University of Mosul
College of science
Department of Physics
Third Stage
Lecture 7

Laser

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Lecture 7: Longitudinal and Transverse Modes

Preparation
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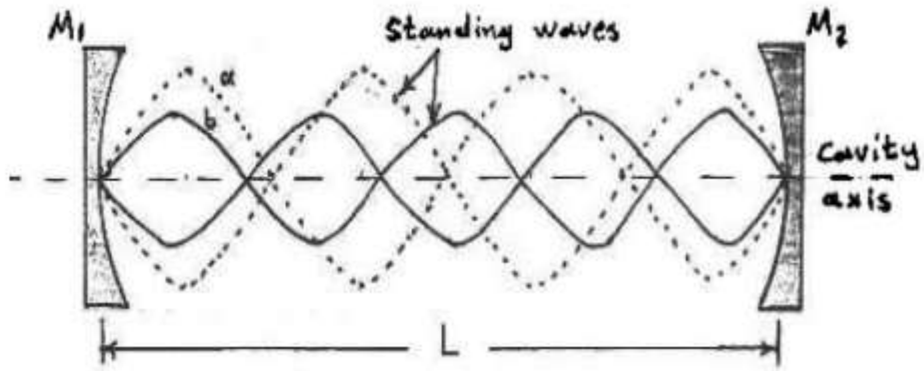
Longitudinal (Axial) Modes

Laser oscillations occur, when the wave within the cavity replicate itself after two reflections so that the electric fields add in phase. In other words, the mirrors form a **resonant cavity** and **standing wave** patterns are setup. The cavity **resonates** when there is an integer number (m) of half wavelengths spanning the region between the mirrors, as shown in figure. That there must be a mode at each mirror and this can only happen when the separation of the mirrors (L) equals a whole number multiple of $\lambda/2$.



Thus,

$$m\frac{\lambda}{2} = L$$

or

$$m = \frac{L}{\lambda/2}$$

and if the refractive index of the active medium is unity, the frequency is given as,

$$\upsilon_m = \frac{c}{\lambda}$$

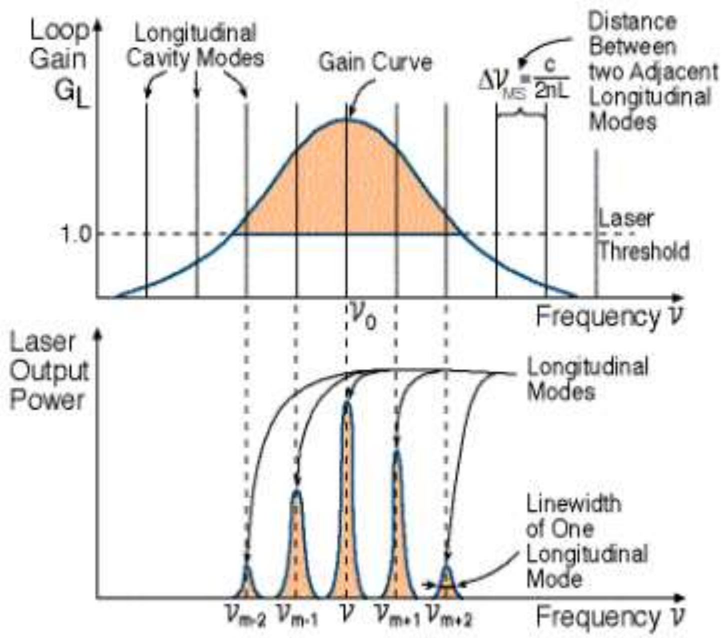
Or

$$\upsilon_m = \frac{mc}{2L}$$

Therefore, there are an infinite number of possible oscillatory longitudinal cavity modes, each with a distinctive frequency υ_m, the frequency separation Δm=1 is given by:

$$\upsilon_{m-1} - \upsilon_m = \Delta\upsilon = \frac{c}{2L}$$

The approximate number of possible laser modes is given by the width of the Laser bandwidth divided by the distance between adjacent modes.



In fact, the number of modes (m) in most lasers is very large. For Example, if the central wavelength is 500nm and the mirror separation is 25cm, n has a value of 1000000, since n can be any integer, there are many possible wavelengths within the laser transition shape.

Example (1)

The length of the optical cavity in He-Ne laser is 30 cm. The emitted wavelength is 0.6328 mm. Calculate:

1. The difference in frequency between adjacent longitudinal modes.

2. The number of the emitted longitudinal mode at this wavelength.

3. The laser frequency.

Solution:

1. The equation for difference in frequency is the same as for the basic mode:

$\Delta\nu = c/(2L) = = 3\text{x}10^8$ [m/s]/(2x0.3 [m]) = $0.5\text{x}10^9$ [Hz] = 0.5 [GHz]

2. From the equation for the wavelength of the n th mode:
   $\lambda_n = 2L/m$
   $m = 2L/\lambda_m = 2\text{x}0.3$ [m]/$0.6328\text{x}10^{-6}$ [m] = $0.948\text{x}10^6$

which means that the laser operates at a frequency which is almost a million times the basic frequency of the cavity.

3. The laser frequency can be calculated in two ways:
   a) By multiplying the mode number from section 2 by the basic mode frequency:
      $\nu = m\Delta\nu = (0.948\text{x}10^6)\ (0.5\text{x}10^9$ [Hz]) = $4.74\text{x}10^{14}$ [Hz]
   b) b) By direct calculation:
      $\nu = c/\lambda = 3\text{x}10^8$ [m/s]/$0.6328\text{x}10^{-6}$ [m] = $4.74\text{x}10^{14}$ [Hz]

Example (2)

The length of the optical cavity in He-Ne laser is 55 cm. The Laser bandwidth is 1.5 [GHz]. Find the approximate number of longitudinal laser modes.

Solution:

The distance between adjacent longitudinal modes is:
$\Delta\nu = c/(2L) = 3\text{x}10^8$ [m/s]/(2x0.55 [m]) = $2.73\text{x}10^8$ [m/s] = 0.273 [GHz]
The approximate number of longitudinal laser modes:
m=Laser bandwidth /$\Delta\nu$= 1.5 [GHz]/0.273 [GHz] = $5.5 \approx 5$

Transverse modes

The longitudinal modes all contribute to a single spot of light in the laser output, whereas in general if the laser beam is shone onto a screen, we observe a pattern of spots. These are due to the transverse modes of the cavity.

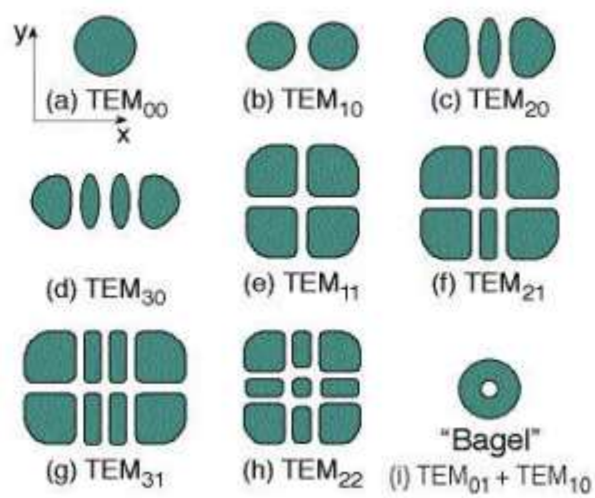
A **transverse mode of electromagnetic** radiation is a particular electromagnetic field pattern of the radiation in the plane perpendicular (i.e., transverse) to the radiation's propagation direction TEM_{mn}. They are characterized by two integers m and n, so that as figure shows, we have TEM_{00}, TEM_{01}, TEM_{11}, … etc. modes (m gives the number of minima as the beam is scanned horizontally and n the number of minima as the beam is scanned vertically).

<u>Or</u>

The intensity distribution across the width of the cavity is the Transverse Mode pattern.

The transverse mode determines the beam shape. The general form for the transverse electric modes is: TEMmn where m is the number of modes in x-direction and n the respectively number of nodes in y- direction

A single transverse mode laser is restricted to give TEM_{00} output.



A small misalignment of the laser mirrors causes different path length for different "rays" inside the cavity. Thus, the distribution of intensity is not the perfect Gaussian distribution.

The TEM_{00} transverse mode is the most wildly used, and this for several resonator the flux density is ideally Gaussian over the beam cross section; and it can be focused down to the smallest – sized spot.

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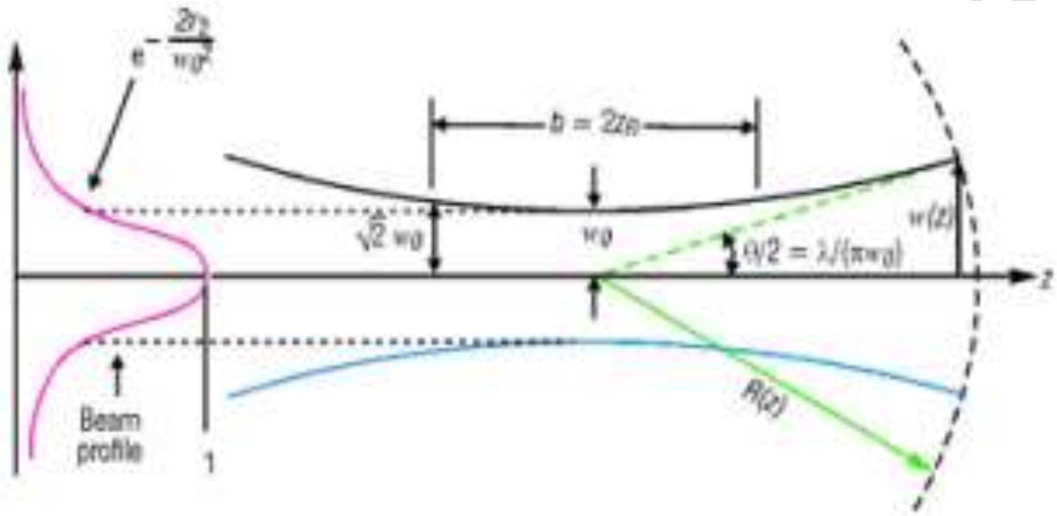
The properties and propagation of a Gaussian laser beam:

In a gain medium located within an optical resonator, the TEM_{00} Gaussian mode that develops when the single – pass gain exceeds the cavity losses have a Gaussian profile at the mirrors in the direction transverse to the direction of propagation of the beam, as shown in Figure.



A Gaussian laser beam has the following properties:

1. The beam has a Gaussian transverse profile at all locations. Such a Gaussian beam can be characterized completely at any spatial location by defining both its "beam waist" and its "wavefront curvature" at a specific location of the beam.
2. A Gaussian beam always has a minimum beam waist (w_o) at one location in space.
3. The transverse distribution of the intensity of a simple Gaussian beam is of the form

$$I = I_o e^{\frac{-2x^2}{w^2}}$$

Where I_o is the maximum intensity and w is the beam radius inside of which 86.5% of the energy is concentrated, as shown in Figure.

The Gaussian beam minimum waist w_o for a typical laser resonator mode occurs in the region between the two mirrors of an optical resonator. For example, the minimum beam waist w_o in a confocal optical resonator ($R_1=R_2=L$) occur halfway between the two mirrors. As the Gaussian beam propagate it expand and diverges from that location, such that the beam waist at a distance of ±z from the minimum beam waist w_o can be described as:

$$w_z = w_o \left[1 + \left(\frac{\lambda z}{\pi w_o^2}\right)^2\right]^{1/2}$$

The beam wavefront curvature of a Gaussian beam at a location z, in term of the minimum beam waist w_o and the wavelength λ, is given by:

$$R_z = z \left[1 + \left(\frac{\pi w_0^2}{\lambda z}\right)^2\right]$$

The angular spread of a Gaussian beam for a value of z is given by:

$$\theta_z = \frac{2\lambda}{\pi w_o}$$

As shown in Figure.

The θ_z term is the full angle, at a given location z, over which the beam reduces to half of its maximum intensity at the center of beam.

H.W

Example

An He-Ne laser operating in a single TEM_{00} mode at 632.8 nm has a mirror separation of 0.5 m with mirrors R1=R2=1m, calculate the radius and wavefront curvature of the Gaussian laser beam at a distance of 10m away from the minimum beam radius (w_o) of 0.3m.

https://www.google.com/search?q=longitudinal+and+transverse+modes%2C+vedio&sca_esv=7319a1d5414ff2b8&ei=ZQutZ4LMKc-qxc8Pz6vViQU&ved=0ahUKEwjCkqGZg7-LAxVPVfEDHc9VNVEQ4dUDCBA&uact=5&oq=longitudinal+and+transverse+modes%2C+vedio&gs_lp=Egxnd3Mtd2l6LXNlcnAiKGxvbmdpdHVkaW5hbCBhbmQgdHJhbnN2ZXJzZSBtb2RlcywgdmVkaW8yBxAhGKABGAoyBxAhGKABGAoyBxAhGKABGApIyCtQ1gpYySRwAXgBkAEAmAGMAqABhwyqAQUwLjEuNrgBA8gBAPgBAZgCCKACzQzCAgoQABiwAxjWBBhHwgIFEAAYgATCAgYQABgWGB7CAgsQABiABBiGAxiKBcICCBAAGIAEGKIEwgIFECEYoAGYAwCIBgGQBgiSBwUxLjAuN6AHmiY&sclient=gws-wiz-serp#fpstate=ive&vld=cid:89eca196,vid:zOxuVbgI0a4,st:0