University of Mosul
College of Science
Department of Physics
Second Stage

Heat and Thermodynamic

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Lecture 10 : The work

Preparation

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Chapter five

The work

Adiabatic processes (ideal gases)

We would have one partial gram of ideal gas well isolated from the ocean. So if work is performed on the gas of dW, then the first law of thermodynamics

$$dQ = dU + dW$$

It is well known that the adiabatic process takes place when no heat enters or leaves the system dQ =0

$$0 = dU + PdV \ldots\ldots\ldots\ldots\ldots(30)$$

As it is known that the external work performed will be at the expense of the internal energy, so this process will be accompanied by a decrease in temperature by dT, meaning that the internal energy

$$dU = C_V \,.\, dT \ldots\ldots\ldots\ldots(31)$$

Substitute equation 31 into equation 30 we get

$$0 = C_V \,.\, dT + PdV \ldots\ldots\ldots\ldots(32)$$

The equation for one mole of an ideal gas is **PV=RT**

After making the differential for this equation we get

$$PdV + VdP = RdT \ldots\ldots\ldots\ldots\ldots(33)$$

$$dT = \frac{PdV + VdP}{R} \ldots\ldots\ldots\ldots(34)$$

We substitute eq.34 into eq.32

$$C_V\left[\frac{PdV + VdP}{R}\right] + PdV = Zero \ldots\ldots\ldots\ldots(35) \qquad \times R$$

$$C_V[PdV + VdP] + PRdV = Zero \ldots\ldots\ldots\ldots(36)$$

From eq.28

$$\mathbf{C_P - C_V = R \ldots\ldots\ldots\ldots\ldots . (28)}$$

$$\mathbf{C_V P dV + C_V V dP + P C_P dV - P C_V dV = 0 \ldots\ldots\ldots . . (37)}$$

Divided eq. 37 on C_V .V.P we get

$$\frac{dP}{P} + \frac{C_P}{C_V}\frac{dV}{V} = 0 \ldots\ldots\ldots\ldots\ldots\ldots . (38)$$

$$\frac{C_P}{C_V} = \gamma$$

$$\mathbf{\frac{dP}{P} + \gamma\frac{dV}{V} = 0 \ldots\ldots\ldots\ldots\ldots\ldots . (39)}$$

When performing the integration of the last equation, it results:

$$\ln P + \gamma \ln V = Constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots . (40)$$

$$\mathbf{\ln P + \ln V^{\gamma} = Constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots . (41)}$$

$$\mathbf{\ln PV^{\gamma} = Constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (42)}$$

Or $\mathbf{PV^{\gamma} = constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)}$

This relationship between pressure and volume in adiabatic processes and there are other equations

Example: car tire is under pressure twice the atmospheric pressure p at an air temperature of 27 C. If the tire suddenly bursts, calculate the resulting temperature. Note that γ for air =1.4

P_1=2p, P_2= 1p, T_1= 27+273 = 300 K, T_2= ?

$\mathbf{P^{\gamma-1}/T^{\gamma} = constant}$

$$\frac{P_1^{\gamma-1}}{T_1^{\gamma}} = \frac{P_2^{\gamma-1}}{T_2^{\gamma}}$$

$$\left(\frac{P_2}{P_1}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)^{\gamma}$$

$$\left(\frac{1}{2}\right)^{1.4-1} = \left(\frac{T_2}{300}\right)^{1.4}$$

$T_2 = 246.1K$, $t = -26.9$ C

The slop of the isothermal and adiabatic curves

In any isothermal process of one ideal gas

Pv = constant

The relationship differential gives us

$PdV + V\,dP = 0$

$$\frac{dP}{dV} = -\frac{P}{V} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(44)$$

Equation 44 represents the slope of the isothermal curve at the point of intersection with the adiabatic curve.

In any adiabatic process of an ideal gas we have a

PV^{γ} = constant

From the differentials of this equation

$P.\gamma.V^{\gamma-1}\,dV + V^{\gamma}\,dP$ =Zero………………(45)

$V^{\gamma}\,dP = -\,P.\gamma.V^{\gamma-1}\,dV$……………………(46)

$$\frac{dP}{dV} = -\gamma.P\frac{V^{\gamma-1}}{V^{\gamma}} = -\gamma.PV^{\gamma-1-\gamma} = -\gamma.PV^{-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(47)$$

$$\frac{dP}{dV} = -\gamma.\frac{P}{V} \dots\dots\dots\dots\dots..(48)$$

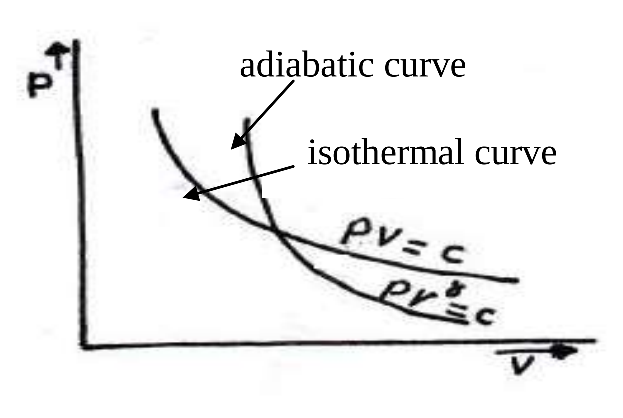
Equation 48 represents the slope of the adiabatic curve at the point of intersection with the isothermal curve.

$$\left(\frac{\partial P}{\partial V}\right)_{adia.} = \text{slop of adiabatic curve} = -\gamma\frac{P}{V}$$

$$\left(\frac{\partial P}{\partial V}\right)_{iso.} = \text{slop of isothermal curve} = -\frac{P}{V}$$

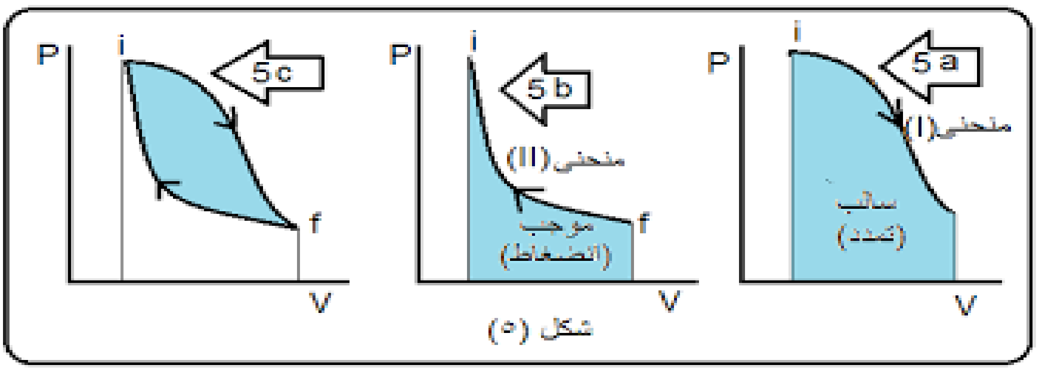
Therefore, the slope of the adiabatic curve is equal to (ɣ) multiplied by the slope of the isothermal curve , i.e.

The slope of the adiabatic curve is greater than the slope of the isothermal curve at the point of intersection of the two curves.



Path function (incomplete differentiation)

The path function is an imperfect differential. To illustrate this, we take the pressure and volume diagram for a given gas at a constant temperature. If the gas state changes from the initial state i to the final state f, then there are a large number of paths or methods to change the state of the gas from i to f or vice versa, and for the sake of ease, we will choose only paths 1 and 2 shown in Figure 1 (pressure and volume diagram for work)



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From the last figure, we notice that the gas is expanding from state i to state f (the first path) and the gas is compressed back from state f to state i (the second path). The process of going from i to f and from f to i is called a cyclic process

When the gas completes a full cycle and returns to its original state, we note the following:

First: The gas pressure returns to its original value, meaning that the dP is a complete differential

$$\oint \mathbf{dP} = \mathbf{0} \ldots \ldots \ldots \ldots \ldots (\mathbf{49})$$

The integral of the change over a complete cycle is zero, which is called the state function.

Second: The volume of gas returns to its original value, ie

$$\oint \mathbf{dV} = \mathbf{0} \ldots \ldots \ldots \ldots \ldots (\mathbf{50})$$

This means that the dV is a complete differential

Third: The work performed after completing the cycle and returning the gas to its initial state is not equal to zero, meaning that

$$\oint \mathbf{\partial W} = \oint \mathbf{P.dV} \neq \mathbf{0} \ldots \ldots \ldots \ldots \ldots (\mathbf{51})$$

This means that δW is an incomplete differential

The physical meaning of incomplete differentiation:

1. In the case of dilation from i to f, the deceptive area under the curve a represents the work done and is symbolized by W_1

$$\mathbf{W_1 = \int_{V_1}^{V_2} PdV \ldots\ldots\ldots (52)}$$

2. The work performed on the gas during its volume shrinking from V_2 to V_1 is equal to the deceptive area under the curve b and symbolized by W_2

$$\mathbf{W_2 = \int_{V_2}^{V_1} PdV \ldots\ldots\ldots (53)}$$

3.The net work performed by the gas during one cycle is equal to the deceptive area between curves a and b in Figure c

$$\mathbf{W = W1 + W2 = \int_{V_1}^{V_2} PdV + \int_{V_2}^{V_1} PdV \ldots\ldots\ldots (54)}$$

$$\mathbf{W = \oint PdV \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (55)}$$

The energy needed to do this work comes from the heat that the system absorbs from the surrounding.

The Work:

The idea of work is of fundamental importance in the topic of thermodynamics, so the existence of thermal machines is for the purpose of completing work and providing effort for humans. To clarify the idea of work, we begin with mechanical work

That is, the amount of work W performed by a certain force F on a body to displace it from its position a certain displacement dx, and from this we conclude the relationship describing work:

$dW = Fdx$ .... (1)

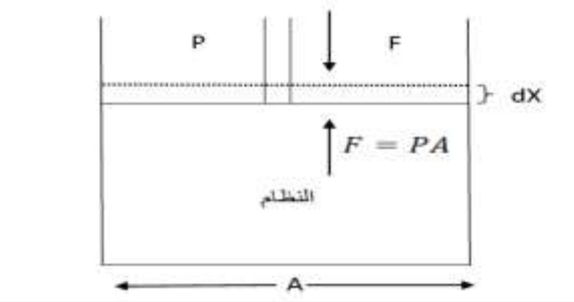


Figure 1

Figure (1) Container is equipped with a movable piston that contains gas. When the gas is allowed to expand, the volume of the piston is changed to the top. If the volume of the gas changes by a dv, the piston is pushed by dx and since the pressure is equal to the product of the division of force by area (force equals the product of pressure in area)

$dW = P.A.\ dx$

$dW = P\ dv$ .... (2)

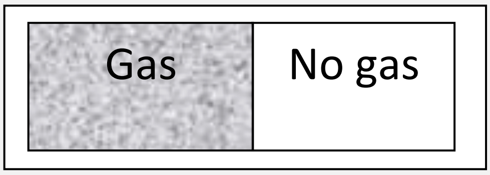
Where dv represents the amount of change in size of the system during the expansion process

1. A work is positive when the work is performed by the system and is accompanied by an increase in volume (expansion process).

2. A work is negative when the work is performed on the system and is accompanied by a decrease in volume (Compression process).

The process of free expansion:

the work performed in this process is zero due to the absence of external forces Because the right part is empty ($P_{opp.} = 0$). It is natural that the gas parts perform work because of the force that exists between the molecules, which is an internal work, that is, between the parts of the system, and we consider it zero because we take into account the external work only.



Work performed through various operations:

1. Work performed in free expansion: We will study work done on both ideal gas and real gas.

A. work performed in the free expansion process of an ideal gas:

In an ideal gas, the attractive forces between the molecules are zero, and therefore when the gas expands freely, the particles diverge from each other without any force affecting them, and thus the work performed during the free expansion process of the ideal gas is zero. W = 0.

B. Work performed in the free expansion process of real gas:

In real gas, consideration must be given to the mutual attraction forces between molecules Therefore, the observed pressure is less than the ideal pressure if there are no attractive forces between the particles, and it must be corrected The observed pressure P is by adding the amount ΔP that represents the internal pressure of the gas resulting from the attraction towards the inside.

$$\Delta P = \frac{a}{V^2} \ldots\ldots\ldots\ldots\ldots(1)$$

Where **a** is constant, and thus we find that when the gas expands freely from volume V_1 to volume V_2, the amount of work that the gas exerts to distance the particles from each other against the forces of attraction between them is W, meaning that

$$W = \int_{V_1}^{V_2} \Delta P dV \ldots\ldots\ldots\ldots(2)$$

$$W = \int_{V_1}^{V_2} \frac{a}{V^2} dV \ldots\ldots\ldots\ldots(3)$$

$$\mathbf{W = a \int_{V_1}^{V_2} \frac{dV}{V^2} \ldots\ldots (4)}$$

$$W = a\left[\frac{V^{-1}}{-1}\right]_{V_1}^{V_2} = -a\left[\frac{1}{V_2} - \frac{1}{V_1}\right] \ldots\ldots\ldots\ldots\ldots (5)$$

$$\therefore W = \frac{a}{V_1} - \frac{a}{V_2} \ldots\ldots\ldots\ldots\ldots (6)$$

2.Derivation of the equation for work performed during an isothermal process (T=constant , dT= 0):

1. Assume that a certain amount of gas was used in the isothermal process and it was under a certain pressure (P=not constant). If the isothermal volume expands, then it performs a work, and if the initial volumes of the gas are V_1, and the final volumes of the gas are V_2 as in the figure 2.

The area shaded in this figure represents the amount of work done by the gas is W, And space ABab in figure 1

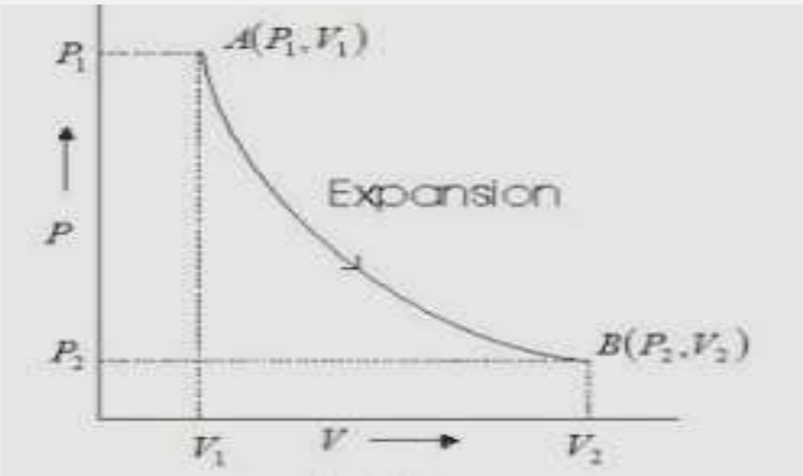


Figure (1)

And for one mole of ideal gas PV=RT

$$W = \oint_{V_1}^{V_2} PdV = ABab \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$\mathbf{W = RT \oint_{V_1}^{V_2} \frac{dV}{V}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

W= RT lin V_2/V_1……………………(9)

for ideal gas PV=constant

$P_1V_1=P_2V_2$………………………….(10)

$V_2/V_1=P_1/P_2$

W= RT lin P_1/P_2……………………(11)

In this process, the amount of change in the internal energy of the gas is zero because its temperature is constant, so the heat transferred is equal to the amount of work performed.

2. If the gas is allowed to expand by a dV, then the work output resulting from the expansion of the gas is equal

dW = PdV ……………… (12)

If the pressure is constant during the process, then equation (12) becomes

$$dW = P\int_{V_1}^{V_2} dV \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

W = P (V_2-V_1) ……………. (14)

Example: One molecular gram of gas at 27 °C is the expansion of the isothermal until its volume is twice the original volume, calculate the work performed during the process, if you know that

$$R = 8.3 * 10^{7} \text{ ergs / Mole. K}$$

$V_2=2V_1$, W= ?

W= RT lin V_2/V_1

W=8.3x 10^{7}x (27+273). Lin $(2V_1/ V_1)$

W=1725.6x 10^{7}erg.