University of Mosul
College of Science
Department of Physics
Second Stage

Heat and Thermodynamic

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Lecture 11 : Work performed during an adiabatic process

Preparation

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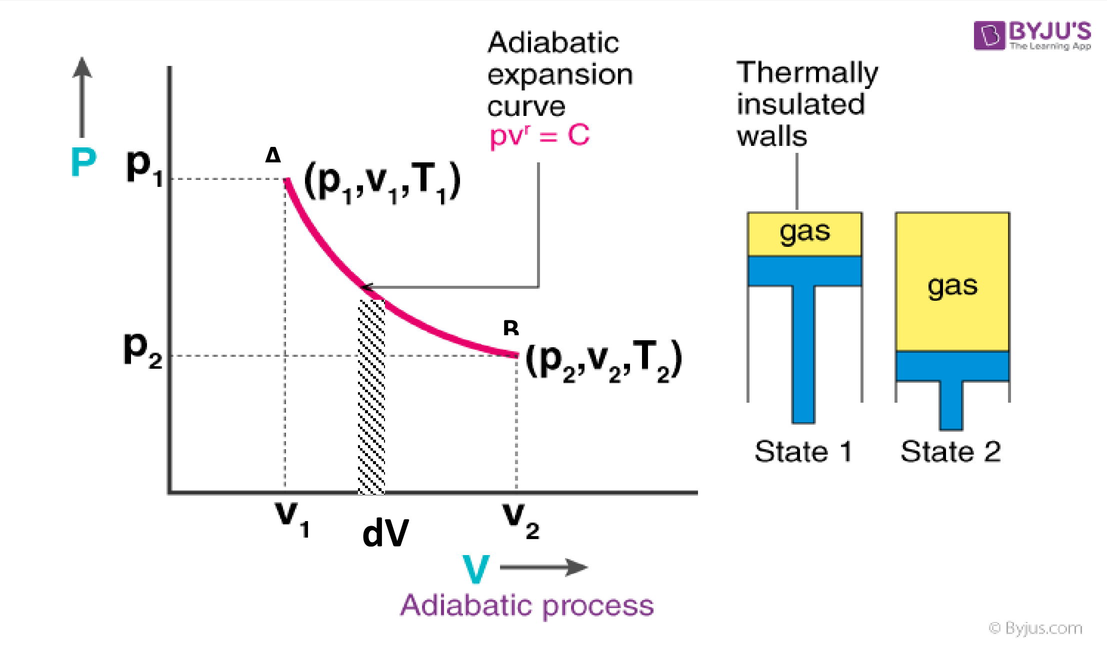
Work performed during an adiabatic process:

During the adiabatic process, the volume of the gas is expanded from volume V_1 to volume V_2, as shown in Fig. 2, so a work of the amount of dW is performed when the volume of the gas increases by a factor of dV.



dW=PdV

$$\mathbf{W} = \int_{V_1}^{V_2} \mathbf{PdV} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathbf{1})$$

The relationship of pressure and volume during the adiabatic process is given by the following equation

PV^{γ}=constant

$P=C/V^{\gamma}$……..(2)

We substitute Equation 2 into Equation 1 yield

$$\mathbf{W} = \int_{V_1}^{V_2} \frac{\mathbf{C}}{\mathbf{V}^{\gamma}} \mathbf{dV} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\mathbf{3})$$

$$\mathbf{W} = \mathbf{C}\int_{\mathbf{V_1}}^{\mathbf{V_2}} \frac{\mathbf{dV}}{\mathbf{V^{\gamma}}} = \mathbf{C}\int_{\mathbf{V_1}}^{\mathbf{V_2}} \mathbf{V^{-\gamma}dV}$$

$$W = \frac{C}{1-\gamma}\left[V^{-[\gamma-1]}\right]_{V_1}^{V_2} \;\ldots\ldots\ldots\ldots..(4)$$

$$\mathbf{W} = \frac{\mathbf{C}}{\mathbf{1-\gamma}}\left[\frac{\mathbf{1}}{\mathbf{V_2^{\gamma-1}}} - \frac{\mathbf{1}}{\mathbf{V_1^{\gamma-1}}}\right] \ldots\ldots\ldots\ldots.(5)$$

Since points A and B lie on the same adiabatic curve, therefore

$$P_1\ V_1^{\gamma} = P_2\ V_2^{\gamma} = C \ldots\ldots\ldots\ldots(6)$$

We substitute the value of C into equation 5, we get

$$\mathbf{W} = \frac{\mathbf{1}}{\mathbf{1-\gamma}}\left[\frac{\mathbf{C}}{\mathbf{V_2^{\gamma-1}}} - \frac{\mathbf{C}}{\mathbf{V_1^{\gamma-1}}}\right] \ldots\ldots\ldots\ldots.(7)$$

$$\mathbf{W} = \frac{\mathbf{1}}{\mathbf{1-\gamma}}\left[\frac{P_2\ V_2^{\gamma}}{\mathbf{V_2^{\gamma-1}}} - \frac{P_1\ V_1^{\gamma}}{\mathbf{V_1^{\gamma-1}}}\right] \ldots\ldots\ldots\ldots.(8)$$

$$\mathbf{W} = \frac{\mathbf{1}}{\mathbf{1-\gamma}}[P_2\ V_2 - P_1\ V_1] \ldots\ldots\ldots\ldots.(9)$$

Equation No. 9 can be written in terms of temperature by considering T_1 and T_2 the temperatures at points A and B respectively(figure 2) , and considering that the gas equation for one mole is:

$P_1V_1=RT_1$

$P_2V_2=RT_2$

Substituting P_1V_1 and P_2V_2 in terms of temperatures T_1 and T_2 into Equation 5, we get

$$\mathbf{W} = \frac{R}{\mathbf{1-\gamma}}[T_2\ -\ T_1] \ldots\ldots\ldots\ldots.(10)$$

Example: A certain amount of air at a temperature of 27 C° was pressurized suddenly until it became one-third of what it was on. Find the work done? If you know = 1.4 and R = $8.3x10^7$ *erg /.mole.K*

$V_2=V_1/3$, W=?

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_1\frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} = T_2\ , \qquad T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_2$$

T_2 = $(27+273)(3)^{1.4-1}$ =465.6K

$$W = \frac{R}{1-\gamma}[T_2 - T_1]$$

$$W = \frac{8.3x107}{1-1.4}[465.6 - 300] = 3.44\times10^{10}\text{erg} = 3.44\times10^{3}\text{J}$$

Work as a function of the volumetric expansion factor β and the volumetric compressibility coefficient K:

To calculate work in terms of β and k, we need dV in terms of pressure and temperature

V=f(T,P)

$$\mathbf{dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP \ldots\ldots\ldots.(1)}$$

$$\because \boldsymbol{\beta} = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \ldots\ldots\ldots.(2)\ , \qquad K = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \ldots\ldots\ldots.(3)$$

We substitute Equations 2 and 3 into Equation 1

dV =βVdT – KVdP…………..(4)

from equ. of work

dW =PdV

We substitute equation 4 in last equation

$dW = P\,\beta V dT - KPVdP$…………..(5)

for an ideal gas $\beta = 1/T$ $K = 1/P$

$$\therefore dW = \frac{PV}{T}dT - PV\frac{dP}{P} \dots\dots\dots\dots (6)$$

$$dW = \frac{PV}{T}dT - VdP \dots\dots\dots.. (7)$$

- From eqe. 7 ,If the process is under constant pressure (isobaric process)

$dP = 0$

$$dW = \frac{PV}{T}dT - 0 \dots\dots\dots. (8)$$

$dW = nRdT$………(9)

- From eqe. 7 , To isothermal process

$dT = 0$

$$dW = 0 - VdP \dots\dots\dots (10)$$

$$W = -nRT\frac{dP}{P} \dots\dots.. (11)$$

$$\boldsymbol{W_T = nRT \ln\frac{P_1}{P_2} \dots\dots.. (12)}$$

- There is another benefit of equation 4, which is obtaining the state equation for the substance by taking the integral of equation

$$4\int_{V_0}^{V} dV = \int_{T_0}^{T} V\beta dT - \int_{P_0}^{P} VKdP \dots\dots\dots. (13)$$

If the changes in pressure and temperature are moderate, then V, β, K can be considered constant

$$\int_{V_0}^{V} dV = V_0\beta\int_{T_0}^{T} dT - V_0K\int_{P_0}^{P} dP \ldots\ldots\ldots (14)$$

$$(V-V_0) = V_0\beta\,(T-T_0) - V_0K(P-P_0)\ldots\ldots\ldots(15)$$

$$V = V_0(1+\beta(T-T_0) - K(P-P_0))\ldots\ldots\ldots\ldots\ (16)$$

Equation 16 represents the equation for the case under conditions where V, β, K does not change.

Example: One mole of an ideal gas completes a closed periodic process consisting of the following sequential stages:

1. From the state (P_1, V_1) under constant pressure to the state (P_1, V_2)

2. Under the volume fixed to the state (P_2, V_2)

3. Under static pressure (P_2, V_1)

4. Under constant volume, back to the initial state (P_1, V_1)

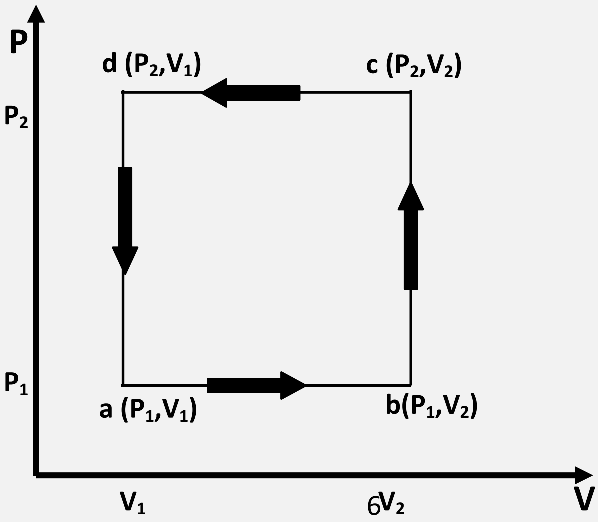
A- Draw a process diagram at (P-V) level.

B- Find the work performed during the operation

C- Find the amount of heat that the gas gained during the process

D- The change in his internal energy

A-



B- Work performed during the process

$W=W(a\rightarrow b)+W(b\rightarrow c)+W(c\rightarrow d)+W(d\rightarrow a)$

Under constant volume, work is zero

$W(b\rightarrow c)=W(d\rightarrow a)=\text{zero}$

$$W=W(a\rightarrow b)=P_1\int_{V_1}^{V_2}dV=P_1(V_2-V_1)\ldots\ldots\ldots\ldots\ldots(1)$$

$$W=W(c\rightarrow d)=P_2\int_{V_2}^{V_1}dV=P_2(V_1-V_2)\ldots\ldots\ldots\ldots\ldots(2)$$

$$W=P_1(V_2-V_1)+P_2(V_1-V_2)\ldots\ldots\ldots\ldots\ldots(3)$$

$W=(V_2-V_1)(P_1-P_2)$……………………………………(4)

C- The amount of heat

$PV=nRT$ For one mole, n = 1, by differential equation

$PdV+VdP=RdT$ …………………….…(1) $dQ=C_PdT$, (at p = constant ,dP = 0) , $dT=PdV/R$

$dQ=C_VdT$, (at V=constant ,dV= 0) , $dT=VdP/R$

$dQ_1(a\rightarrow b)=C_P\,P_1dV/R$, $dQ_2(b\rightarrow c)=C_V\,V_2dP/R$

$dQ_3(c\rightarrow d)=C_P\,P_2dV/R$, $dQ_4(d\rightarrow a)=C_V\,V_1dP/R$

$dQ=dQ_1+dQ_2+dQ_3+dQ_4$……………………………………..…..(2)

$dQ=\underline{C_P\,P_1dV/R}+C_V\,V_2dP/R+\underline{C_P\,P_2dV/R}+C_V\,V_1dP/R$…….(3)

$dQ=C_P\,dV/R(P_1+P_2)+C_V\,dP/R\,(V_2+V_1)$ ……………………(4)

D. Change in internal energy

$dU = dQ - dW$

$dU_1 = dQ_1 - dW_1$

$dU_1 = C_P P_1 dV/R - P_1(V_2-V_1)$…………….(1)

$dU_2 = dQ_2 - 0 = C_V V_2 dP/R$ ………………(2)

$dU_3 = dQ_3 - dW_3$……………………………(3)

$dU_3 = C_P P_2 dV/R - P_2(V_2-V_1)$…………….(4)

$dU_4 = dQ_4 - 0 = C_V V_1 dP/R$ ………………(5)

$dU = dU_1 + dU_2 + dU_3 + dU_3$……………….(6)

$dU = \underline{C_P P_1 dV/R - P_1(V_2-V_1)} + C_V V_2 dP/R +$

$\underline{C_P P_2 dV/R - P_2(V_2-V_1)} + C_V V_1 dP/R$

$dU = C_P dV/R (P_1 + P_2) + C_V dP/R (V_1+V_2) + (V_2-V_1)(P_2 - P_1)$…..(7)