University of Mosul

College of Science

Department of Physics

Second stage

Lecture 9

Sound and wave Motion

2024-2025

Lecture 9: Damping 2

Preparation

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2--The second case: It represents the state of incomplete decay movement $r^2 < w^2$:

This case means that the resistance experienced by the vibrator is light, that is, the damping in vibration is relatively small decay coefficient r is small compared to the angular frequency ω the magnitude is under the root $(r^2 - w^2)^{\frac{1}{2}}$ is negative and therefore the root is an imaginary quantity. We suppose that in the case of $r < \omega$ that $(r^2 - w^2)^{\frac{1}{2}} = i\omega_o$, where ω_o is the frequency angular decay. Substitute into equation (7) and the solution becomes as follows:

$$x = D_1 e^{(-r + iw_0 t)} + D_2 e^{(-r - iw_0)t}$$

$$x = e^{-rt}(D_1 e^{iw_0 t} + D_2 e^{-iw_0 t}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 15$$

In exactly the same way as was followed in the first case, it can be proved that:

$$D_1 e^{iw_0 t} + D_2 e^{-iw_0 t} = C \sin(w_0 + \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 16$$

Thus, the general solution in this case becomes as follows:

$$X = C e^{-rt} \sin(w_0 + \theta) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 17$$

Where θ and C are two optional constants whose value can be found from the initial conditions of motion. This equation represents motion The decaying simple harmonic can be represented graphically as shown in Figure .

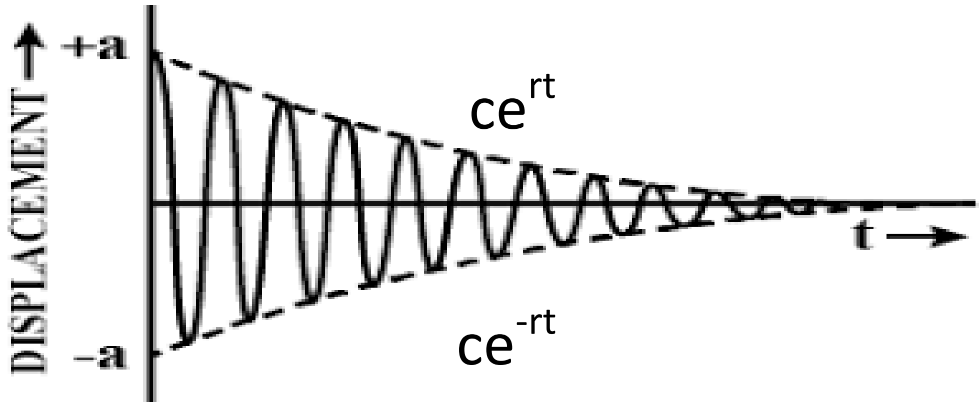

Figure shows the displacement and time curves in the fading vibration

In the previous figure it is clear that the amplitude of the movement decreases with time. This amplitude can be determined when the value of $\sin(\omega_o t+\theta)$ is at its maximum value, that is, when $\sin(\omega_o t+\theta)=\pm 1$ Therefore, the effective amplitude of the movement is ($\pm Ce^{-rt}$). It is noted that it is a variable quantity and depends on the decay factor r and the time t. This indicates that the capacitance decreases exponentially with time until it ceases to exist when the value of t is infinity. This condition means that the resistance encountered by the oscillator is too small allow oscillations to occur around the equilibrium position, although the amplitude of these oscillations decreases with time as shown in the previous figure. The difference in time separating two successive peaks (or troughs) is called the decay period of the free oscillation and is usually denoted by the symbol To and can be found from the relationship:

$$\sqrt{r^2 - w^2} = iw_0$$

from which we find that

$$w_0 = \sqrt{w^2 - r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 18$$

and whereas $w_0 = \frac{2\pi}{T_0}$

$$T_0 = \frac{2\pi}{\sqrt{w^2 - r^2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 19$$

When comparing the periodic time of non-decaying free vibration, T= 2π/ω, with To, we observe that To is always greater than T. This indicates that the resistance experienced by the vibrating particle slows down its movement. Moreover, the greater the measured frictional resistance as a function of the decay factor r, the longer the periodic time, To, increases, even if the value of r becomes equal to the value of ωThe length of the periodic time t0 becomes infinity as it grows. Which indicates that the movement is non-oscillatory and infinite in size, and that the particle will return to its original equilibrium position if left free after any initial displacement. This indicates the presence of resistance. A small amount of resistance in front of any vibrator leads to a decrease in vibration, which manifests itself in the form of two effects. The first is a gradual decrease (in vibration) in amplitude and the second is an increase in the length of the periodic time. These two effects are related to equations (17) and (19). This type of vibration represents most cases of vibration in nature, where energy is gradually dissipated as a result of resistance, and the oscillator suffers from it until there is no movement and it stops vibrating completely.

When sin(ωot+ θ)=∓1, the oscillating curve (solid line) in the previous figure intersects the Ce-rt exponential curve (dotted line) at points that are not aligned with the maximum amplitude values, but are shifted slightly to the right, Therefore, the tangents at those points are horizontal, parallel to the time axis. Specifically, the positions of the extreme values of x can be determined by finding the derivative of the displacement x with respect to time t and then applying conditions to the extreme values.

Overall The difference between the two cases is so small that it can be neglected. So the value of capacitance at the point of tangency can be taken to be equal to the maximum capacitance value.

3- The third case:It represents the critical movement condition $(r^2 = w^2)$:

This special case represents the boundary between two completely different behaviors of the oscillator. The first behavior is vibrational behavior It starts when the value of r is slightly less than the value of ω, and this case has been studied in the previous section. The second behavior is behavior non-vibrational and occurs when the value of r is equal to or greater than the value of ω and in this item we will analyze the case when r=ω ,r= 2 and in the next item we will analyze the case when r < ω. So replace us $\omega^2 = r^2$ in the general solution of equation (7) of the equation of motion We get two terms that are exactly the same, and the resulting solution is as follows.

$$x = D_1\, e^{-rt} + D_2\, e^{-rt}$$

$$x = (D_1 + D_2)e^{-rt}$$

$$x = De^{-rt}$$

Where D equals (D_1+D_2) , This solution contains one optional constant. While the equation of motion is from The second order and its general solution must include two arbitrary constants in order to satisfy the two initial conditions for motion. for this purpose We will try to find the appropriate solution that contains two optional constants, so we will resort to the previous case (Imperfect motion decay state), and we assume that the decay coefficient r increases gradually until it approaches a limit of ω, that is, $(r^2 - w^2)^2 = i\delta\omega_\circ$, whereas $\delta\omega_\circ = 0$ We won't get far from it it $r^2 = w^2$ condition of this case, that is At any given time t will be a value very small. We substitute that in Equation (7),which represents the general solution, is obtained.

$$x = D_1 e^{(-r+i\delta w_0)t} + D_2 e^{(-r-i\delta w_0)t}$$

$$x = e^{-rt}(D_1 e^{i\delta w_0 t} + D_2 e^{-i\delta w_0 t})$$

In the same way as we followed in the first case, it can be proved that

$$D_1 e^{i\delta w_0 t} + D_2 e^{-i\delta w_0 t} = A\cos\delta W_0 t + B\sin\delta W_0 t$$

whereas A=(D_1+D_2)and B=i(D_1 - D_2) Thus, the solution in this case is

$x = e^{-rt}(A\cos\delta W_0 t + B\sin\delta W_0 t)$ ………………………..20

But $\sin\delta W_0 t \to 0$ Therefore the following approximations can be made with a high degree of accuracy$\cos\delta W_0 t \approx 1$, $\sin\delta W_0 t \approx \delta W_0 t$ Substituting into equation (20), we find that:

$x = e^{-rt}(A + B\,\delta w_0 t)$

If we consider $Ɓ = B\delta W_0 t$ The general solution in this case becomes

$x = e^{-rt}(A + Ɓt)$ ……………………………………….21

where A ,Ɓ They represent two arbitrary constants that can be found from the initial conditions of motion. Notes from this solution It represents the boundary case of equation (7) when the decay coefficient r increases approaching the value of ω, that is, when the time approaches The periodic To of the vibration decaying from infinity i.e. To →∞. This experimental solution can be confirmed Substituting it into the equation of motion (3), we find that the two sides are congruent. This solution describes the motion of the particle in the critical state and can be represented graphically as shown in Figure 5

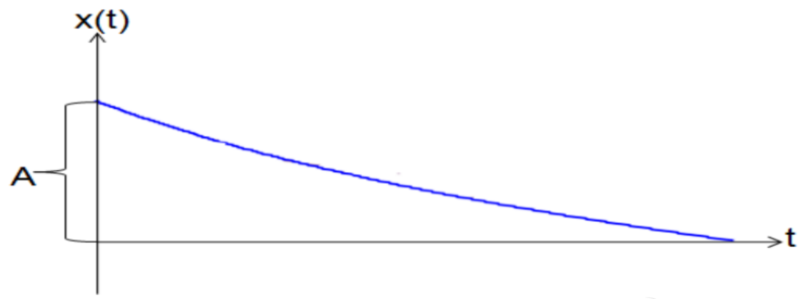

Figure (5) shows the movement of the particle in the critical condition if it is displaced by an initial displacement of the amount A and then left free.

The figure shows that the particle returns to its original position when left free after being displaced. The value is A, indicating that the solution does not depict any vibrational behavior due to significant frictional resistance, i.e. $\omega^2 = r^2$. When the frictional resistance, a function of the dissipation factor r, exceeds $\omega^2 < r^2$, it hinders vibrational behavior and slows down the particle's movement, prolonging its return to equilibrium. This scenario will be further examined in the next section. Conversely, if the frictional resistance, as a function of the factor r, decreases to $\omega^2 > r^2$, the vibrator moves to the state previously studied.

It is evident that the critical state implies that a particle will quickly return to its original position when displaced from a balanced position and left free from any accompanying vibratory motion. However, if the particle is displaced from its equilibrium position, it will overshoot the equilibrium position before returning to it quickly without oscillation. This critical motion condition is highly significant in the design of practical measuring devices with moving parts, such as indicators in electrical measuring instruments like galvanometers, ammeters, voltmeters, and others. For instance, when an impulse is applied immediately after closing the circuit, the indicator undergoes sudden movement. If the damping coefficient is not suitable, the indicator will either vibrate around its equilibrium position if $\omega^2 > r^2$ or move slowly towards the equilibrium position if it does not , When $\omega^2 < r^2$ or $\omega^2 = r^2$, neither case is preferred. However, if $\omega^2 = r^2$, the pointer quickly reaches its equilibrium point without oscillating around it, allowing for an accurate and fast reading when connecting the measuring device to the circuit.

4- It represents the state of excessive movement and decay$r^2 > w^2$:

In this case, the oscillator suffers from strong frictional resistance, and the value of the decay coefficient r is large compared to the natural angular frequency of the oscillator, ω. Which means the expression is under the root$(r^2 - w^2)^{\frac{1}{2}}$ is a positive real quantity Thus, the general solution to equation (7) for this case is as follows :

$$x = e^{-rt}\left(D_1 e^{+\sqrt{r^2-w^2}t} + D_2 e^{(-\sqrt{r^2-w^2})t}\right) \text{ ....22}$$

Whereas D_1 and D_2 are optional constants that can be found from the initial conditions of motion. notes in these The equation does not contain a factor whose value fluctuates with time, which indicates that the oscillator does not behave vibrating. This solution consists of two parts, the first of which is represented by the first term, which represents the slow-decaying part, and the second term The fast decay part represents as shown in Figure (6).

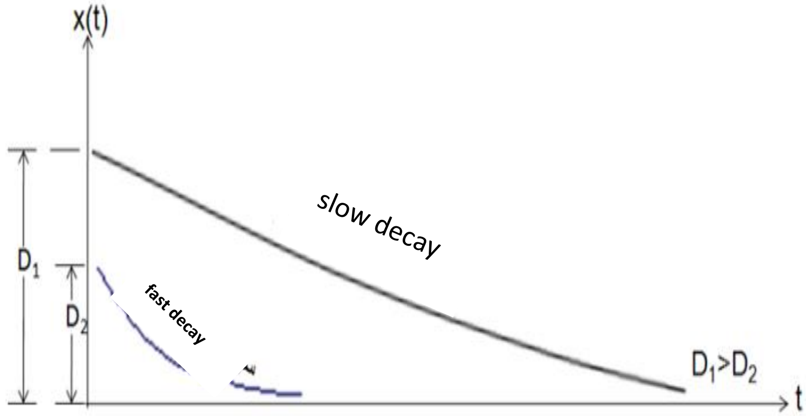

Figure (6) shows that the rate of change of displacement with time differs in the two terms that make up the general solution to Equation 22.

The actual movement of the vibrator is represented by the sum of these two parts, which can be represented graphically as shown in Figure (7). This figure indicates that if the particle had an initial displacement of (D1+D2) in time t = 0 from its equilibrium position ,Then left free, the resultant displacement will fade exponentially with the passage of time until the particle reaches its equilibrium position At the time of the final length, that is, the particle will very slowly return to its equilibrium position after a very long period of time and stop It moved completely in that position without wobbling. This indicates that the resistance experienced by the oscillator is large so as not to allow any vibration to occur.

Figure (7) shows the change of displacement with time in the oscillator, which suffers from severely diminished motion .

Ninth lecture