College of Science
Department of Physics
Fourth Class
Lecture 16

Quantum Mechanics

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Lecture 16: Problems

Preparation

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Problem 7.1

If a beam of electrons impinges on an energy barrier of height 0.03 ev and infinity width, find the fraction of electrons reflected and transmitted at the barrier if the energy of the impinging electrons is 0.04 ev. ($m_e = 9.1 \times 10^{-31} kg$)

Solution:

E=0.04 ev and V_o= 0.03

$\therefore E > V_o \qquad R = \frac{(P_1-P_2)^2}{(P_1+P_2)^2} \qquad P_1 = \sqrt{2mE}\ ,\ P_2 = \sqrt{2m(E-V_o)}$

$$P_1 = \sqrt{2 \times 9.1 \times 10^{-31} \times 0.04 \times 1.6 \times 10^{-19}} = 1.079 \times 10^{-25} Kg.m/sec$$

$$P_2 = \sqrt{2 \times 9.1 \times 10^{-31} \times 0.01 \times 1.6 \times 10^{-19}} = 0.539 \times 10^{-25} Kg.m/sec$$

$$R = \frac{(P_1-P_2)^2}{(P_1+P_2)^2} = \frac{[(1.079-0.539)\times 10^{-25}]^2}{[(1.079+0.539)\times 10^{-25}]^2} = \frac{0.29}{2.61} = 0.11$$

T + R = 1 , **T**= 1- 0.11= 0.89

Problem 7.2

Calculate the probability of transmission of a particle through the rectangular barrier indicate below: $V_o = 2ev\ , E = 1ev$ and barrier width =1°A , $m_e = 9.1 \times 10^{-24} gm$

Solution

$$T = \frac{16E(V_o-E)}{V_o^2} \exp\left(-2\sqrt{\frac{2m(V_o-E)}{\hbar^2}}\ a\right)$$

$$T = \frac{16 \times 1 \times 1.6 \times 10^{-12} \times (2-1) \times 1.6 \times 10^{-12}}{(2 \times 1.6 \times 10^{-12})^2} \times \exp\left(-2\frac{\sqrt{2 \times 1.6 \times 10^{-24} \times 1.6 \times 10^{-12}}}{1.05 \times 10^{-27}} \times 1 \times 10^{-8}\right)$$

$$T = 4\, exp\left(-\frac{2 \times 1.6 \times \sqrt{2} \times 10^{-18} \times 10^{-8}}{1.05 \times 10^{-27}}\right) = 1.9 \times 10^{-19}$$

Problem 7.3

- **a-** Evaluate the transmission coefficient for an electron of total energy 2 ev incident up on a rectangular potential barrier of high 4 ev and thickness $10^{-10}m$.
- **b-** Repeat the evaluation for a barrier thickness of $10^{-9}m$.

Solution:

a- E =2 ev , V_o= 4 ev , a = $10^{-10}m$

$$\boldsymbol{T} = \frac{-4P_1^2P_2^2 sech^2\left(\frac{iP_2 a}{\hbar}\right)}{\left(P_1^2+P_2^2\right)^2 \tanh^2\left(\frac{iP_2 a}{\hbar}\right) - 4P_1^2P_2^2}$$

$$P_1 = \sqrt{2mE} = \sqrt{2\times 9.1\times 10^{-31}\times 2\times 1.6\times 10^{-19}}$$

$$= 7.63\times 10^{-25} kg.\frac{m}{sec}$$

$$P_2 = \sqrt{2m(E-V_o)}$$

$$= \sqrt{2\times 9.1\times 10^{-31}\times (2-4)\times 1.6\times 10^{-19}}$$

$$= \boldsymbol{i}\sqrt{4\times 9.1\times 1.6\times 10^{-50}} = i\times 7.63\times 10^{-25} kg.\frac{m}{sec}$$

$$(P_1^2+P_2^2)^2 = [(7.63\times 10^{-25})^2 + (i\times 7.63\times 10^{-25})^2]^2$$

= zero

$$\therefore \quad \boldsymbol{T} = \frac{-4P_1^2P_2^2 sech^2\left(\frac{iP_2 a}{\hbar}\right)}{-4P_1^2P_2^2} = \boldsymbol{sech^2}\left(\frac{iP_2 a}{\hbar}\right)$$

$$T = sech^2\left(\frac{\boldsymbol{i}(i\times 7.63\times 10^{-25})\times 10^{-10}}{1.05\times 10^{-34}}\right) = \mathbf{0.634}$$

b- $\frac{\boldsymbol{i}P_2 a}{\hbar} = \frac{\boldsymbol{i}\times i\times 7.63\times 10^{-25}\times 10^{-9}}{1.05\times 10^{-34}} = -7.26 \quad , \quad |-7.26| > 1$

$$\therefore \quad \boldsymbol{T} = \frac{16E(V_o-E)}{V_o^2}\ \mathbf{e}^{\frac{2iP_2 a}{\hbar}} = \frac{16\times 2\times 2}{16} e^{-2\times 7.26} = 1.9\times 10^{-6}$$

Problem 7.4

With out bearing with detailed algebra. Show that when $E << V_o$ the transmission probability for a particle approaching a rectangular potential barrier, falls off exponentially with increasing barrier width and $\sqrt{V_o - E}$.

Solution :

For $E << V_o$

$$T = \frac{-4P_1^2 P_2^2 sech^2\left(\frac{iP_2 a}{\hbar}\right)}{(P_1^2 + P_2^2)^2 \tanh^2\left(\frac{iP_2 a}{\hbar}\right) - 4P_1^2 P_2^2}$$

$$\tan h\theta = \frac{\sin h\theta}{\cos h\theta} = \frac{\frac{e^{\theta} - e^{-\theta}}{2}}{\frac{e^{\theta} + e^{-\theta}}{2}} = \frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}}$$

$$\theta = \frac{iP_2 a}{\hbar} \quad for\ large\ a \ \rightarrow e^{-\theta}\ neglected \ \ e^{-\infty} \rightarrow 0$$

$$\tan h\theta = \frac{e^{\theta}}{e^{\theta}} = 1 \ \rightarrow for\ large\ \ a \quad , \quad \sec h\theta = \frac{1}{\cos h\theta} = \frac{1}{\frac{e^{\theta} + e^{-\theta}}{2}} = \frac{2}{e^{\theta} + e^{-\theta}}$$

For large θ, $\frac{1}{e^{\theta} + e^{-\theta}} \ \rightarrow \frac{1}{e^{-\theta}} \quad becuse \ \frac{1}{e^{\theta}}\ is\ neglected$ and $\sec h\theta = 2e^{\theta}$

$$\therefore \quad T = \frac{-4P_1^2 P_2^2 (2e^{\frac{iP_2 a}{\hbar}})^2}{(P_1^2 + P_2^2)^2 - 4P_1^2 P_2^2} = \frac{-4P_1^2 P_2^2 (4e^{\frac{2iP_2 a}{\hbar}})}{(P_1^2 - P_2^2)^2}$$

$$P_1 = \sqrt{2mE} \qquad , \quad P_2 = \sqrt{2m(E - V_o}$$

$$T = \frac{-4(2mE)\big(2m(E - V_o)\big)\left(4e^{\frac{2iP_2 a}{\hbar}}\right)}{[(2mE) - 22m(E - V_o)]^2} = \frac{-16(2m)^2 E(E - V_o) e^{\frac{2iP_2 a}{\hbar}}}{(2m)^2 [E - E + V_o]^2}$$

$$= \frac{-16E(V_o - E) e^{\frac{2iP_2 a}{\hbar}}}{V_o^2} = \frac{-16E(V_o - E) e^{\frac{2i\sqrt{2m(E - V_o}}{\hbar} \ \ a}}{V_o^2}$$

$$T = \frac{-16E(V_o - E)}{V_o^2} \exp\left[\frac{-2\sqrt{2m(V_o - E)}}{\hbar} \ \ a\right]$$