**College of Science**
**Department of Physics**
**Fourth Class**
**Lecture 4**

# Quantum Mechanics

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## Lecture 4: The basic postulate of quantum

**Preparation**

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## Unit 3

## The basic postulate of quantum

### 3.1 Postulate 1

To every system there correspond a set of functions (state function or wave functions) which completely describe the states of the system.

Definition:

A state is a function of certain variables, a function from which by the rules of quantum theory significant information can be obtained.

By state or state function we thus mean a mathematical construct

$\Psi(x_1,y_1,z_1;x_2,y_2,z_2;\text{------------}x_n,y_n,z_n)$

$\Psi=\sum \Psi_n$

حيث $\Psi_n$ مجموعة الاحوال الممكنة للنظام الذي تمثله $\Psi$ وهذا مايسمى مبدأ التراكب

### 3.2 Postulate 2

To every physically measurable quantity (called an observable or dynamical variable there corresponds to a linear Hermitian operator.

In the following table we give a summary of the four most important operators of Q.M.

| Name of observable | Classical representation | Quantum operator |
|---|---|---|
| Cartesian component of line momentum of Jth partical | $Px_j=m_j\dot{x}_j$ | $\frac{ħ}{i}\frac{\partial}{\partial x_j} or -iħ\frac{\partial}{\partial x_j}$ |
| Cartesian coordinate | $X_j$ | $\hat{x}_j$ |
| X-component of angular momentum of Jth particle | $\vec{L}=\vec{r}\times\vec{p}$ <br> $m_j(y_j\dot{z}_j-z_j\dot{y}_j)$ | $\frac{ħ}{i}(y_j\frac{\partial}{\partial z_j}-z_j\frac{\partial}{\partial y_j})$ |
| Total energy | T=E+V=1/2mv$^2$+V <br> $\frac{1}{2}\sum\frac{1}{m_j}(P_{x_j}^2+P_{y_j}^2+P_{z_j}^2)+V(x_1,------,z_n)$ | $-\frac{ħ^2}{2}\sum_{j=1}^{n}\frac{1}{m_j}(\frac{\partial^2}{\partial x_j^2}+\frac{\partial^2}{\partial y_j^2}+\frac{\partial^2}{\partial z_j^2})+V(x_1,-----,z_n)$ |

To show: $P_x = \frac{ħ}{i}\frac{\partial}{\partial x_j}$

$\Psi(x) = e^{ikx}$

$\frac{\partial \Psi(x)}{\partial x} = ike^{ikx}$ k: wave vector

$\frac{\partial}{\partial x} = i\frac{2\pi}{\lambda}$ p=h/λ

$\frac{\partial}{\partial x} = i\frac{2\pi}{h/p_x} \rightarrow \frac{\partial}{\partial x} = ip_x\frac{2\pi}{h}$

$$\frac{h}{i2\pi}\frac{\partial}{\partial x} = p_x \quad \rightarrow \quad p_x = \frac{ħ}{i}\frac{\partial}{\partial x} = \frac{i^4 ħ}{i}\frac{\partial}{\partial x} = i^3 ħ\frac{\partial}{\partial x} = -iħ\frac{\partial}{\partial x}$$

Note :$i = \sqrt{-1} \rightarrow i.i = -1$

To show: $\hat{H} = -\frac{ħ^2}{2m}\frac{\partial^2}{\partial x^2} + V$

$$\hat{H} = \frac{{p_x}^2}{2m} + V(x)$$

$\frac{{p_x}^2}{2m} = (\frac{ħ}{i}\frac{\partial}{\partial x_j})^2\frac{1}{2m} = \frac{1}{2m} \times \frac{ħ^2}{i^2}\frac{\partial^2}{\partial x^2} = -\frac{ħ^2}{2m}\frac{\partial^2}{\partial x^2}$

$\hat{H} = -\frac{ħ^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$

- The operator for the Cartesian coordinate X is identical with its classical representation and has been included only for formal reasons.
- When the operator corresponding to the linear momentum (p) of a single particle is written in the vector from $-iħ\nabla$ ,those corresponding to angular momentum and energy of this particle may be constructed according to classical formula

  Angular momentum = $\vec{r} \times \vec{p}$
  $= -iħ\,\vec{r} \times \nabla$

  Total energy =$\frac{p^2}{2m} + V$
  $= -\frac{ħ^2}{2m}\nabla^2 + V$

**Note:** $\vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}} = -\boldsymbol{i}\hbar \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$

These vector forms are valid in all other system of coordinates and should be used as the basis of transformations.

The total energy operator is just the sum of the kinetic energy written as $\frac{p^2}{2m}$ in Cartesian components form and the potential energy labeled as V.

It will not be possible for us to specify the form of V until we decide which problem we have to solve. knowing V is same what like knowing the law of force in problem in classical mechanics.

### 3.3 Postulate 3

The only possible values which a measurement of the observable whose operator $\hat{A}$ is can yield are the eigen values α of the equation

$\hat{A}\Psi$(x)=α Ψ(x) ---------------1

Provided that Ψ(x) obey the following conditions

1. Ψ is a single valued.
2. $\int \Psi^* \Psi \, d\vartheta < \infty$

$$\int_{-\infty}^{\infty} \Psi_m^* \Psi_n d\vartheta = \delta_{mn} = \begin{cases} 0 \rightarrow m \neq n \ \ orthogonal \\ 1 \rightarrow m = n \ \ normalized \end{cases}$$

$d\vartheta$- is the volume element of configuration space

$\Psi^*$- complex conjugate of $\Psi$.

The range of integration depend on the particular problem under consideration. We illustrate the meaning of this postulate by an example.

Let us find the measurable value of linear momentum of a particle ,known to same where on the x-axis between the finite point x=a and x=b

$\hat{p}$ linear momentum operator $= -i\hbar \frac{\partial}{\partial x}$

$\hat{A}\Psi$(x)=$\alpha\Psi$(x) becomes $-i\hbar \frac{\partial}{\partial x} = p\Psi$

Which has the solution $\Psi = ce^{\frac{i}{\hbar}px}$

From point of view of conditions (i) and (ii)

(i) $\Psi$ is single valued
(ii) $\int \Psi^* \Psi \, d\vartheta < \infty$ finite

$\Psi$ is single valued $\Psi = ce^{\frac{i}{\hbar}px}$ and more over

$$\int_a^b \Psi^* \Psi dx = \int_a^b C^* \, e^{-\frac{i}{\hbar}px} \, Ce^{\frac{i}{\hbar}px} dx$$

$$= C^* C \int_a^b e^0 dx = C^* C \, (b - a)$$ is finite for every finite C.

No restriction up on (p) all values of linear momentum, it form a continuous spectrum (p is not discrete) and every function of the form $ce^{\frac{i}{\hbar}px}$ with constant (p) is an eigen function.