



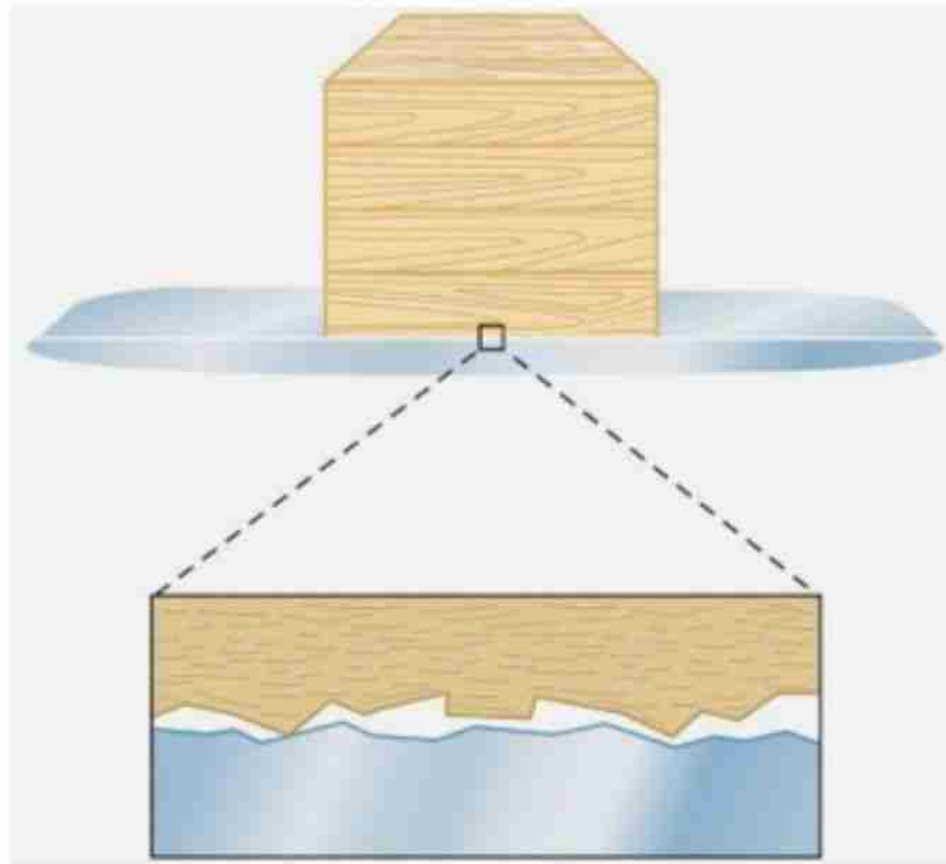
# lecture 7 Frictions

**<https://youtu.be/g550H4e5FCY?t=8>**

# Frictional Forces

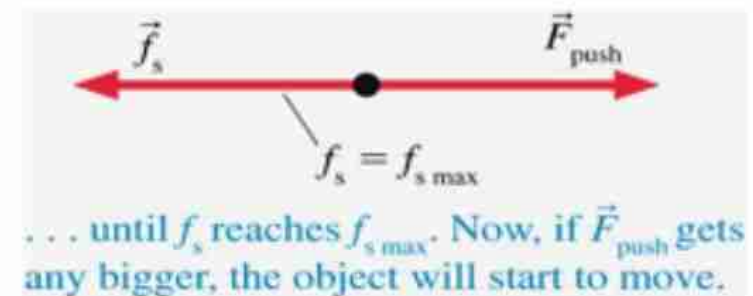
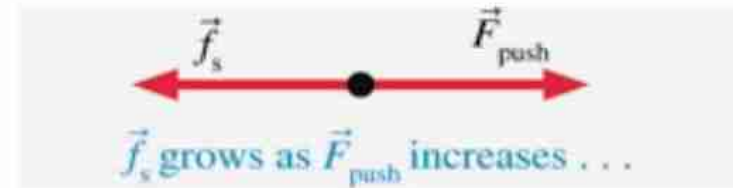
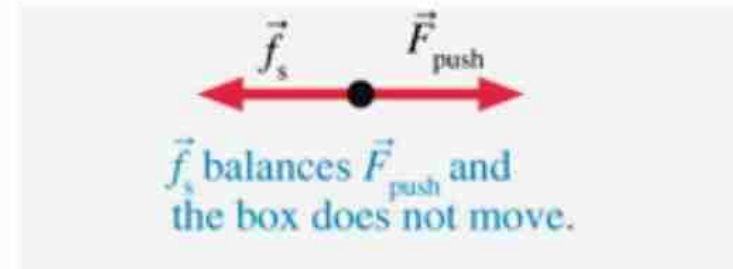
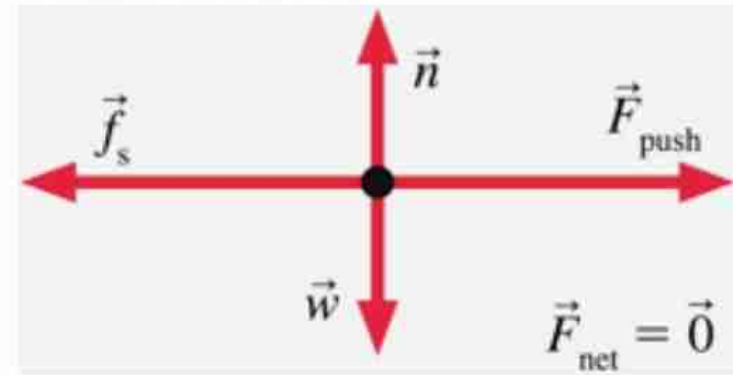
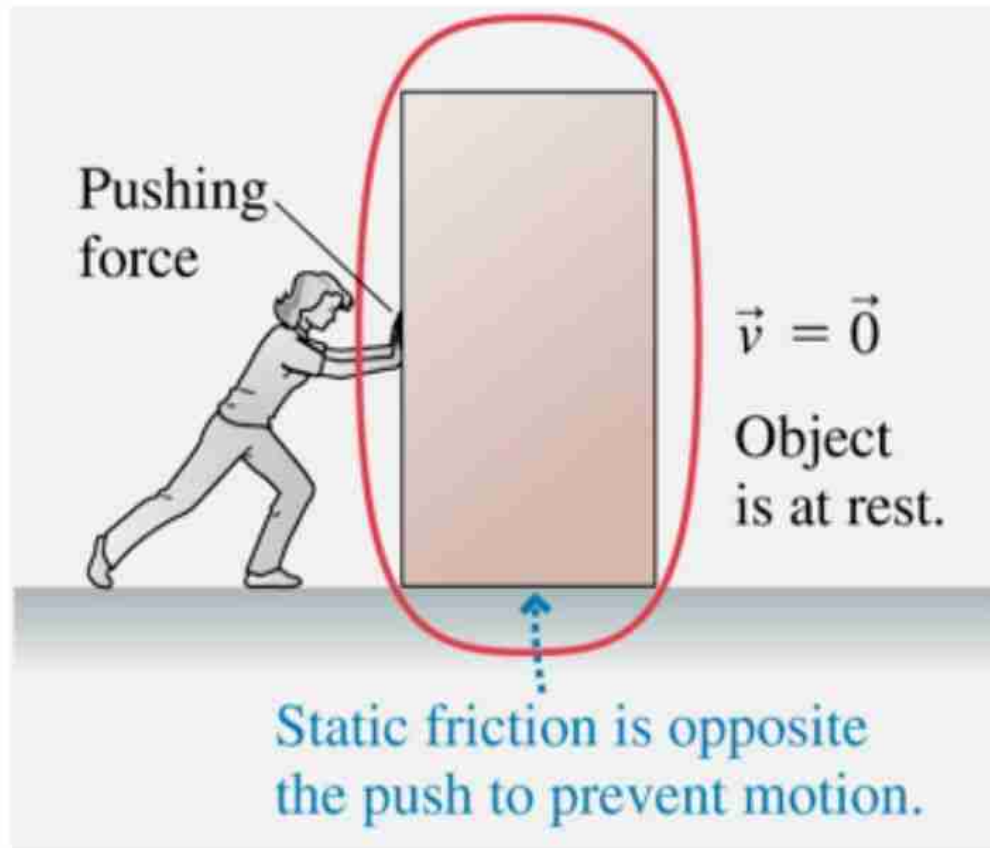


Friction has its basis in surfaces that are not completely smooth:

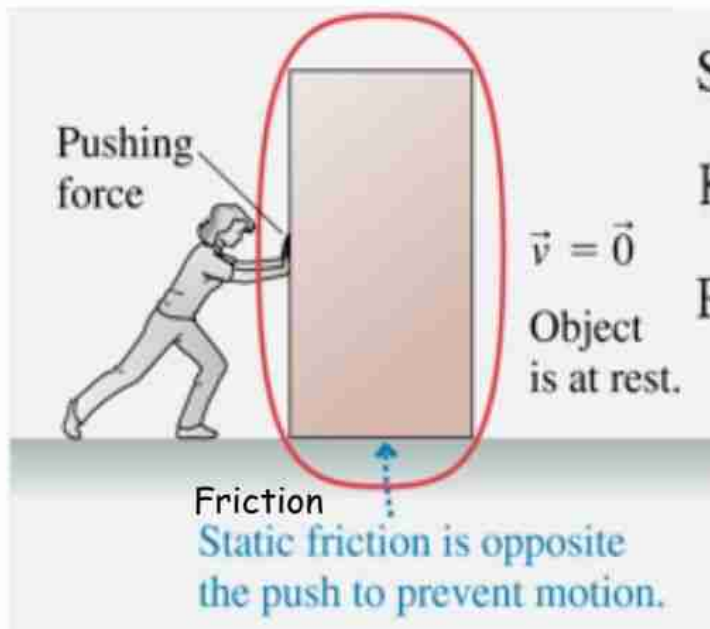




# Static Friction



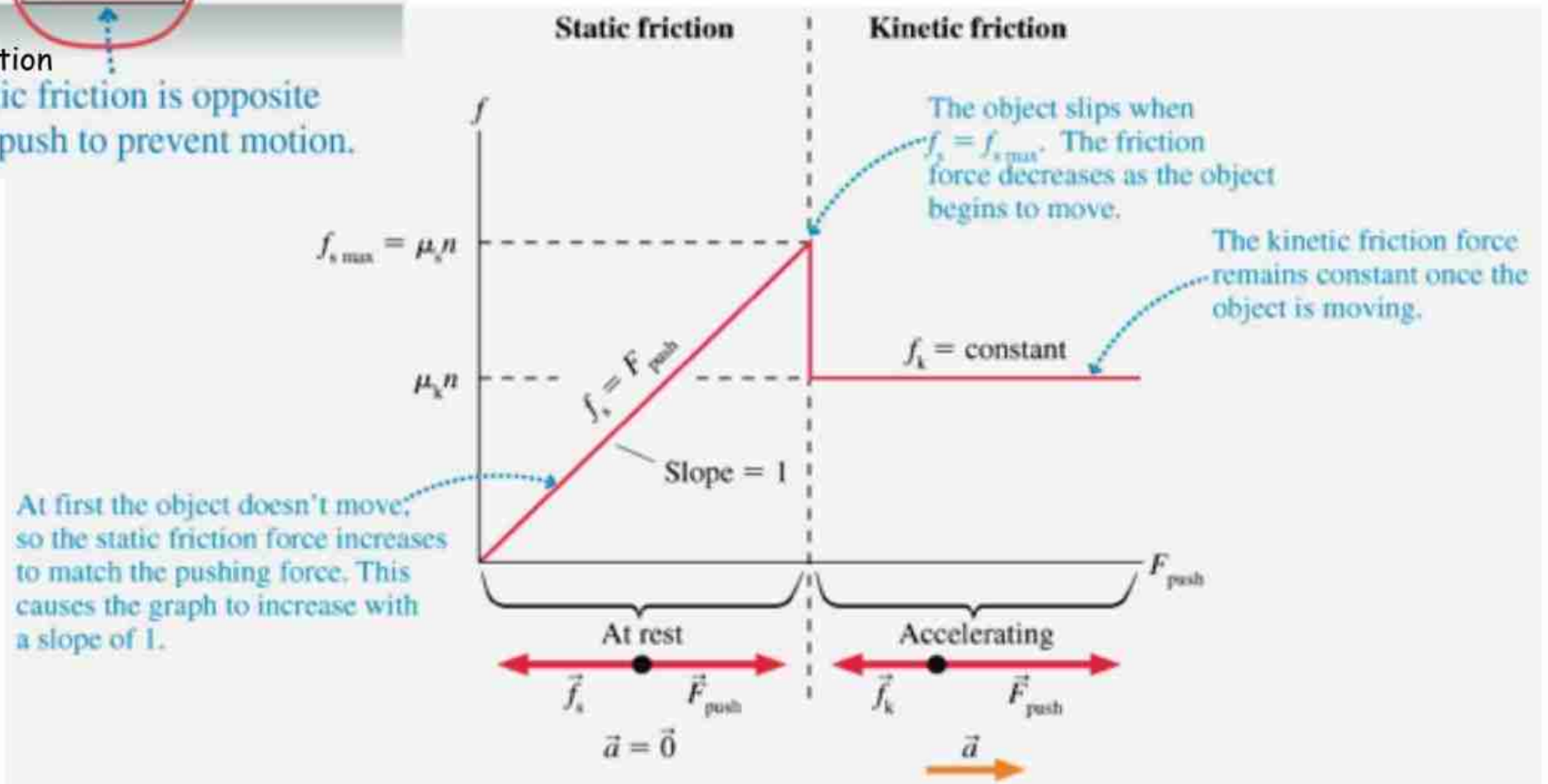
# A Model of Friction



Static:  $\vec{f}_s \leq (\mu_s n, \text{direction as necessary to prevent motion})$

Kinetic:  $\vec{f}_k = (\mu_k n, \text{direction opposite the motion})$

Rolling:  $\vec{f}_r = (\mu_r n, \text{direction opposite the motion})$



# Static Frictional Forces



$$0 \leq f_s \leq f_{s,\max}$$

where

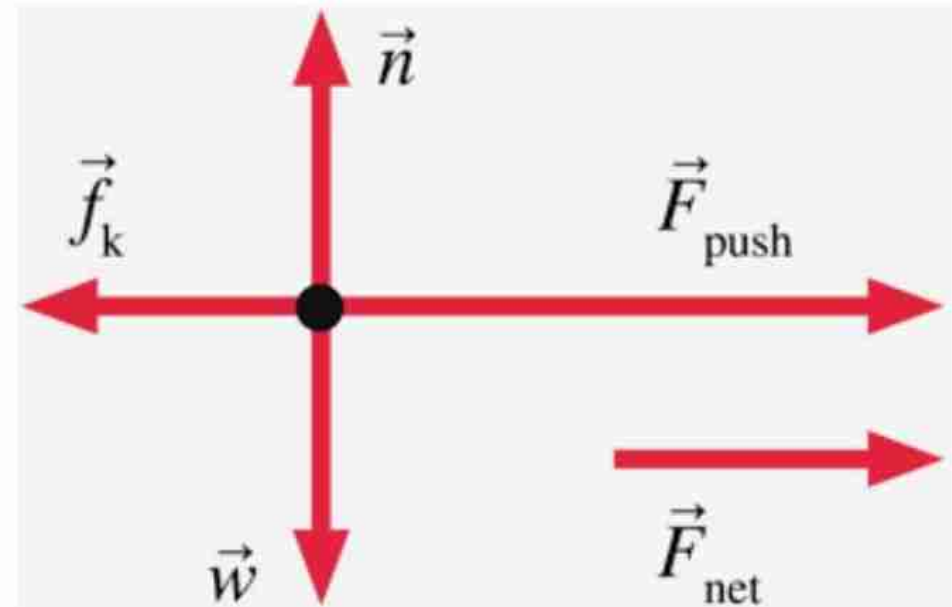
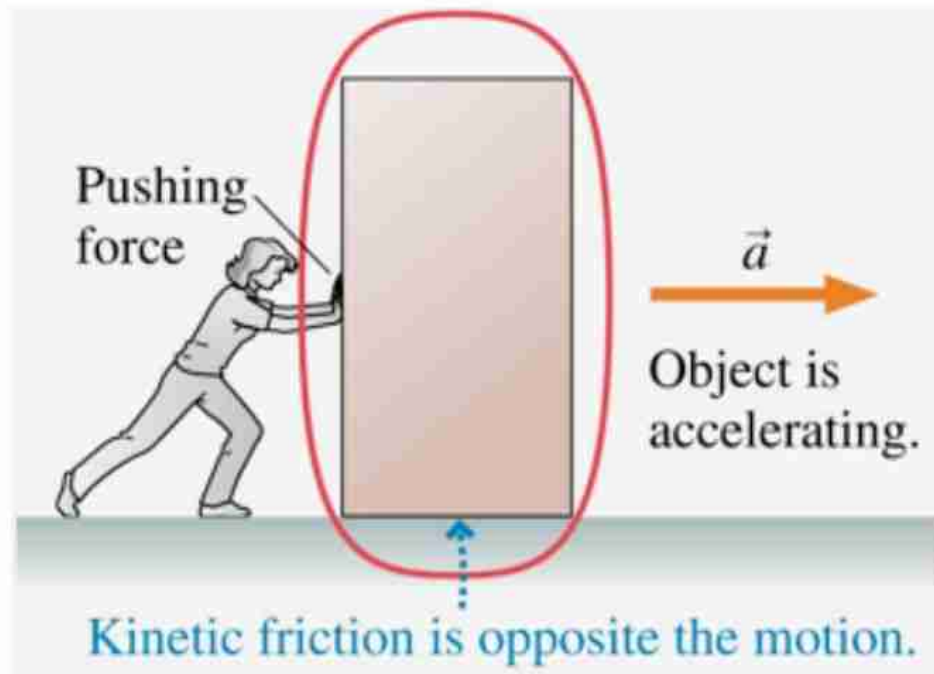
$$f_{s,\max} = \mu_s N$$

The static frictional force is also independent of the area of contact and the relative speed of the surfaces.





# Kinetic Friction



$$F_{\text{push}} \geq f_k = \mu_k n$$



# Kinetic Friction

**Kinetic friction:** the friction experienced by surfaces sliding against one another

The kinetic frictional force depends on the normal force:

$$f_k = \mu_k N$$

The constant  $\mu_k$  is called the coefficient of kinetic friction.

# Coefficients of Friction



**TABLE 6-1** Typical Coefficients of Friction

Materials	Kinetic, $\mu_k$	Static, $\mu_s$
Rubber on concrete (dry)	0.80	1–4
Steel on steel	0.57	0.74
Glass on glass	0.40	0.94
Wood on leather	0.40	0.50
Copper on steel	0.36	0.53
Rubber on concrete (wet)	0.25	0.30
Steel on ice	0.06	0.10
Waxed ski on snow	0.05	0.10
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.003	0.01

$$\mathbf{f}_s \leq \mu_s \mathbf{F}_n$$

$$\mathbf{f}_{s \max} = \mu_s \mathbf{F}_n$$

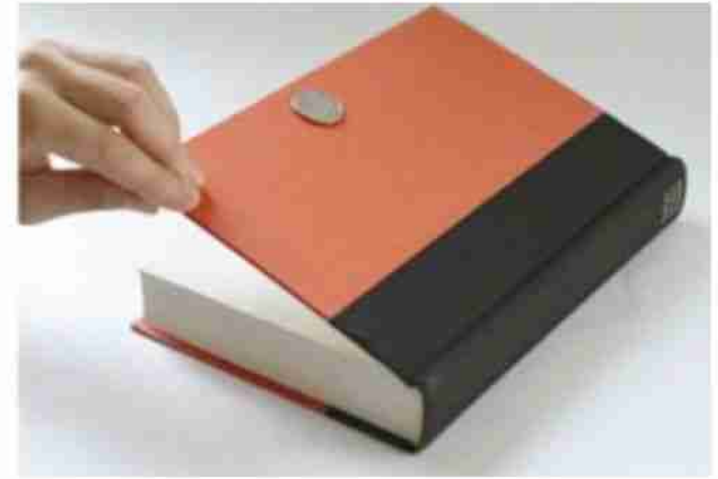
$$\mathbf{f}_k = \mu_k \mathbf{F}_n$$

Typically,  $\mu_s > \mu_k$



# Example: A Sliding Coin

A hardcover book is resting on a tabletop with its front cover facing upward. You place a coin on the cover and very slowly open the book until the coin starts to slide. The angle  $\theta_{\max}$  is the angle of the cover just before the coin begins to slide.



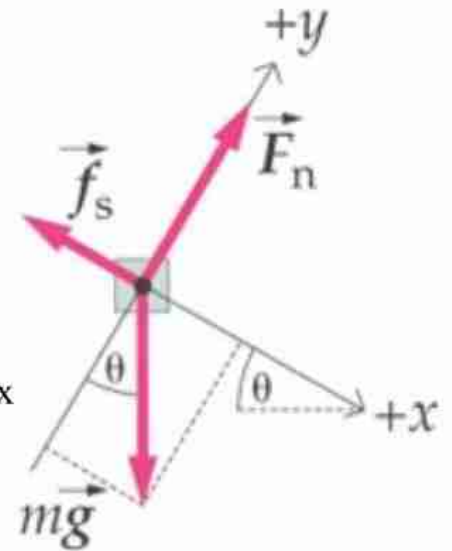
Find the coefficient of static friction  $\mu_s$  between the coin and book.

$$\sum F_y = ma_y \Rightarrow F_n - mg \cos \theta = 0 \text{ or } F_n = mg \cos \theta$$

$$f_s = \mu_s F_n \text{ at } \theta_{\max}, \text{ so } f_s = \mu_s mg \cos \theta_{\max}$$

$$\sum F_x = ma_x \Rightarrow -f_s + mg \sin \theta_{\max} = 0 \text{ or } f_s = mg \sin \theta_{\max}$$

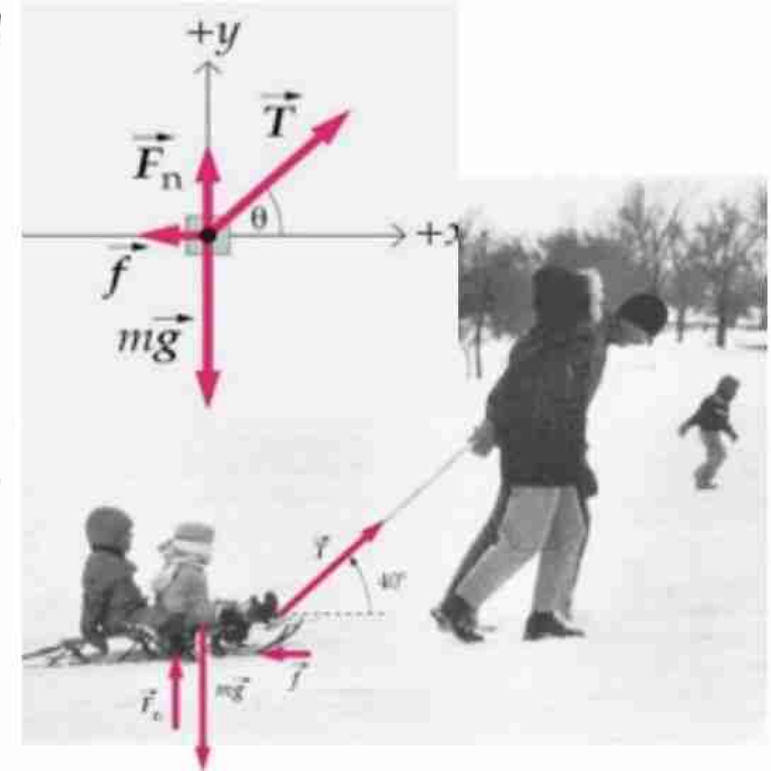
$$\text{Therefore, } \mu_s \cos \theta_{\max} = \sin \theta_{\max} \text{ or } \mu_s = \tan \theta_{\max}$$



# Example: Pulling A Sled

Two children sitting on a sled at rest in the snow ask you to pull them. You oblige by pulling on the sled's rope, which makes an angle of  $40^\circ$  with the horizontal. The children have a combined mass of 45 kg, and the sled has a mass of 5.0 kg. The coefficients of static and kinetic friction are  $\mu_s=0.20$  and  $\mu_k=0.15$ , and the sled is initially at rest.

Find the acceleration of the sled and children  
(a) if the rope tension is 100 N, and  
(b) if it is 140 N.



$$\sum F_y = ma_y \Rightarrow F_n - mg + T \sin \theta = 0 \text{ or } F_n = mg - T \sin \theta$$

$$\text{If sled does not move, } \sum F_x = ma_x \Rightarrow -f_s + T \cos \theta = 0 \text{ or } f_s = \mu_s F_n = T \cos \theta$$

$$\mu_s (mg - T \sin \theta) = T \cos \theta; \text{ to move sled, } T_{\min} = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta} = 109.7 \text{ N, so } a_a = 0$$

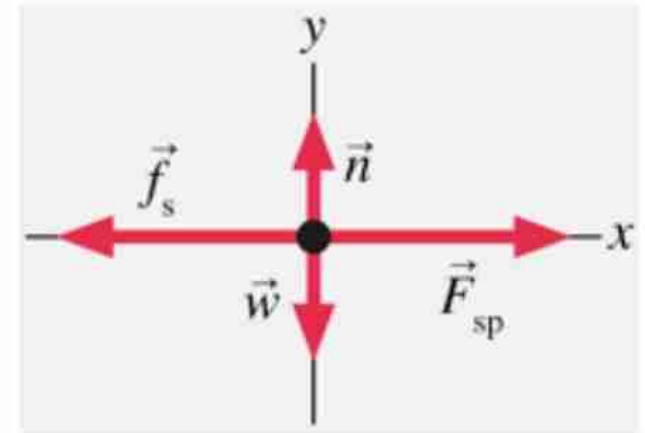
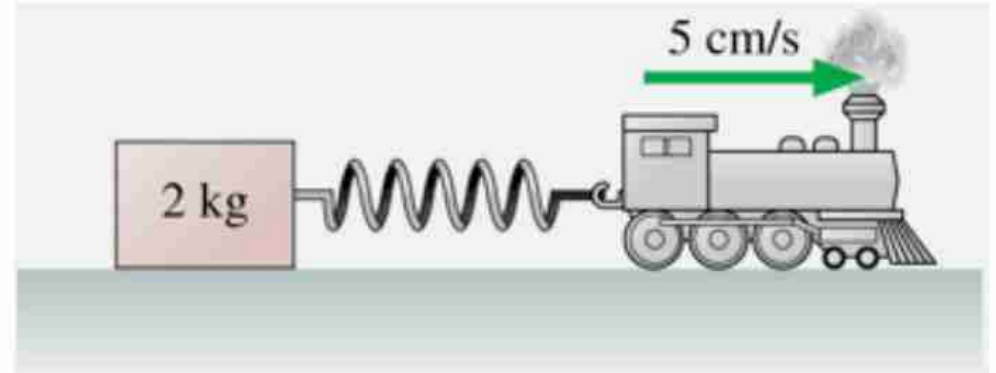
$$\text{If sled moves, } \sum F_x = ma_x \Rightarrow -f_k + T \cos \theta = ma_b$$

$$a_b = [-\mu_k (mg - T \sin \theta) + T \cos \theta] / m = 0.943 \text{ m/s}^2$$

# Example: Dragging a Block (1)

A spring is attached to a 2 kg block. The other end is pulled by a motorized toy train that moves forward at 5.0 cm/s. The spring constant is  $k=50$  N/m and the coefficient of static friction between the block and the surface is  $\mu_s=0.6$ . The spring is in equilibrium at  $t=0$  s when the train starts to move.

At what time does the block start to slip?





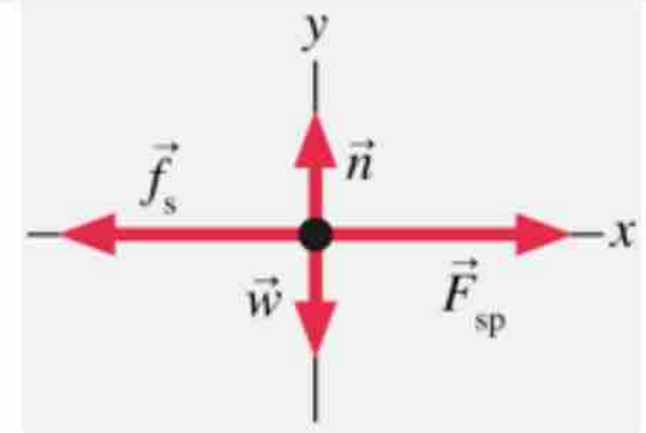
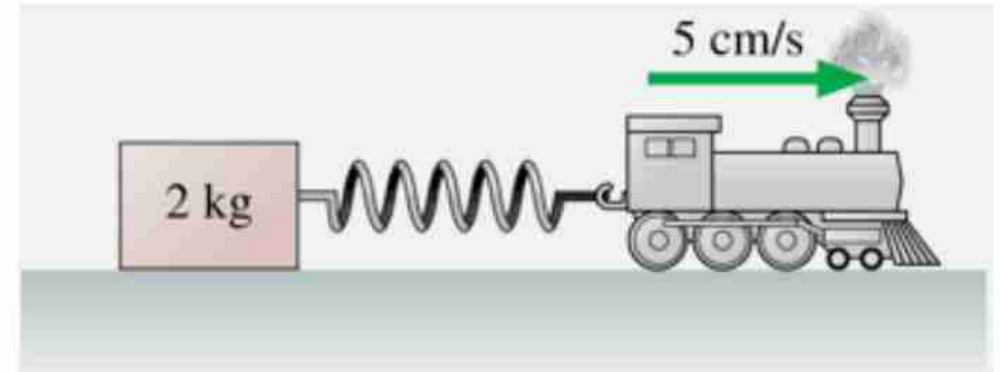
# Example: Dragging a Block (2)

$$\sum (F_{\text{net}})_x = (F_{\text{sp}})_x + (f_s)_x = F_{\text{sp}} - f_s = 0$$

$$f_s = \mu_s mg = F_{\text{sp}} = k\Delta x$$

$$\Delta x = \frac{\mu_s mg}{k} = \frac{(0.60)(2.0 \text{ kg})(9.80 \text{ m/s}^2)}{(50 \text{ N/m})}$$
$$= 0.235 \text{ m} = 23.5 \text{ cm}$$

$$t = \frac{\Delta x}{v} = \frac{(23.5 \text{ cm})}{(5.0 \text{ cm/s})} = 4.7 \text{ s}$$



This is an example of "stick-slip motion", which is common in nature.

**Example:** behavior of rocks during seismic activity and earthquakes.