University of Mosul

College of Science

Department of Physics

Second stage

Lecture 4

Sound and wave Motion

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Lecture 4: Applications of S.H.M

Preparation

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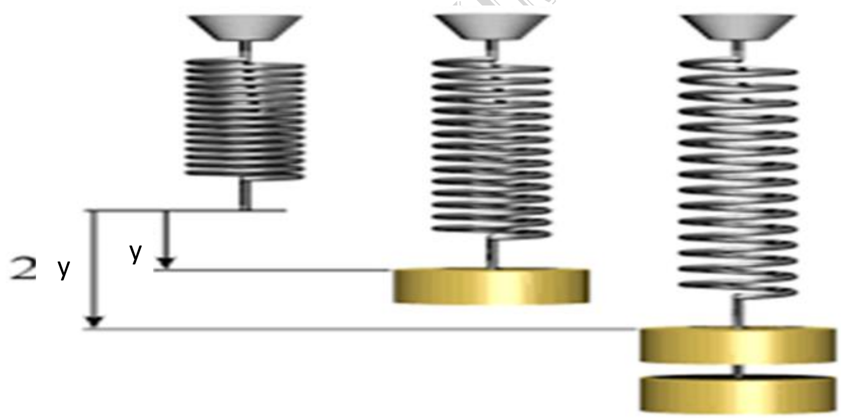
Applications of simple harmonic motion

1. *spiral spring*; If a body of mass m is attached to a spring, it will hang in equilibrium with the spring that has expanded by an amount ΔL, such that the force directed upwards, which does not affect the spring, is equal to the weight of the body, mg. Where k is the spring constant, $k\Delta L = mg$.

Assuming the body is pulled downward by a distance (y) from the equilibrium position and then left to oscillate, the resultant force F acting on the body according to Newton's second law is:



$F = ma = m\ddot{y} = mg - k\Delta L - KY$ Applying Newton's second law

$F = -ky$, $m\ddot{y} + ky = 0$

$\ddot{y} + \frac{k}{m}y = 0 \longrightarrow \frac{d^2y}{dt^2} + \frac{k}{m}y = 0$

And this is the equation for simple

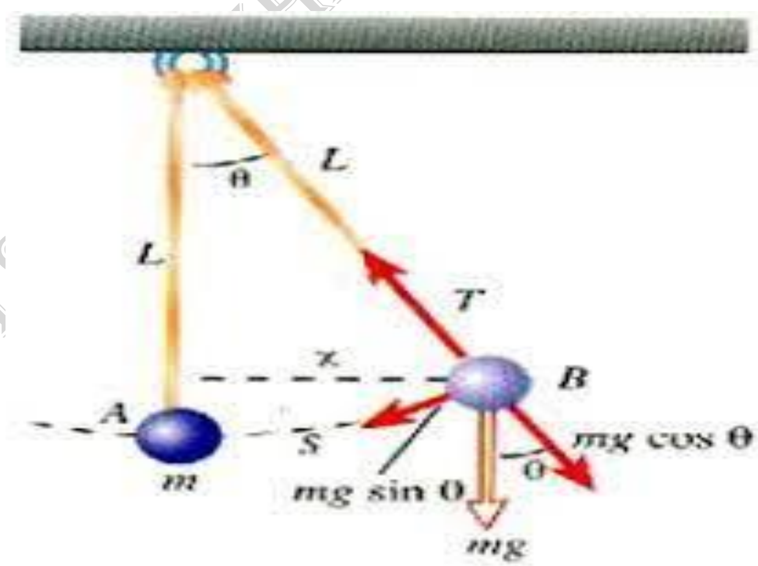
$$\frac{d^2y}{dt^2} + \omega^2 y = 0$$

$y = y_0 \sin(\omega t + \theta)$ harmonic motion

$T = 2\pi\sqrt{\frac{K}{M}}$

2. simple pendulum ;

The simple pendulum is considered one of the mechanical systems that undergo periodic motion. It consists of a body with a mass (m) suspended from a string of length (L) at one end, while the other end remains fixed, as shown in the figure. The motion occurs in the vertical plane and continues due to the influence of gravitational force. It will be shown that when the angle is less than 10 degrees, the motion of the pendulum corresponds to simple harmonic motion.



The forces acting on the suspended body are: the tension force T generated in the string, and the gravitational force mg. The tangential component $mg\sin\theta$ always acts in a direction that makes the angle ($\theta = 0$), and in the opposite direction to the displacement of the body relative to its equilibrium position. Therefore, the tangential component is the restoring force. Thus we can apply Newton's second law to motion in the tangential direction

$$F_t = ma \to -mgsin\ \theta = m\frac{d^2s}{dt^2}$$

In the case where S is the displacement of the body, measured by the length of the arc, it is indicated by a negative sign to indicate that the tangential force points towards the equilibrium point. Since $S = \theta L$, and since L is constant, the previous equation becomes as follows:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

Since θ is the position, is this equation an equation of simple harmonic motion and does it have the same mathematical form? This leads us to expect that this movement does not follow the laws of simple harmonic motion. But if we assume that θ is small, we can make use of the approximation sinθ = θ. Therefore, the approximation makes the equation as follows.

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

Now we have an equation of harmonic motion, from which we infer that the motion of the pendulum with small displacements is a simple harmonic motion.

Where “θ_{max}” is the largest angular position of the pendulum and the angular frequency w is given as follows:

$$w = \sqrt{\frac{g}{L}}$$

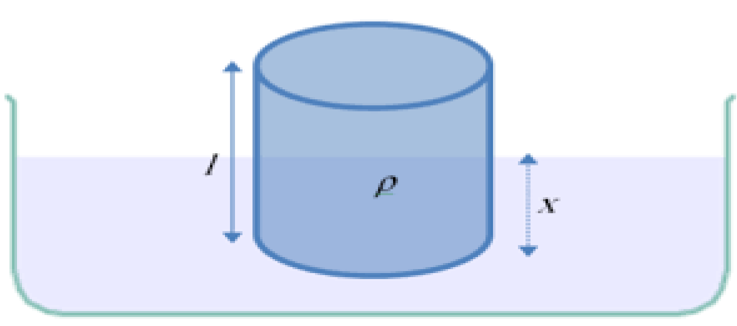
The periodic time is given as follows:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

From here we conclude that the periodic time and frequency of the simple pendulum depend on the length of the string and the acceleration of gravity. Since the periodic time does not depend on the mass of the body suspended in the pendulum, We conclude that every simple pendulum of the same length would, of course, have the same period of time if it were on the surface of the Earth under the influence of gravity .A simple pendulum can be used to determine time because [1]the period of its period depends only on the length of the pendulum and on the Earth's gravitational wheel. These measurements are very important for monitoring changes in the Earth's gravitational wheel in different areas on the Earth's surface, [2]and these measurements may sometimes help in oil exploration.

3-floating body:

Any object floating on the surface of a liquid, when pushed slightly downwards and then released, will oscillate with a vertical up and down movement on the surface of the liquid. Let's assume we have a regular cylindrical body floating in a fluid such that the cylinder is vertical to the surface of the liquid. Observe the figure.



Floating object weight = floating object mass x ground acceleration

$= m_{object} \times g$

$m_{object} = A_{object} * l_{object} * \rho_{object}$

So The weight of the displaced fluid

$g * A * l_{object} * \rho_{object}$

where A ،l_{object} و ، ρ_{object} ,It is the cross-sectional area of the body (cylinder = the area of the displaced fluid), its height and its density, respectively.

Likewise, the weight of the displaced fluid is: ⟶ $g * A_{Fliud} * l_{fluid} * \rho_{fluid}$

In the case of equilibrium, according to Archimedes' principle, the floating body is balanced.

The weight of the floating body = Displaced Liquid Weight

$g * A * l_{object} * \rho_{object} = g * A * l_{fluid} * \rho_{fluid}$

It is produced

$l_{object} / l_{fluid} = \rho_{fluid} / \rho_{object}$

As for when pushing the cylinder down

Weight of additional fluid displaced = force of fluid pushing the cylinder upward ($A \times \rho_{fluid}$ g) This is the only force acting on the cylinder.

According to Newton's second law:

$\sum F = ma$ ……..(1)

$m \frac{d^2x}{dt^2} = -(A x \rho_{fluid}) g$ … . . (2)

The negative sign indicates that the direction of fluid push is opposite to the increasing displacement.

$mg = A l_{object} \rho_{object} g$ ===> $m = A l_{object} \rho_{object}$ …. (3)

Substituting (3) into (2), we get:

$$A l_{object}\, \rho_{object} \frac{d^2x}{dt^2} = -A x \rho_{fluid}\, g \Longrightarrow \frac{d^2x}{dt^2} = -\frac{\rho_{fluid}\ g}{\rho_{object} l_{object}}\, x \ \ldots\ \ldots\ . (4)$$

And compared with the general equation for simple harmonic motion.

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

We conclude that the motion of the floating body, then, will move in harmonic motion simple angular frequency:

$$\omega = \sqrt{\frac{\rho_{fluid}\ g}{\rho_{object} l_{object}}}$$

Example 1: A body with a mass of 25 grams and a spring constant of k = 400 dyne/cm begins its motion with a displacement of 10 cm to the right from its equilibrium position, and with a speed V = 40 cm/s. Find F, w, V, A and x? Then find x,v,a at t=π/8?

Example 2_ When a 10-gram object is suspended, the spring extends by 3.9 cm. Calculate the time when a body with a mass of 25 grams is suspended?

Example_3: A mass of 20 grams is moving in simple harmonic motion with a frequency of 3 hertz and an amplitude of 5 centimeters.

- ***What is the total distance traveled by the moving object during one complete revolution?***
- ***What is the maximum speed reached by the body?***
- ***What is the maximum acceleration of the body?***

Example_4 A mass m was placed on a spring with a spring constant k=6.5 N/m and allowed to undergo(move) simple harmonic motion with an amplitude of 10 cm. The velocity was measured when the mass was at the midpoint between the equilibrium position and the maximum displacement30cm/sec?

- ***body mass***
- ***Movement time***
- ***The highest acceleration of the body***

the solution: