(2-6) The variation of Pressure in a Fluid

As shown in Figure 2-3, let us determine the pressure at any height y above some reference point (such as the ocean floor or the bottom of a tank or swimming pool). Within this fluid, at the height y, we consider a tiny, flat, slab like volume of the fluid whose area is A and whose (infinitesimal) thickness is dy, as shown.

The pressure acting upward on its lower surface (at height y) $= P$.

The pressure acting downward on the top surface of our tiny slab (at height $y + dy$) $= P + dP$.

The fluid pressure acting on our slab thus exerts a force $= P\,A$ upward on our slab and a force $= (P + dP)A$

The mass of the element $dm = \rho\, dv = \rho\, A\, dy$

The weight of the element $= dw = (dm)g = \rho\, g\, A\, dy$

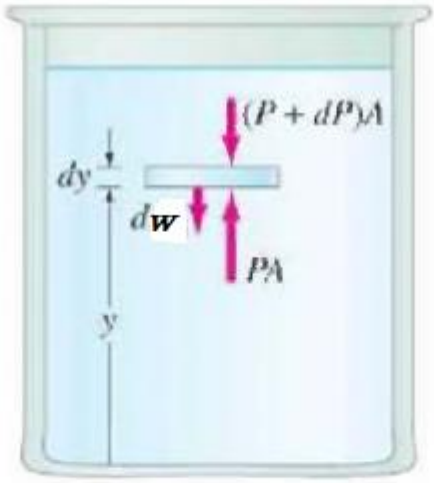

Figure 2-3

Hence for vertical equilibrium:

$P\,A = (P + dP)\,A + dw$

$P A = P A + dp\, A + \rho\, g\, A\, dy$

$-\rho\, g\, dy = dP$

$$\frac{dP}{dy} = -\rho\, g \qquad \dots\dots\dots\dots\dots\dots\dots (2-9)$$

This equation represents the **hydrostatic equilibrium condition** and tells us how the pressure varies with height above any reference point. The minus sign indicates that the pressure decreases with an increase the height.

If the pressure at a height y_1 in the fluid is P_1, and at height y_2 it is P_2, then we can integrate Eq. (2-9) to obtain

$$\int_{P_1}^{P_2} dP = -\int_{y_1}^{y_2} \rho g dy$$

$$P_2 - P_1 = -\int_{y_1}^{y_2} \rho g dy$$

Taking ρ and g as constants, we obtain

$$P_2 - P_1 = -\rho g(y_2 - y_1)$$

Let y_2 be the height of the surface at which point the pressure (P_2) acting on the fluid is usually the atmospheric pressure (P_o).

y_1 is any level in the fluid when the pressure at them is (P), then,

$$P_o - P = -\rho g(y_2 - y_1)$$

But $y_2 - y_1 = h$ the depth below the surface (figure 2-4) so that

$$P = P_o + \rho g h$$

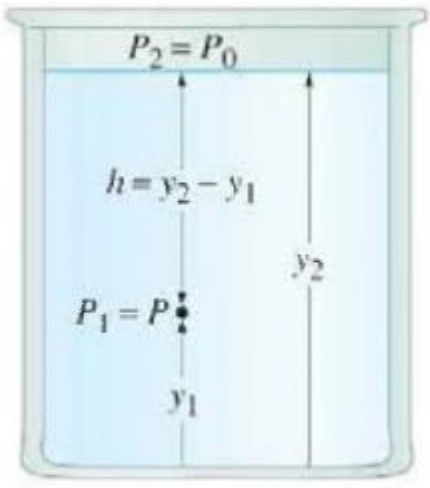

Figure 2-4

(2-7) The variation of Pressure in the atmosphere

1) If the air is considered as an ideal gas

P V = n R T (ideal gas equation)

$$\frac{n}{V} = \frac{P}{RT} \quad \dots\dots\dots\dots\dots\dots\dots (2-10)$$

$$\rho = \frac{m}{V}, \quad m = n\,M$$

$$\rho = \frac{nM}{V} = \frac{n}{V}M \quad \dots\dots\dots\dots\dots\dots\dots (2-11)$$

Substituting equation (2-10) back into equation (2-11), gives

$$\rho = \frac{P}{RT}M = \frac{M}{RT}\ P \quad \dots\dots\dots\dots\dots\dots (2-12)$$

Where: R=gas constant=8.31Pa.m^3/mol.K

n= number of moles of the gas

M= molecular weight for ideal gas

T= absolute temperature for the gas

Put (ρ) from equation (2-12) in equation (2-9) we will get:

$$\frac{dP}{dy} = -\frac{M}{RT}Pg$$

$$\frac{dP}{P} = -\frac{M}{RT}gdy$$

$$\int_{P_1}^{P_2}\frac{dP}{P} = -\frac{Mg}{RT}\int_{y_1}^{y_2}dy$$

$$[lnP]_{P_1}^{P_2} = -\frac{Mg}{RT}(y_2 - y_1)$$

$$ln\frac{P_2}{P_1} = -\frac{Mg}{RT}(y_2 - y_1)$$

Take the exp. Of two sides

$$\frac{P_2}{P_1} = e^{-\frac{Mg}{RT}(y_2 - y_1)}$$

OR $\quad P_2 = P_1 e^{-\frac{Mg}{RT}(y_2 - y_1)} \quad \dots\dots\dots\dots\dots\dots\dots (2-13)$

This equation is called the **barometric equation.** This equation used to model how the pressure of the air changes with altitudes.