***University of Mosul***

***College of Science***

***Department of Physics***


***Second stage***

***Lecture 5***



# *Sound and wave Motion*

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***Lecture 5:*** Synthesis **of S.H.M**


***Preparation***

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Chapter three

## Synthesis of simple harmonic motions

The second chapter discussed examples of a body moving with one simple harmonic motion. However, it is necessary to realize that there are many cases in physics where two or more simple harmonic motions combine simultaneously. For example, the eardrum, located inside the ear, is often affected by multiple simple harmonic motions due to the ear capturing multiple sounds with different frequencies at the same time. Additionally, a simple pendulum suspended on a ship's surface is subjected to the simultaneous influence of both the pendulum and the ship's motion if both oscillate together. The effect of these movements on the particle may be in one straight line, or in two perpendicular lines, or in any other direction. In all of these cases, we will try to find the resulting movement resulting from the effect of these movements using the principle of composition.

The principle of superposition: It has special importance in all types of waves and vibrations found in nature, as it is a phenomenon that can be observed and verified to prove the validity of our daily experiences. This principle assumes that multiple wave motions or vibrations can coexist in a specific location without exerting any influence on each other. In other words, it is possible for two or more waves to propagate across the same spatial point simultaneously without undergoing any changes due to the presence of other waves.

Examples of mounting(superposition )rule:

1- When you throw several stones in different locations within a calm body of water, each stone acts as a source of waves that spread across the surface of the water. Once these waves are generated, they converge and interfere with each other, then exit the interference area and continue forward without affecting each other.

2- Sound waves are detected by our hearing system as coming from a particular source, even when they travel through a medium filled with many other sound waves.

3- We perceive surrounding objects clearly through light waves, even though the light reaching our eyes comes from a specific object in a space filled with many light waves travelling in different directions.

This experimental observation indicates that the different waves behave independently of each other. When a group of waves passes through a point in space, the total displacement at that point is equal to the sum of the individual displacements of the waves individually.

This principle only applies to linear wave and vibrational movements, specifically in cases that adhere to Hooke's law within elastic limits, and can be expressed through linear mathematical equations to depict wave or vibrational movements. One of the most prominent equations among these is the equation of simple harmonic motion. The significance of the composition principle becomes evident as it provides a means to analyze complex wave and vibrational movements into their basic components. which are simple harmonic motions, French mathematician G. Fourier was instrumental in asserting that complex periodic waves and vibrational motions are essentially composed of simple harmonic motions. It is worth mentioning that this principle is comprehensive, but it only applies to small-scale movements that can be described by linear equations, where the wave amplitude or vibration is at its minimum. This is indeed the case in most practical cases. However, for movements described by non-linear equations, such as violent vibrations and shock waves, this principle does not apply. In order to analyze and express this principle mathematically, we will use the differential equation of simple harmonic motion, which was derived in the previous chapter.

$\frac{d^2x}{dt^2} = -w^2x$ ……………………..1

This equation shows that the acceleration $((d^2x)/ dt^2 )$ is linearly proportional to the displacement from the equilibrium position x. The two terms in the equation are raised to the power of one, which indicates that it is a linear equation The same variable in both terms It indicates that it is homogeneous, and its solution provides a comprehensive description of the motion. However, if the equation is not linear, meaning that its terms do not include the same variable, the solution becomes difficult and requires advanced mathematics to analyze the resulting movement. This is not required now. Our analysis will now be limited to the homogeneous linear equation (1) only.

Suppose the first appropriate solution to this equation is:.

$x = x_1$ …………………………2

Where $x_1$ can take any form be it Asinωt for example. Let us assume that the appropriate second solution to this equation is:

$x = x_2$ …………………………3

Where $x_2$ can take any other form so be it Bcosωt for example. Substituting the first term into equation (1), we get:

$\frac{d^2x_1}{dt^2} + w^2x_1 = 0$ ……………….4

Substituting the second term into equation (1), we find that

$\frac{d^2x_2}{dt^2} + w^2x_2 = 0$ ……………….5

We add the equations for two (4) and (5) and it turns out

$$\frac{d^2}{dt^2}(x_1+x_2)+w^2(x_2+x_1)=0 \quad \ldots\ldots..6$$

This indicates that there are three solutions to equation (1).

$$x = x_1$$

$$x = x_2$$

$$x = x_1 + x_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

From this it can be concluded that there is an important property that distinguishes the homogeneous linear differential equation, which is that the linear combination of any two solutions to such an equation is considered an appropriate solution to it. Specifically, simply combining two solutions yields a third solution to the homogeneous linear equation. It should be noted that this property does not apply to nonlinear equations. This property represents the configuration rule. Since all simple harmonic motions are governed by a homogeneous linear equation, this means that they are all subject to the rule of addition. In simpler terms, the result of two or more harmonic vibrations is equal to the sum of the individual vibrations affected by the body. Therefore, this rule can be applied to a particle that is subject to more than simple harmonic motion.



## Synthesis of two simple harmonic motions in the same direction:

Suppose we have a particle that simultaneously undergoes two simple harmonic motions with the same frequency along the x-axis. The first simple harmonic motion is represented by the equation:

$$x_1 = a_1 \sin(wt + \theta_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots..8$$

The second simple harmonic motion is given by Eq:

$$x_2 = a_2 \sin(wt + \theta_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots.. 9$$

Where $x_1$ and $x_2$ represent the two displacements of the particle due to the effects of the two harmonic motions, and $a_1$ and $a_2$ represent the amplitudes of the motions. $\theta_1$ and $\theta_2$ represent the initial phase angles, and w represents the angular frequency.

$$x = x_1 + x_2$$

$$\sin(A - B) = sinAcosB - sinBcosA$$

$$\sin(A + B) = sinAcosB + sinBcosA$$

$$\cos(A - B) = cosAcosB + sinBsinA$$

$$\cos(A + B) = cosAco\square\square B - sinBsinA$$

$$x = a_1 \sin(wt + \theta_1) + a_2 \sin(wt + \theta_2)$$

$$x = a_1 [\sin(wt) \cos(\theta_1) + \cos(wt) \sin(\theta_1)] + a_2 [\sin(wt) \cos(\theta_2) + \cos(wt) \sin(\theta_2)]$$

$$x = [\{a_1 \cos(\theta_1) + a2 \cos(\theta_2)\} \sin(wt)] + [\{a_1 \sin(\theta_1) + a_2 \sin(\theta_2)\} \cos(wt)]$$

Since $a_1$ , $a_2$ , $\theta_1$, $\theta_2$ are constants that we can assume equal to

$$a1 \cos(\theta 1) + a2 \cos(\theta 2) = Acos(\theta) \;*$$

$$a_1 \sin(\theta_1) + a_2 \sin(\theta_2) = Asin(\theta) \;**$$

Thus, the above equation is as follows

$$x = Acos(\theta) \sin(wt) + Asin(\theta) \cos(wt)$$

$$x = A \sin(wt + \theta) \quad \text{.............1}$$

By squaring and adding the two equations (* and **), we get

$$[\{Acos(\theta)\}^2 + \{Asin(\theta)\}^2] = [a_1{}^2 cos^2\theta_1 + a_2{}^2 cos^2\theta_2 + 2a_1 a_2 cos\,\theta_1 cos\,\theta_2 + a_1{}^2 sin^2\theta_1 + a_2{}^2 sin^2\theta_2 + 2a_1 a_2 sin\,\theta_1 sin\,\theta_2]$$

$$A^2[cos(\theta)^2 + sin(\theta)^2] = [a_1{}^2 + a_2{}^2 + 2a_1 a_2 (cos\,\theta_1 cos\,\theta_2 + sin\,\theta_1 sin\,\theta_2)]$$

$$A^2 = [a_1{}^2 + a_2{}^2 + 2a_1 a_2 cos\,(\theta_2 - \theta_1)] \quad \text{.....2}$$

Now by dividing the two equations (* and **) we get

$$\frac{A\sin(\theta)}{A\cos(\theta)} = \frac{a_1 \sin(\theta_1) + a_2 \sin(\theta_2)}{a_1 \cos(\theta_1) + a_2 \cos(\theta_2)}$$

$$\tan\theta = \frac{a_1 \sin(\theta_1) + a_2 \sin(\theta_2)}{a_1 \cos(\theta_1) + a_2 \cos(\theta_2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

Equation 1 represents the simultaneous displacement x for the two simple harmonic motions. It is noted that they are similar to equations $x_2$ and $x_1$, which indicates that they also represent simple harmonic motion with the same common angular frequency for the two components of motion.

A represents the amplitude of the simple harmonic motion resulting from the summation of the two motions and can be obtained from Relation 2. θ represents the initial phase angle of the resultant motion obtained using Relation 3. There are several important results related to the deduction of simple harmonic motion from Relation 1, especially regarding the interference between Any two movements

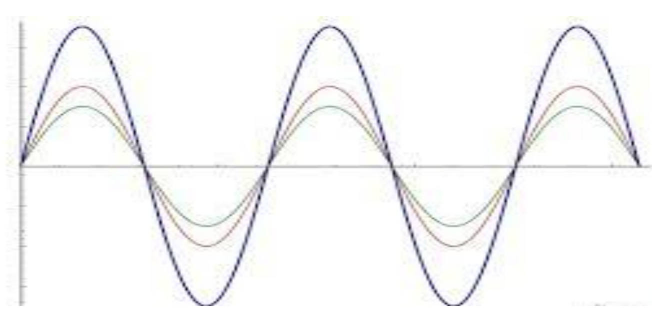
1- The overlap between two simple harmonic motions of the same frequency and phase and different amplitudes:

i.e

$\theta_1 = \theta_2 = \theta$

The equation becomes as follows:

$x = (a_1 + a_2)\sin(wt + \theta)$



From the above graph, it can be seen that the amplitude of the resulting vibrating motion is equal to the sum of the amplitudes of the two overlapping motions having the same frequency and phase. This means that the two harmonic movements in this case reinforce each other and it is called constructive interference, and when the two amplitudes are equal, their sum is twice the original amplitude.

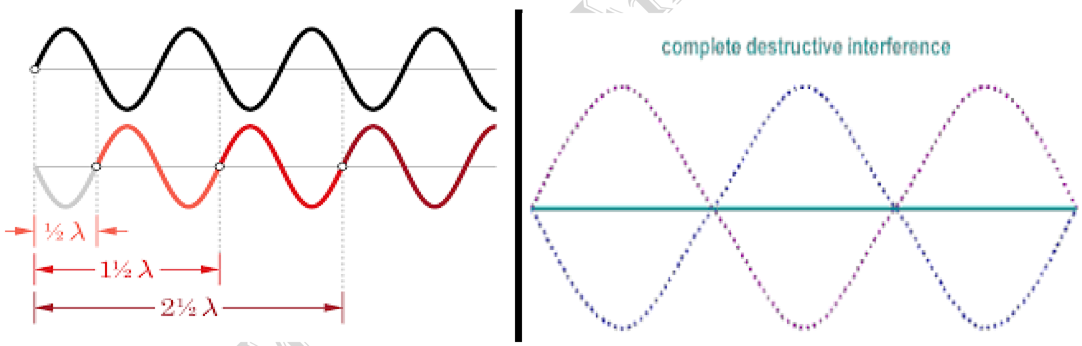
2-The interference between two simple harmonic motions of the same frequency but different amplitude and phase.

Let's take the case is that

$\theta_2 = \pi$ ,  $\theta_1 = 0$

The above equation becomes as follows:

$x = (a_1 - a_2)\sin(wt)$




It is evident from the figure that the resultant of the capacitance equals the difference between the capacities of the two overlapping compatible movements, and the peak of one of them is above the bottom of the other. The movements oppose each other, and in this case, they destruct each other and are called destructive interference. In the case of $a_1$=$a_2$, the resultant is zero, as shown in the above figure.