

University of Mosul

College of Science

Department of Physics

Second stage

Lecture 7

Sound and wave Motion

2024-2025

Lecture 7: beats(Strokes)

Preparation

M. Maysam Shihab Ahmed

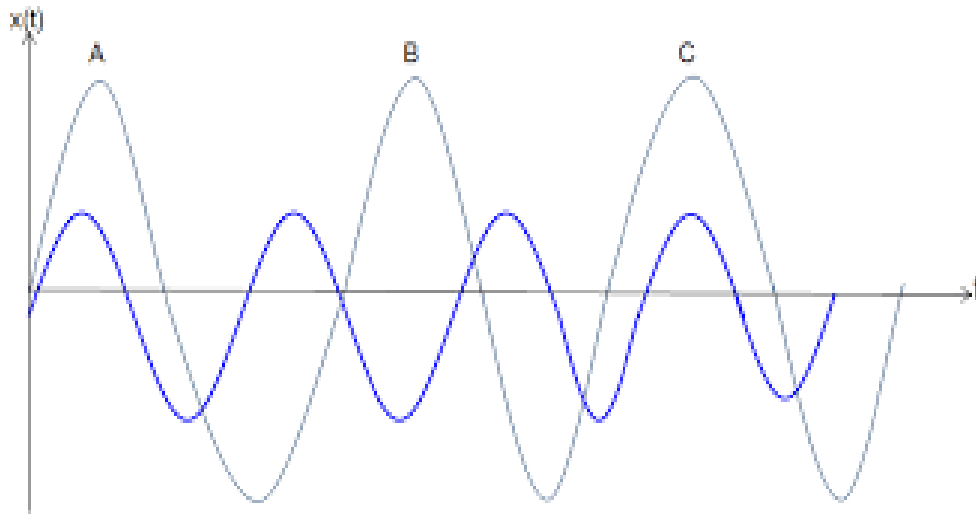
The beats(Strikes) or combination of two vibrations slightly different in frequency:

When a particle is affected by two simple harmonic motions, the difference between their frequencies is very small, the amplitude of the resulting vibrational motion of the particle alternates between maximum and minimum with the passage of time. This pattern of periodic motion is known as strokes.

When the two synchronous movements occur along a specific axis as a result of the slight difference in their frequencies, a gradual change in phase difference between the movements occurs over time. At a specific moment, such as the one corresponding to point A in Figure (1), the movements are in the same phase, meaning the displacements occur in the same direction. As a result, the amplitude of the vibrational motion is at its peak, representing constructive interference. The resultant displacement of the particle at that moment is equal to the sum of the displacements. Over time, the movements diverge in phase, leading to an increase in the phase difference between them until it reaches π (180), as shown at point B in Figure (1). At this point, the displacements are opposite, each seeking to cancel the other out, resulting in the minimum amplitude of the vibrational motion, representing destructive interference. The resultant displacement of the particle is equal to the difference between the displacements.

As time passes, the phase difference between the two motions increases until it reaches a value of 2π , as shown by the time instant corresponding to C, resulting in constructive interference again followed by destructive interference after a specified period of time. This sequence of events repeats, causing the total amplitude of the vibrational motion to alternate

between its highest and lowest values over time at a constant frequency. It is called the phenomenon of Beats frequency, and it is equal to the difference between the frequencies of the two harmonic movements.



This phenomenon can be explained analytically. Suppose there is a particle in a medium subject to vibration under the influence of two harmonic motions of slightly different frequencies. As a result, the phase difference between the two movements is constantly changing. Therefore, it becomes unnecessary to specify an initial value for the phase difference between movements. If the instantaneous displacement of the particle at time t , due to first harmonic motion of amplitude A_1 and frequency f_1 , is denoted by x_1 , where in.

$$x_1 = A_1 \sin \omega_1 t = A_1 \sin 2\pi f_1 t \dots\dots\dots 1$$

The instantaneous displacement of the same particle at time t, caused by the influence of the second harmonic motion with an amplitude of A_2 and frequency f_2 , is denoted as x_2 .

$$x_2 = A_2 \sin \omega_2 t = A_1 \sin 2\pi f_2 t \dots\dots\dots 2$$

The resultant displacement x at time t results from the combination of the two motions, that is,

$$x = x_1 + x_2$$

$$x = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t \dots\dots\dots 3$$

Examination of the effects resulting from the strikes can be easily evaluated by considering the two movements of equal amplitude.

If we assume that A is identical to A_1 and A_2 , the equation can be formulated as follows:

$$x = A \sin \omega_1 t + A \sin \omega_2 t \dots\dots\dots 4$$

$$x = 2A \cos\left(\frac{\omega_2 - \omega_1}{2}\right) t \sin\left(\frac{\omega_2 + \omega_1}{2}\right) t \dots\dots\dots 5$$

The above equation provides a comprehensive explanation of a pure mathematical result that is applicable and general for all values of ω_1 and ω_2 . However, it does not provide a comprehensive representation of the multiplication phenomenon unless the difference between ω_1 and ω_2 is small.

$$f_2 - f_1 = \Delta f \dots\dots\dots 6$$

The frequency difference, referred to as Δf , must not exceed 10 hertz. This value depends on the time interval between two consecutive beats and the human ear's ability to distinguish that. Graphically, equation (5) can be represented as shown in figure. Specifically, the red part represents the first harmonic motion at frequency f_1 . On the other hand, the part in dotted blue represents the second harmonic motion, which has frequency f_2 . Finally, the part represents the result of the combination of the two movements, which includes two frequencies. The first frequency is relatively high and falls within the dotted envelope, while the second frequency is lower and represents the dotted envelope itself. It is noted from the figure that the capacitance changes sinusoidally and this phenomenon is called capacitance modulation or modulation, which plays an important role in electromagnetic communications, electronics and acoustics.

A physical interpretation of equation (5) can be readily provided by presenting it in the following model.

$$x = B \sin\left(\frac{\omega_2 + \omega_1}{2}\right)t \dots\dots\dots 7$$

$$B = 2A \cos\left(\frac{\omega_2 - \omega_1}{2}\right)t \dots\dots\dots 8$$

Equation (7) indicates a periodic movement with amplitude B , oscillating at a high frequency. The mentioned frequency is equivalent to the arithmetic average of the two original frequencies, i.e. $((f_1 + f_2)/2)$, which represents the actual frequency of the resultant movement, which falls within the discontinuous boundaries shown in the figure. This oscillating component (factor) $\sin((\omega_2 + \omega_1)/2)t$ always lies between the two limits ± 1 . Equation

(8) gives the amplitude of movement B, and it is noted that it changes periodically with time.

