(2-3) Pressure

Pressure is defined as the magnitude of the normal force acting per unit surface area. The pressure is thus a scalar quantity. We write this mathematically as

$$P = \frac{F}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-2)$$

The SI unit for pressure is **newton/meter^2 (N/m^2),** which is given the special name **Pascal (Pa)**, in honor of the French mathematician, physicist, and philosopher, Blaise Pascal. Hence, **1 Pa = 1 N/m^2**.

There are others used units like **lb/in^2 (psi), dyne/cm^2, bar, atm,** and **mm.Hg (torr)** the relation between them is:

1 N/m^2= 1 Pa

1 N/m^2= 1.45 x 10^{-4} lb/in^2

1 N/m^2= 9.87 x 10^{-6} atm

1 N/m^2= 10^{-5} bar

1 N/m^2= 10 dyne/cm^2

1 atm= 760 mm.Hg= 1.013 x 10^5 Pa

Pressures are not limited to fluids, as the following examples show.

Example 5: A man weighs **200 lb**. At one particular moment when he walks, his right heel is the only part of his body that touches the ground. If the heel of his shoe measures **9 cm** by **8 cm**, what pressure does the man exert on the ground in **Pa**?

Answer:

1 cm = 0.393 in

So, 9 cm = 3.5 in

8 cm = 3.25 in

$$P = \frac{F}{A}$$

$$= \frac{200}{3.5 \times 3.25} = 17.6\ lb/in^2$$

$$= 1.21 \times 10^5\ Pa$$

Example 6: A **100 lb** woman is wearing “high-heel” shoes. The cross section of her high-heel shoe measures **1.27 cm** by **1.59 cm.** At a particular moment when she is walking, only one heel of her shoe makes contact with the ground. What is the pressure exerted on the ground by the woman in **Pa**?

Example 7: The two feet of a **60 kg** person cover an area of **500 cm²**, (a) Determine the pressure exerted by the two feet on the ground. (b) If the person stands on one foot, what will the pressure be under that foot?

Example 8: A man has a **140 kg** regular bed that is supported by four legs each leg has a circular cross section of radius **2 cm**. What pressure does this bed exert on the floor?

Example 9: A child weighs **364 N** and sits on a three-legged stool, which weighs **41 N**. The bottoms of the stool's legs touch the ground over a total area of **19.3 cm²**.

a) What is the average pressure that the child and stool exert on the ground?

b) How does the pressure change when the child leans over so that only two legs of the stool touch the floor?

(2-4) Pressure in Fluid of uniform density

Pressure exerted by a fluid is easily determined with the aid of figure 2-1, which represents a pool of water. We want to determine the pressure p at the bottom of the pool caused by the water in the pool. By our definition, equation (2-2), the pressure at the bottom of the pool is the magnitude of the force acting on a unit area of the bottom of the pool. But the force acting on the bottom of the pool is caused by the weight of all the water above it. Thus,

$$P = \frac{F}{A} = \frac{weight\ of\ water}{Area} = \frac{w}{A} = \frac{mg}{A} \quad \dots\dots\dots\dots.(2-3)$$

The mass of the water in the pool, given by equation (2-1), is (m=ρv). The volume of all the water in the pool is just equal to the area $\boldsymbol{A}$ of the bottom of the pool times the depth $\boldsymbol{h}$ of the water in the pool, that is,

$$\mathrm{v} = Ah \quad \dots\dots\dots\dots\dots\dots\dots(2-4)$$

Substituting equations (2-1) and (2-4) into equation (2-3) gives for the pressure at the bottom of the pool:

$$P = \frac{mg}{A} = \frac{\rho \mathrm{v} g}{A} = \frac{\rho A h g}{A}$$

Thus,

$$P = \rho g h \quad \dots\dots\dots\dots\dots\dots\dots (2-5)$$

Equation (2-5) says that the water pressure at any depth h in any pool is given by the product of the density of the water in the pool, the acceleration due to gravity g, and the depth h in the pool. Equation (2-5) is sometimes called **the hydrostatic equation.**

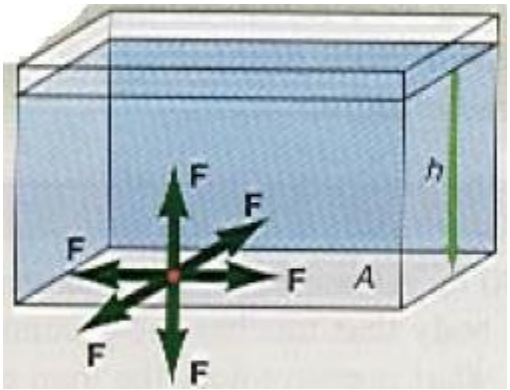

Figure 2-1 water pool

(2-5) Measurement of Pressure

(2-5-1) Atmospheric Pressure

The pressure of the air in the atmosphere was first measured by Evangelista Torricelli (1608-1647), a student of Galileo, by the use of a mercury **barometer.** A long narrow tube is filled to the top with mercury, chemical symbol Hg. It is then placed upside down into a reservoir filled with mercury, as shown in figure (2-2).

The mercury in the tube starts to flow out into the reservoir, but it comes to a stop when the top of the mercury column is at a height **h** above the top of the mercury reservoir, as also shown in figure (2-2). The mercury does not empty completely because the normal pressure of the atmosphere p_o pushes downward on the mercury reservoir. Because the force caused by the pressure of a fluid is the same in all directions, there is also force acting upward inside the tube at the height of the mercury reservoir, and hence there is also a pressure p_o acting upward as shown in figure (2-2). This force upward is capable of holding the weight of the mercury in the tube up to a height h. Thus, the pressure exerted by the mercury in the tube is exactly balanced by the normal atmospheric pressure on the reservoir, that is,

$$P_o = P_{Hg} \quad \dots\dots\dots\dots\dots\dots (2-6)$$

But the pressure of the mercury in the tube P_{Hg}, given by equation (2-5),

$$P_{Hg} = \rho_{Hg} g h \quad \dots\dots\dots\dots\dots\dots (2-7)$$

Substituting equation (2-7) back into equation (2-6), gives

$$P_o = \rho_{Hg} g h \quad \dots\dots\dots\dots\dots\dots (2-8)$$

Equation (2-8) says that normal atmospheric pressure can be determined by measuring the height h of the column of mercury in the tube. It is found experimentally, that on the average, normal atmospheric pressure can support a column of mercury 76.0 cm high, or 760 mm high. The unit of 1.00 mm of Hg is sometimes called a torr in honor of Torricelli. Hence, normal atmospheric pressure can also be given as 760 torr. Using the value of the density of mercury of 1.360×10^4 kg/m^3, normal atmospheric pressure, determined from equation (2-8), is

$$P_o = \rho_{Hg} g h = \left(1.36 \times 10^4 \frac{kg}{m^3}\right)\left(9.8 \frac{m}{s^2}\right)(0.760\, m)$$
$$= 1.013 \times 10^5\, N/m^2 (Pa)$$

Thus, the average or normal atmospheric pressure acting on us at the surface of the earth is 1.013×10^5 Pa.

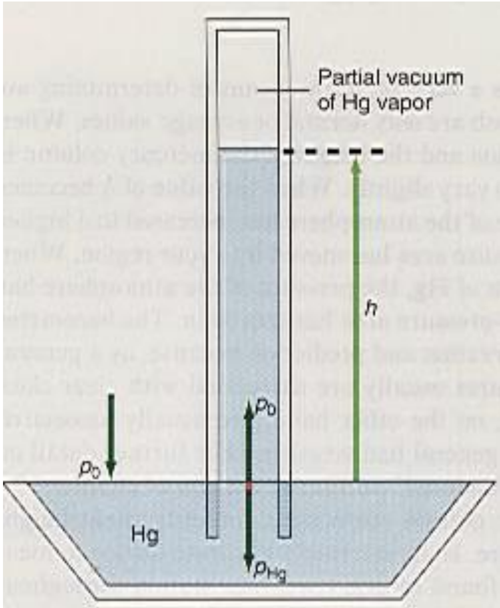

Figure 2-2 A mercury barometer