{..............60}$$

$$\frac{N_u}{N_l} = \frac{\Gamma_{ou}\gamma_{lo}}{\gamma_{uo}\Gamma_{ol}+\gamma_{ul}[\Gamma_{ol}+\Gamma_{ou}]} > 1 \qquad \ldots\ldots\ldots\ldots 61$$

We can say that, in atomic systems (gas laser) the collisional decay are negligible compared to radiative decay. i.e , γul=Aul , γuo = Auo i.e., the collisional decay processes are negligible compared to radiative decay.

$$\therefore \frac{N_u}{N_l} = \frac{\Gamma_{ou}A_{lo}}{A_{uo}\Gamma_{ol}+A_{ul}[\Gamma_{ol}+\Gamma_{ou}]} > 1 \qquad \ldots\ldots\ldots\ldots 62$$

The population inversion can be obtained if the decay from level l is greater than the decay from level u, provided that the pumping to level l is not highly favored over that to level u. For an atomic system in which A_{lo} is large, A_{uo} would be very small because level u and o would have to be of the same parity, thus

$$\frac{N_u}{N_l} = \frac{1}{\left[1+\frac{\Gamma_{ol}}{\Gamma_{ou}}\right]}\frac{A_{lo}}{A_{ul}} > 1 \qquad \Gamma_{ou} > \frac{A_{ul}}{A_{lo}}(\,\Gamma_{ou} + \Gamma_{ol}) \qquad \ldots\ldots\ldots\ldots 63$$

If $\frac{\Gamma ol}{\Gamma ou}$ =1 then $A_{10} > 2A_{ul}$, i.e., it most desirable to have a fast decay out of the lower laser and a higher - pumping flux to the upper laser level.

https://www.youtube.com/watch?v=T3YkrHjBNWk