University of Mosul

College of Science

Department of Physics

Second stage

Lecture 8

Sound and wave Motion

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Lecture 8: Damping

Preparation

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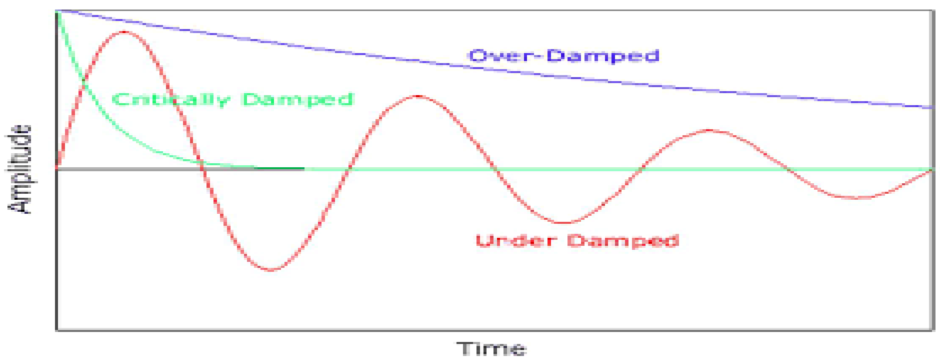
Chapter Four

Damping(Decay)

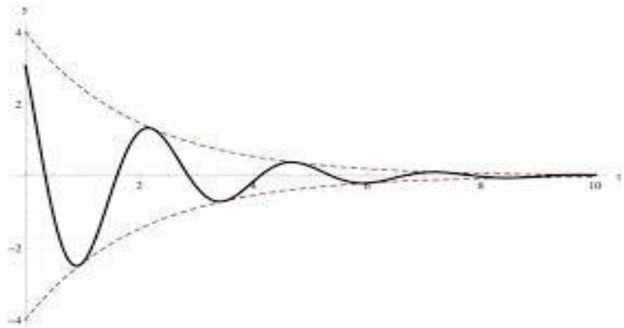
The aforementioned phenomenon refers to the gradual reduction in wave amplitude as the wave progresses over time, ultimately resulting in its eventual decay after a specific duration, denoted as time t.



Decay simulation: The phenomenon of decay is an undeniable reality in the natural world. Waves, characterized by a constant amplitude, are unable to persist in actuality due to the dissipation of energy. This dissipation occurs as a consequence of various hindrances encountered by the wave, such as air resistance, friction, and the distinct characteristics of different media.

Types of damping

1. Light damping, also known as under damping, is characterized by the gradual fading away of vibrations and the subsequent slow decrease in amplitude, as depicted in the second curve illustrated in the preceding figure. A classic example that exemplifies this type of damping is the motion exhibited by a pendulum.

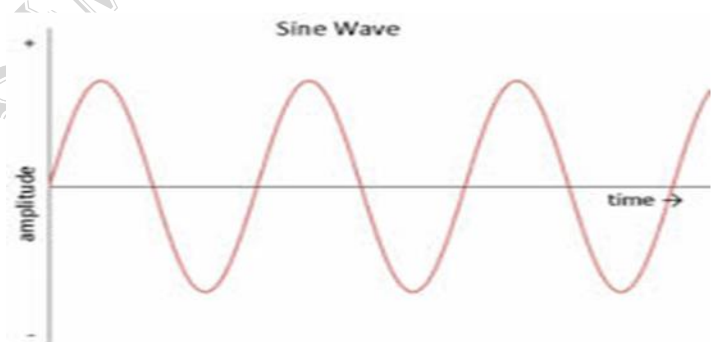


2-Heavy Damping (Over damping):The system has no oscillations and returns very slowly to the equilibrium state. Note curve 3 in the figure as an example of systems like doors connected to a spring (resembling a spring) and a braking system.

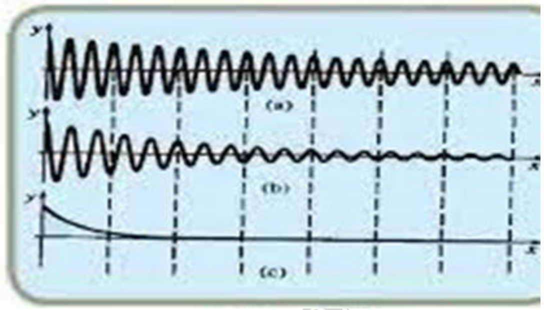
3-Critical Damping: In this type of decay, the system returns to its equilibrium state in the shortest possible time frame (about a quarter of the cycle period). Examples include shock absorbers in cars (dampers), as well as in electric current measuring devices, ballistic galvanometers, and electric scales.

Damped vibration:

In our study of simple harmonic motion, we considered that the oscillation is free and undamped. This type of oscillation occurs when a particle is displaced slightly from its equilibrium position and is then allowed to oscillate freely under the influence of only the restoring force arising from its elastic property. The oscillation is not hindered by any external resistance or energy dissipation in any form. Therefore, the amplitude of the oscillatory motion remains constant, indicating that the oscillation continues indefinitely over time, as shown in Figure.



In fact, the described vibration refers to a completely ideal circumstance, in which no tangible vibrating body continues to oscillate permanently without restrictions. More precisely, any vibrating tangible object must undergo some form of energy dissipation in some way. Therefore, the range of oscillatory motion gradually decreases with time, as shown in Figure (2). This type of vibration is referred to as decaying vibration, which provides a more realistic state compared to non-vanishing vibration.



Forces causing vibration decay:

In general, vibration is a form of energy dissipation, often undesirable due to various factors. One of these factors is the potential breakdown of vibrating components, which can be harmful. Additionally, generating annoying sounds or transferring unwanted forces and motions to nearby parts or bodies is also an undesirable result of vibration. As a result, energy is consumed by the accompanying vibration. In fact, every vibrating particle faces resistance that hinders its movement and leads to the gradual decay of its vibrational motion over time. The magnitude of these resisting forces can be significant enough to prevent vibration from occurring altogether.

Describing the precise forces that contribute to the dissipation of energy through vibration is challenging. However, the force opposing the motion of the vibrating particle can be constrained by one or more decay factors. These factors can arise from the viscosity of the fluid (such as air or liquid) through which the vibrating particle moves or from internal friction between molecules that experience relative motion as a result of vibration. Electrostatic forces, such as those caused by dry friction (Coulomb friction) or induced forces resulting from electromagnetic induction due to the vibration of a magnet containing magnetizable materials near a natural or electric magnet, can also contribute to the resistance. If one or more of these factors, or others, are consistently present in any vibrating process, work must be continually expended to overcome them. This expended work is gradually dissipated as heat to the surrounding medium. This reduction in energy is referred to as energy decay or dissipation. Such a diminishing of vibrational amplitude is known as the decay of harmonics. All natural vibrators that undergo free vibration experience this type of decay, albeit to varying degrees. For instance, a simple pendulum vibrating in the air encounters relatively little resistance, resulting in a gradual decrease in amplitude by a small amount. Consequently, it can continue to vibrate for a relatively long time if left undisturbed. However, if the pendulum is immersed in water, the resistance it faces increases significantly, leading to a noticeable decrease in amplitude.

Accordingly, the vibration ceases after a brief duration. However, if the object is submerged in a liquid with high viscosity, such as honey, it does not vibrate but instead returns to its original equilibrium position upon removal. The same phenomenon occurs with another vibrator under similar conditions. It is worth noting that any object vibrating in a medium, such as air, inevitably loses energy. During the vibration process, the surrounding air particles also vibrate, resulting in the transfer of vibrational energy from the object to the particles. This acoustic emission, exemplified by the sound produced by a tuning fork, represents the transmission of vibrational energy through the air to our auditory system. Consequently, this energy represents a depletion of the vibrating object's energy. To comprehensively examine the decay of vibration, all dissipative forces must be considered. However, it is challenging to enumerate all the factors contributing to energy dissipation, particularly since these forces may vary depending on factors such as displacement, velocity, and stress. Consequently, our investigation in this chapter will focus on the most salient factors influencing the decay of oscillator motion. One such factor is the resistance exerted by the fluid due to its viscosity, impeding particle movement. The level of resistance encountered by a particle in motion within a fluid relies on its velocity, shape, and fluid properties. For a given particle moving in a specific fluid, the relationship between the fluid's resistance and the particle's motion strength varies with speed. At high speeds, the resistive force is nearly directly proportional to the square of the velocity, whereas at low speeds or under normal circumstances, the magnitude of the resistive force, FR, is linearly proportional to the particle's instantaneous velocity. Mathematically, this can be expressed as a proportionality relationship.

$$F_R \; \alpha \; \vartheta$$

From this proportion it follows that

$F_R = -R\vartheta$ …………………………………1

Where R represents the constant of proportionality and is denoted as the resistance constant or decay constant. The inclusion of the negative sign signifies the opposing direction. The resistive force consistently opposes the motion of the object. In other words, this force perpetually endeavors to diminish the particle's vibratory movement.

We can express the instantaneous velocity v of the particle as dt/dx if the particle is moving towards the x-axis, Equation (1) becomes as follows:

$F_R = -R(\, dx \,/\, dt \,)$………………………2

The linear relationship, in addition to being close to reality, leads to the simplest mathematical analyses. Therefore, our research in this chapter will be limited to studying the force of resistance experienced by the vibrating particle, which is directly related and resulting from viscosity or friction and is directly proportional to the instantaneous velocity of the particle.

Damping equation for harmonic motion:

Here we will consider the motion of a simple harmonic oscillator composed of a spherical particle with mass m. This particle is connected to a helical spring with a spring constant k and one end fixed securely to a stationary support, as shown in figure.

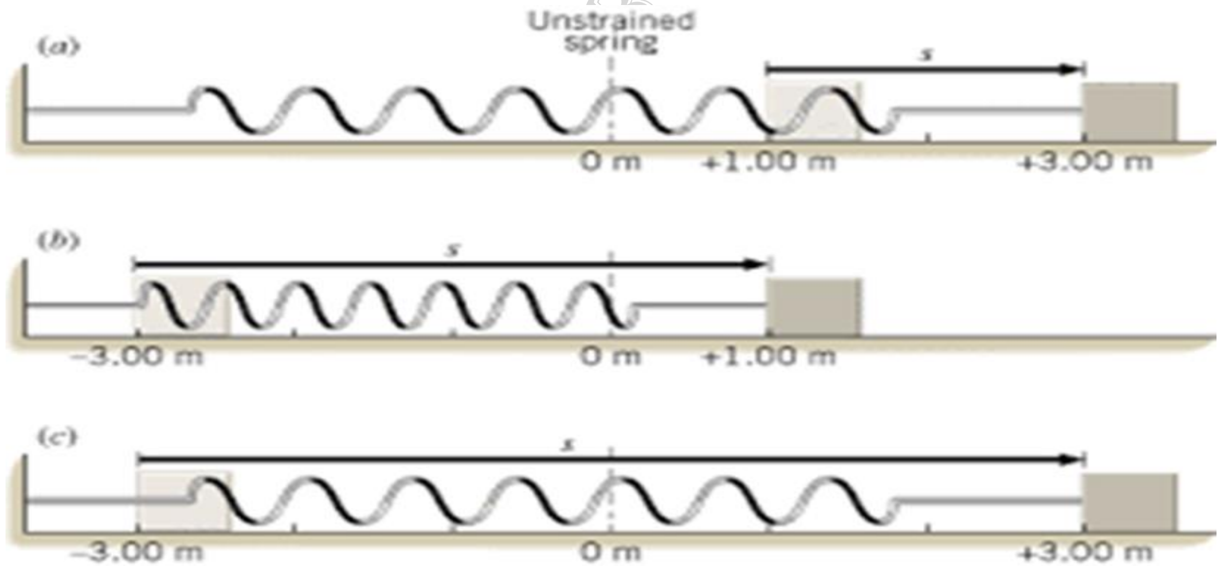


Figure It shows a harmonic oscillator on which two forces act: the restoring force and the fluid resistance force

When a mass m is displaced by a small amount x, a restoring force of magnitude (-kx) appears. When the mass returns to its equilibrium position, it experiences a resistive force due to friction or fluid viscosity, with a magnitude of (-R $(\frac{dX}{dT})$). The negative sign indicates that this force always opposes the direction of the relative speed of the vibrating mass. The net force acting on the mass at any time t is (-R $(\frac{dX}{dT})$ –Kx). Applying Newton's second law of motion, we get:

$m\frac{d^2x}{dt^2} = -R\,(\frac{dX}{dT}) - Kx$ ……………………….....1

Divide both sides of equation (1) by m and arrange the terms to become:

$\frac{d^2x}{dt^2} = -\frac{R}{M}(\frac{dX}{dt}) - \frac{k}{M}x = 0$ ……………………..2

In order to facilitate the form of the solution to equation (2), we assume that

2r= R /m, and since ω^2= k/m , then equation (2) becomes:

$\frac{d^2x}{dt^2} = -2r\,(\frac{dX}{dt}) - w^2 x = 0$

$\frac{d^2x}{dt^2} + 2r\frac{dX}{dt} + w^2 x = 0$…………………………….....3

This is the differential equation for free decaying harmonic motion, and it is noted that it is a differential equation of the second order.

Solve the decaying harmonic motion equation:

Given the absence of a direct integration of equation (3), it becomes imperative to explore an alternate approach in search of the suitable solution. Upon careful consideration, it is evident that the sought-after solution must take the form of a function that exhibits identical mathematical characteristics across all its terms. Subsequently, the appropriate function in question is one that shares the same properties for its first and second derivatives as the function itself. The mathematical function that meets these criteria is the exponential function $e^{\alpha t}$. Hence, it is reasonable to assume that the fitting solution is:

$X=D\, e^{\alpha t}$ …………………………………..1

Where D is an optional constant, we substitute this solution into equation (3) in the previous paragraph, and it results.

$$\frac{d^2De^{\alpha t}}{dt^2} = -\, 2r \left(\frac{dDe^{\alpha t}}{dt}\right) - w^2De^{\alpha t} = 0$$

$\alpha^2\, De^{\alpha t} + 2r\,\alpha\;\; De^{\alpha t} + w^2De^{\alpha t} = 0$ …………………..2

And equation (2) becomes:

$De^{\alpha t}(\alpha^2 + 2r\,\alpha\;\; + w^2 = 0$ ……………………………..3

This means either $De^{\alpha t}=0$, which is not possible because it represents the imposed solution and is not equal to zero unless the value of the variable t is a negative amount that is infinitely large. Or that $\alpha^2 + 2r\alpha + w^2 = 0$ The correct way to solve this equation is to use the constitution:

$\alpha = \frac{-b \pm \sqrt{b^2 - 4\alpha c}}{2\alpha}$……………………………4

Substituting, we find that

$\alpha = \frac{-2r \pm \sqrt{4r^2 - 4w^2}}{2}$…………………………5

$\alpha_1 = -r + \sqrt{r^2 - w^2}$

$\alpha_2 = -r - \sqrt{r^2 - w^2}$

By substituting the two previous results into equation (1), we find that the general solution to the equation of motion is:

$$x = D_1 e^{t} + D_2 e^{\alpha_2 t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

$$x = D_1 e^{-r + \sqrt{r^2 - w^2}\; t} + D_2 e^{-r - \sqrt{r^2 - w^2} t} \ldots 7$$

Whereas D_1 and D_2 are arbitrary constants that can be found from the initial conditions of motion. It is known that the solution The generic of a second order differential equation must include two optional constants. That the physical interpretation of equation (7) It indicates that there are four states of motion, each of which depends on the value of r with respect to ω, and these states are :

1-The first case: it represents the state of non-decay (r =0):

This condition means that the resistance experienced by the oscillator during its motion is completely non-existent, i.e. (R = 0) (and this case corresponds to non-decaying simple harmonic motion. In this case, the solution to equation (7) becomes as follows:

$$x = D_1 e^{+\sqrt{-w^2}\; t} + D_2 e^{-\sqrt{-w^2} t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

$$x = D_1 e^{+iwt} + D_2 e^{-iwt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9$$

Where(i)is an imaginary number and is equal to$\sqrt{-1}$, But we have

$$e^{i\omega t} = \cos wt + i \sin wt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 10$$

$$e^{-i\omega t} = \cos wt - i \sin wt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 11$$

We substitute equations (10) and (11) into equation (9), and we find that:

$$x = D_1(\cos\omega t + i\sin\omega t) + D_2(\cos\omega t - i\sin\omega t)$$

$x = (D_1 + D_2)cos\omega t + i(D_1 + D_2)sin\omega t$ ……………………12

If we assume that A=(D_1+D_2)and B=i(D_1-D_2), then by substituting in equation (12), we find that the last solution becomes:

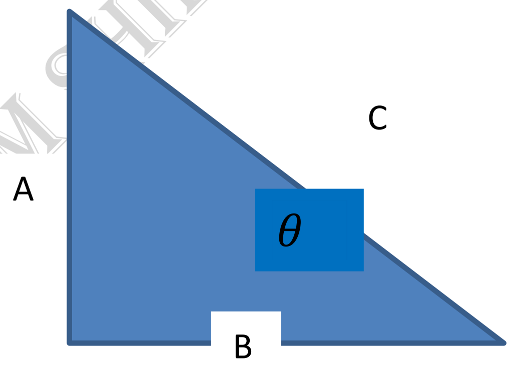
$x = Acos\omega t + Bsin\omega t$ …………………………………13

This solution can be further simplified if we multiply and divide the right-hand side by C , whereas: C=$\sqrt{A^2 + B^2}$,$\tan\theta = \frac{A}{B}$ Then the final solution becomes as follows:

$x = C\{(A/C)cos\omega t + (B/C)sin\omega t\}$

$x = C(sin\theta cos\omega t + cos\theta sin\omega t)$

$x = Csin(\omega t + \theta)$ …………………………………14



Where θ represents the initial phase angle of the motion and is equal (arc tanA/B). This equation represents the harmonic motion Simple and indicates that the amplitude of motion during vibration remains constant over time.

Eight Lecture