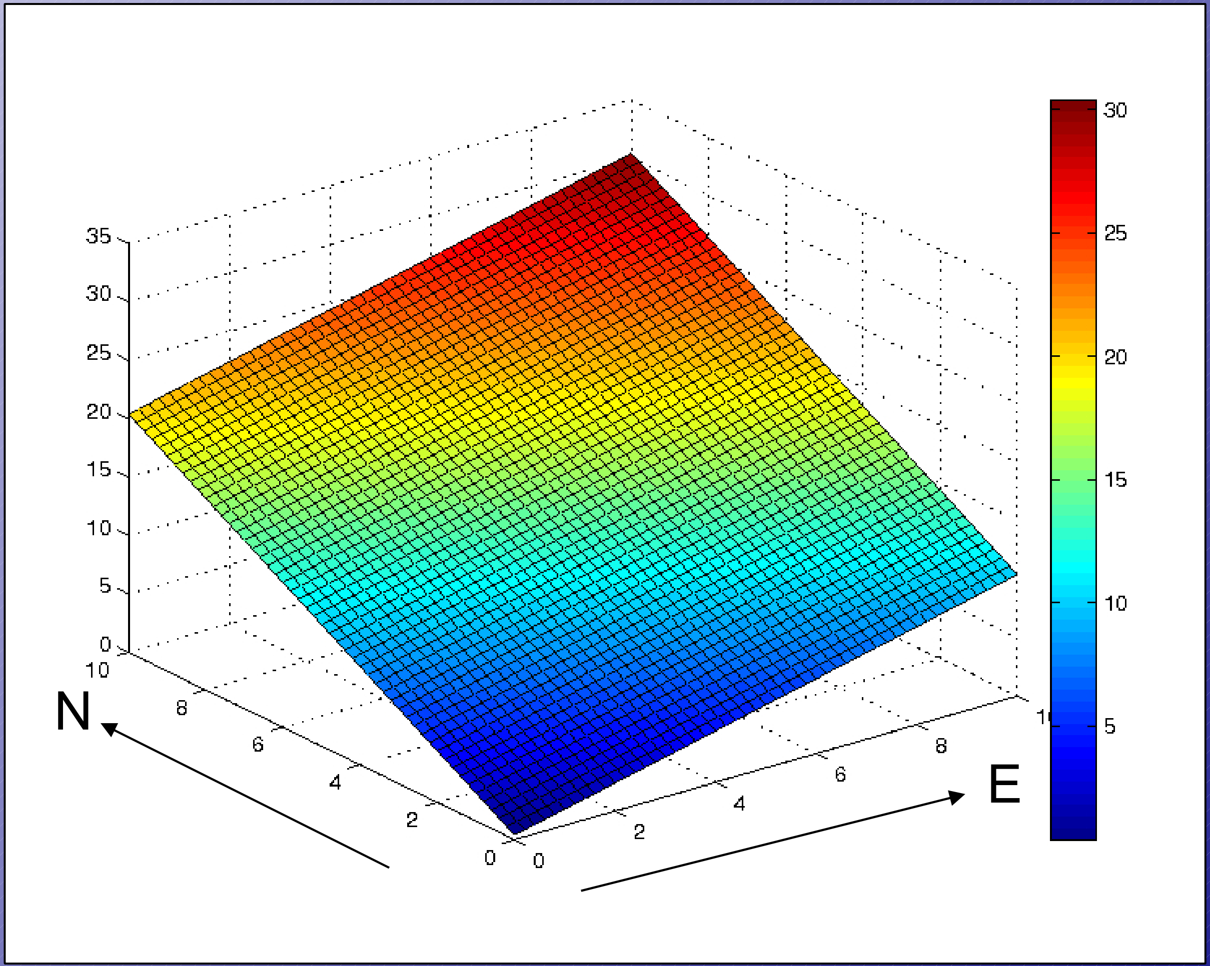
The Three-Point Problem



Describing a plane in 3-D space

- Graphical
- Cramer's Rule (2-D and 3-D)

The Three-Point Problem

- Given the elevation of 3 points on a geologic surface
- What is the attitude (strike and dip) of that surface ?

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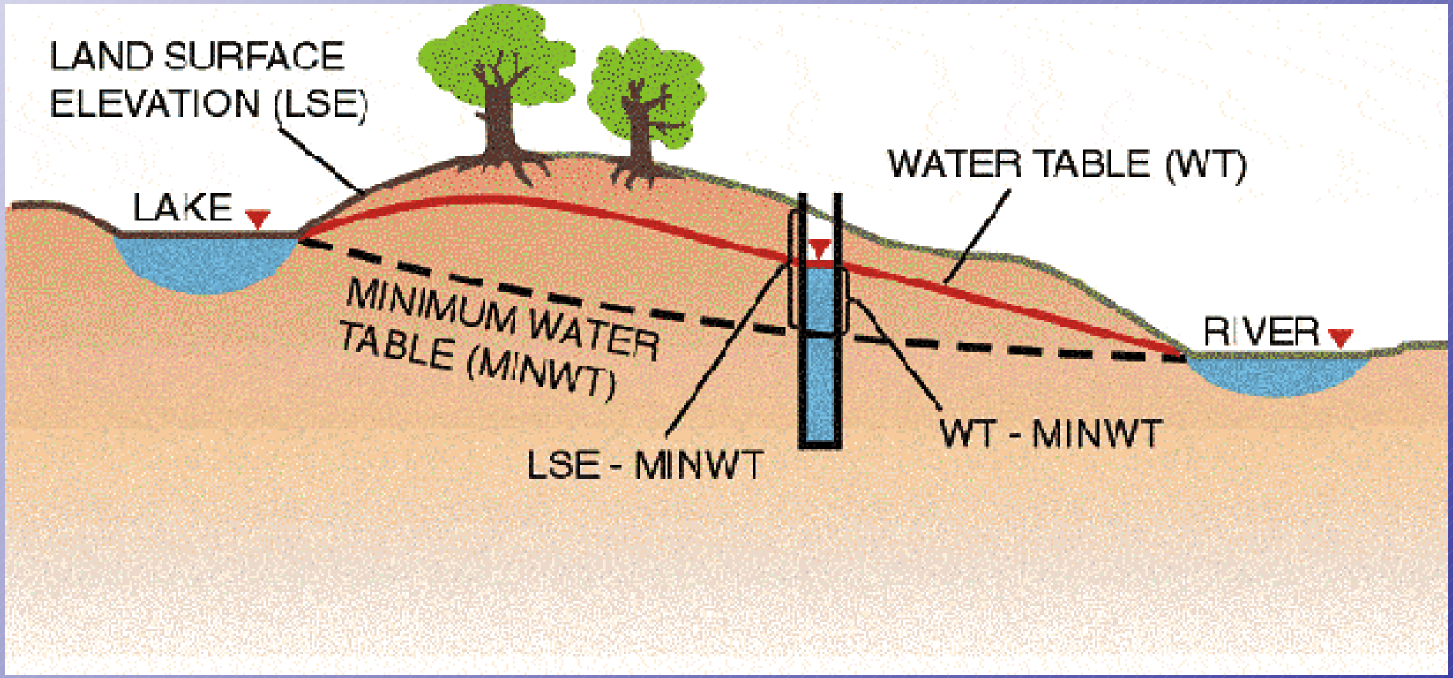

- Given the water level in 3 wells
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- The 3 point problem is also a gateway to useful mathematics !
- We will study 2 solutions to this problem using Cramer's Rule

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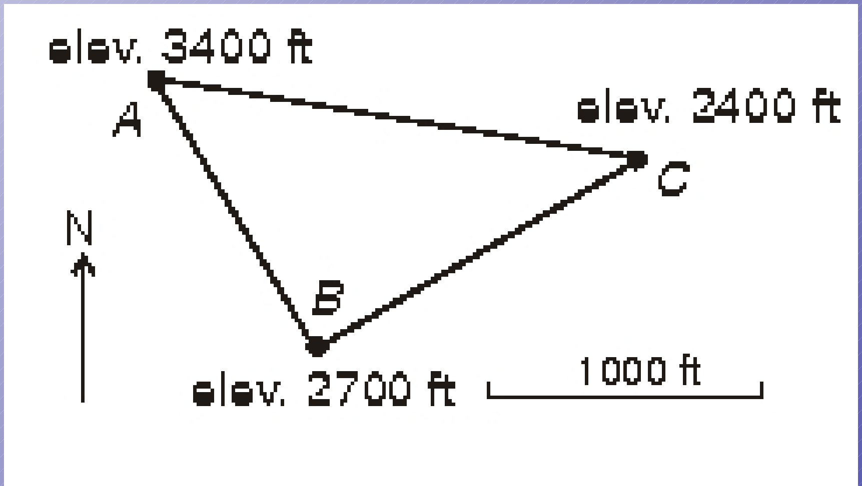
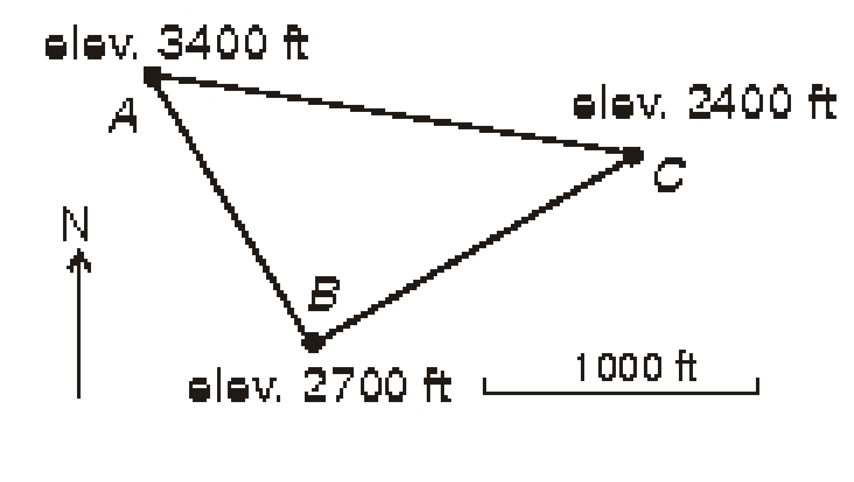


- The figure above represents an unconformity surface
- We want to find the strike and dip of the unconformity

The Three-Point Problem: Graphical Solution



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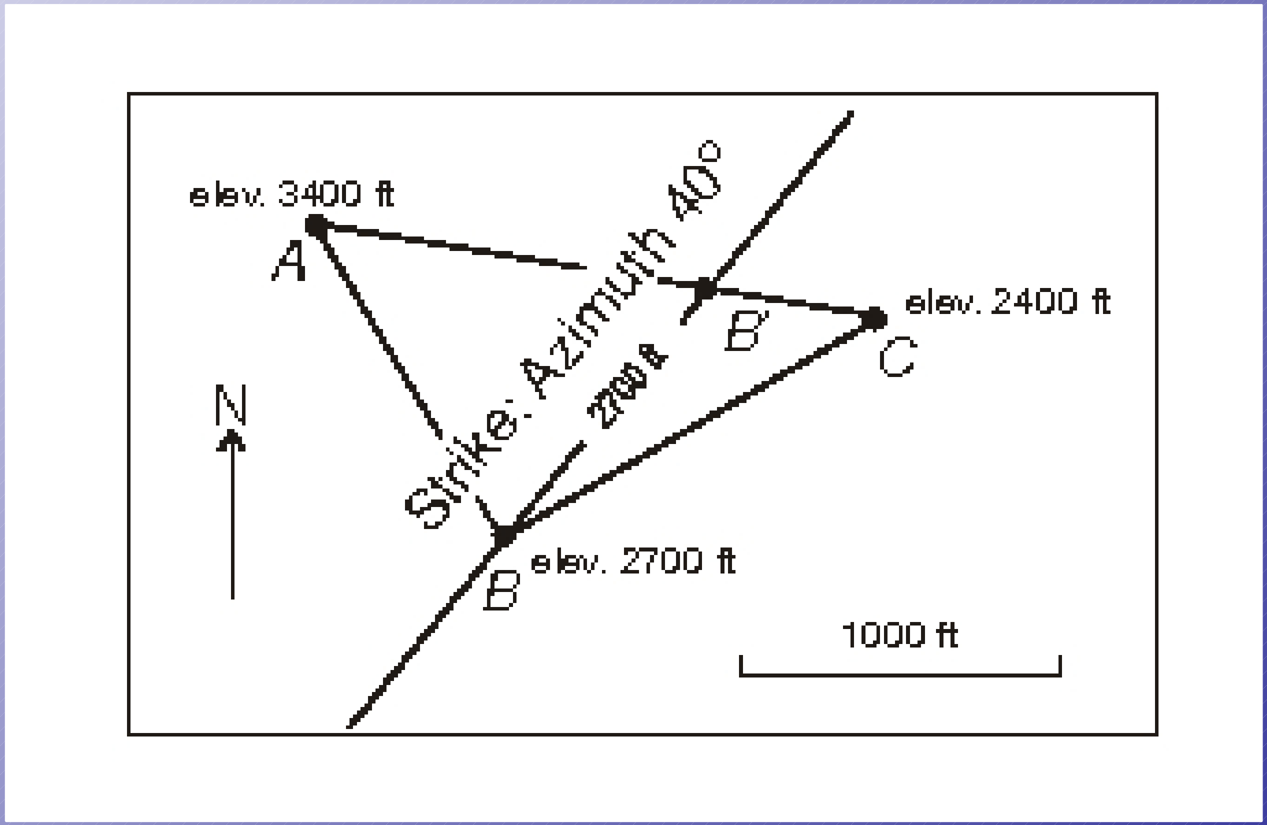
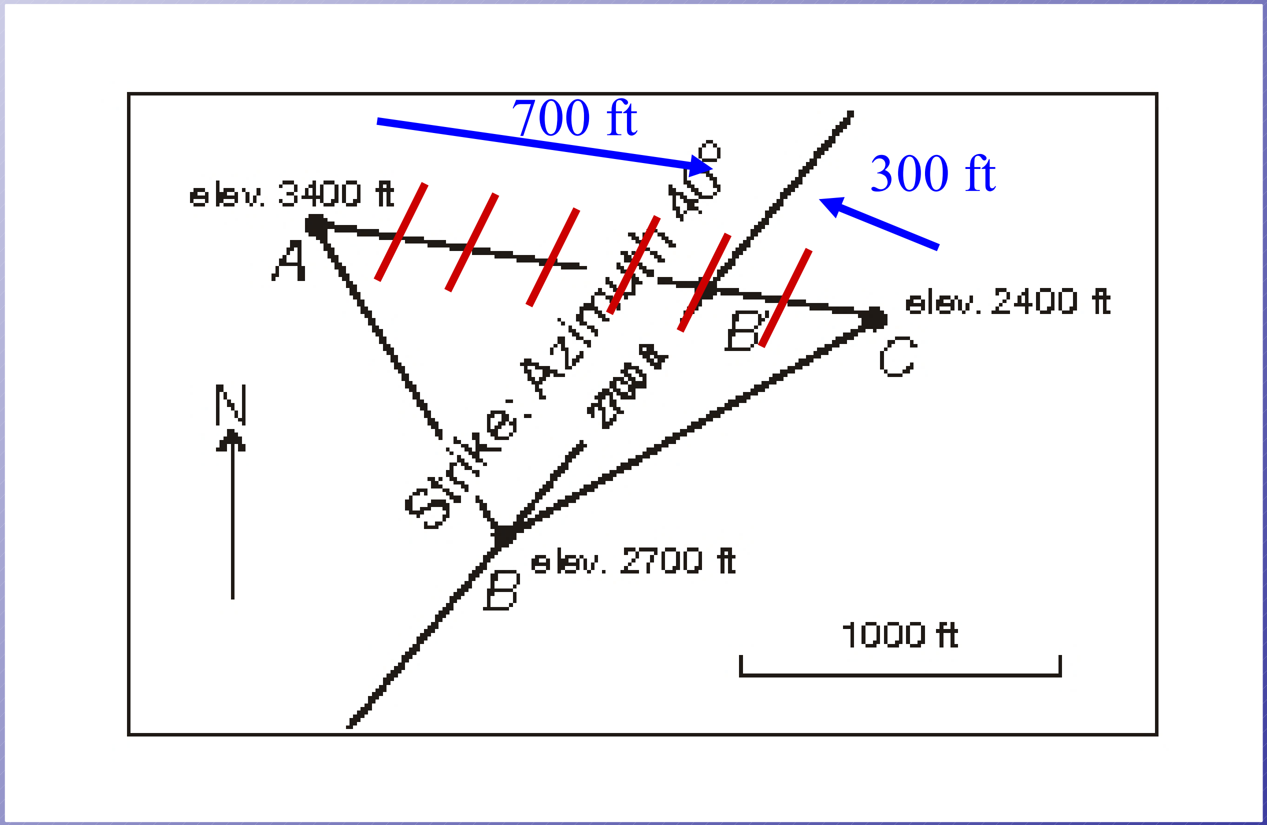


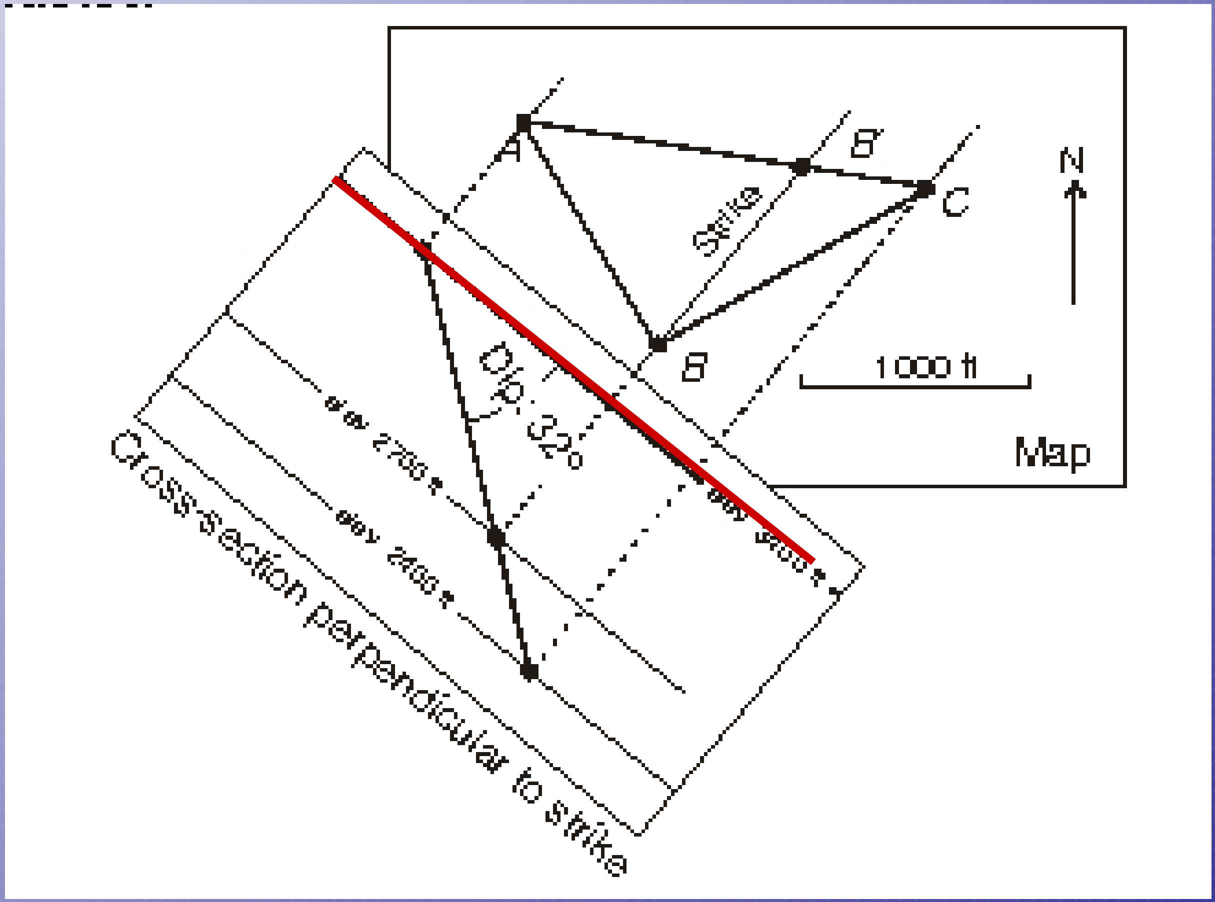
- A contour line passing through B must cross the line segment AC
- By the definition of ***strike,*** the direction of this contour is the strike of the unconformity surface

The Three-Point Problem: Graphical Solution - *Strike*



- Locate the contour: by dividing segment AC into increments
- This unconformity drops 1000 ft between A and C
- Therefore B' is 70% of the distance from A to C
- We can measure the azimuth of the strike with a protractor

The Three-Point Problem: Graphical Solution - *Dip*



- Draw a cross-section perpendicular to BB'

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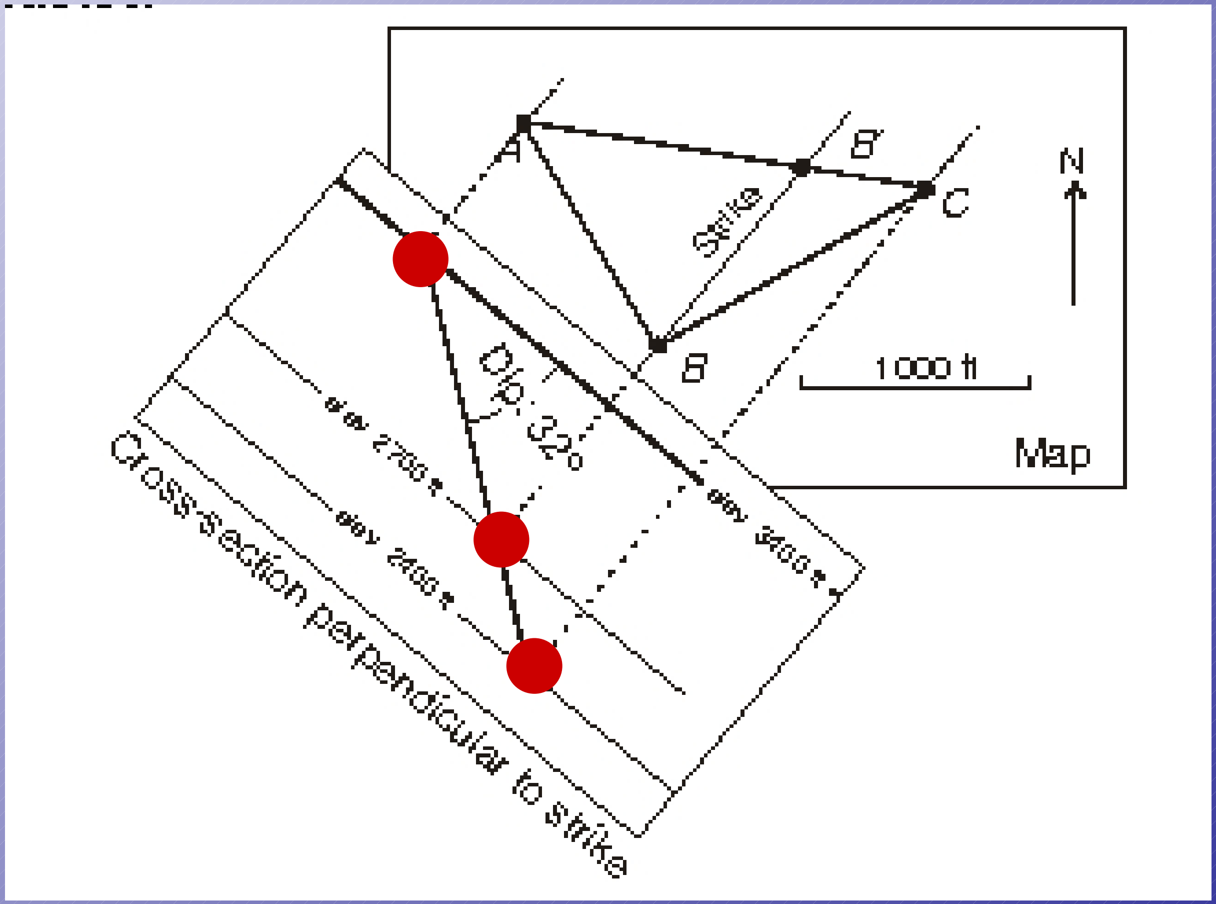

- Draw a cross-section perpendicular to BB'
- Then use a vertical scale = horizontal scale
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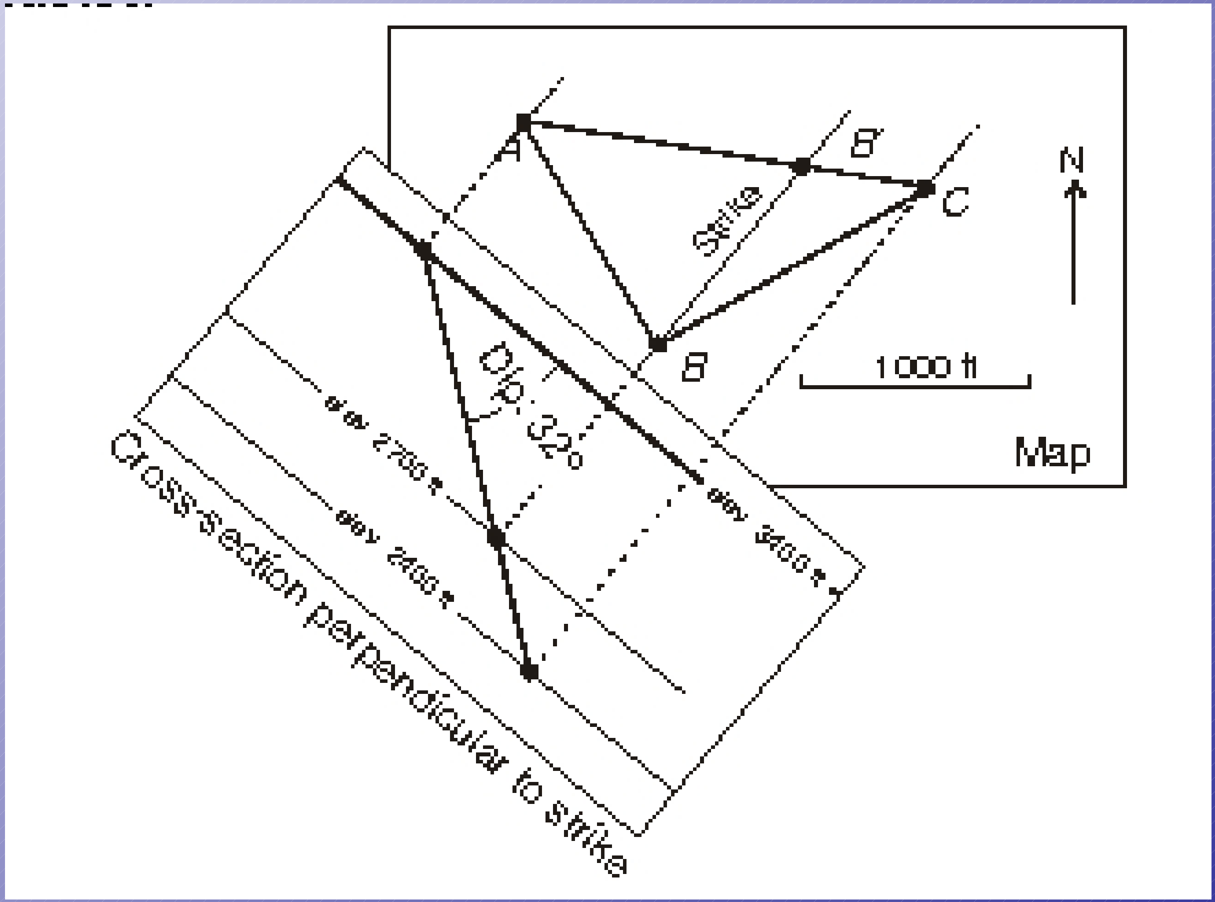
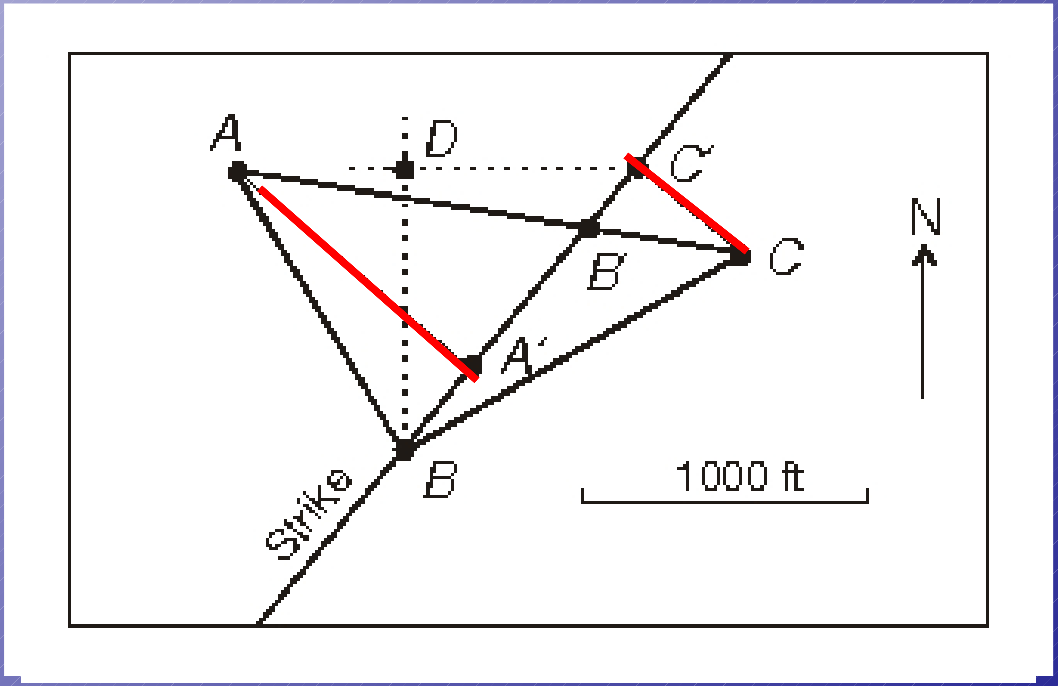


- Because the cross-section is perpendicular to ***strike***
- The included angle is the ***true dip***.
- You can measure the dip angle with a protractor (32°)

The Three-Point Problem: Graphical Solution - Modified

- Drawing parallels and perpendiculars with triangles
- First draw 2 lines which are perpendicular to the ***strike line*** (AA' and CC')



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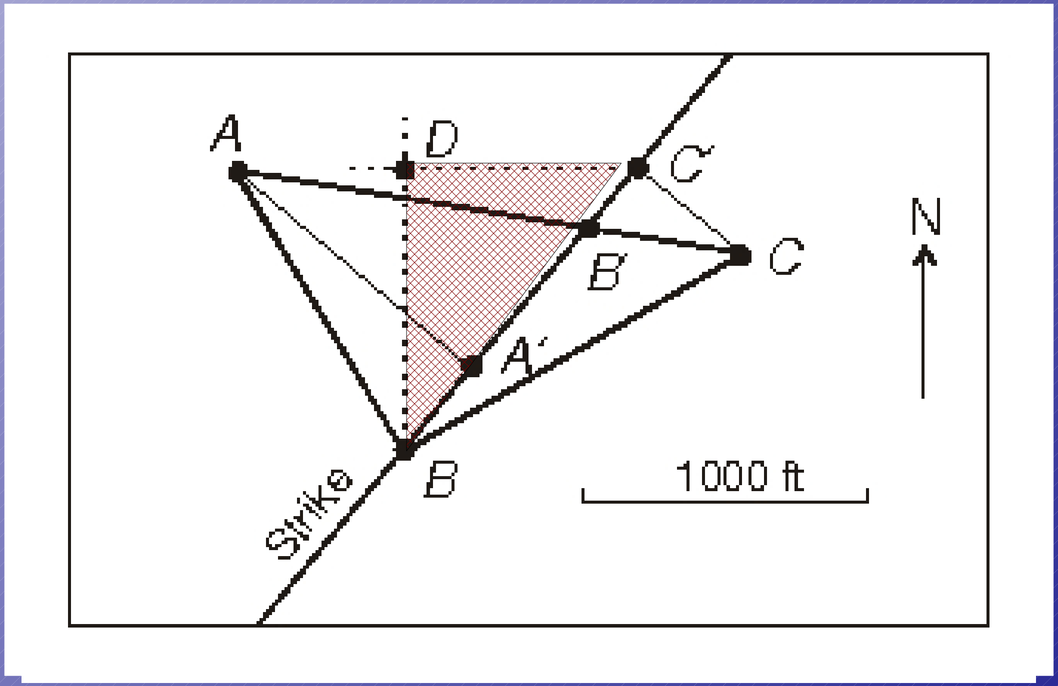

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- Measure distances BD, DC', AA', and CC'.

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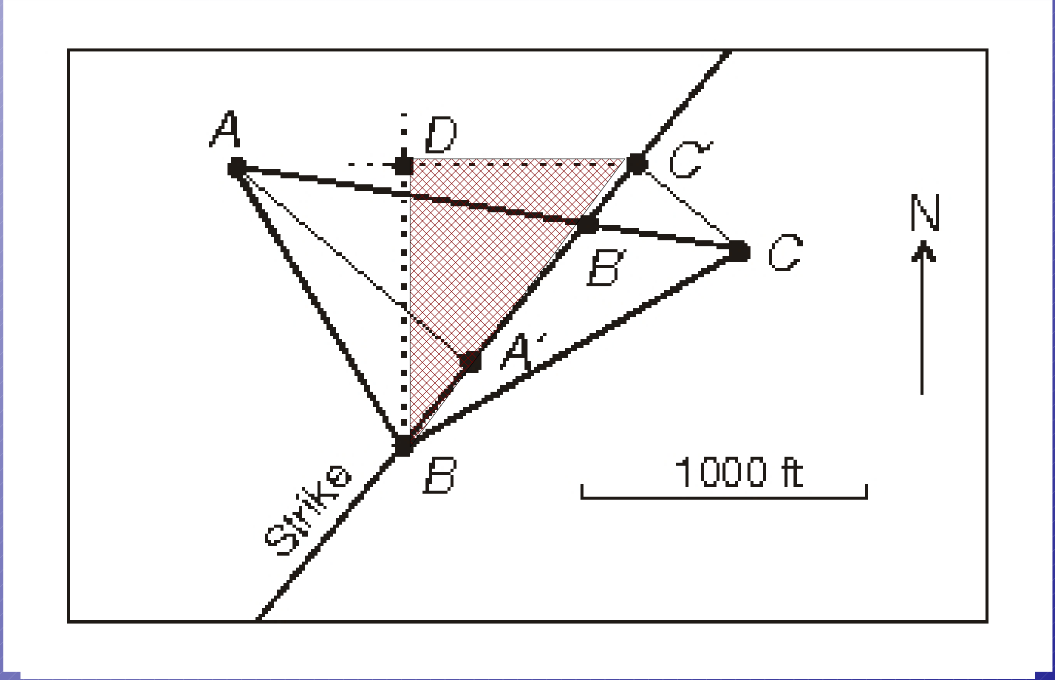

- The azimuth of ***strike*** θ_{strike} *is*:

$$\Theta_{strike} = \arctan\ (DC' / BD)$$

- The angle of dip θ_{dip} *is:*

$$\Theta_{dip} = \arctan\ (h_A - h_{A'} / AA')$$

where h_A, $h_{A'}$, h_C, *and* $h_{C'}$ *are the elevation at each location*

The Three-Point Problem: Graphical Solutions

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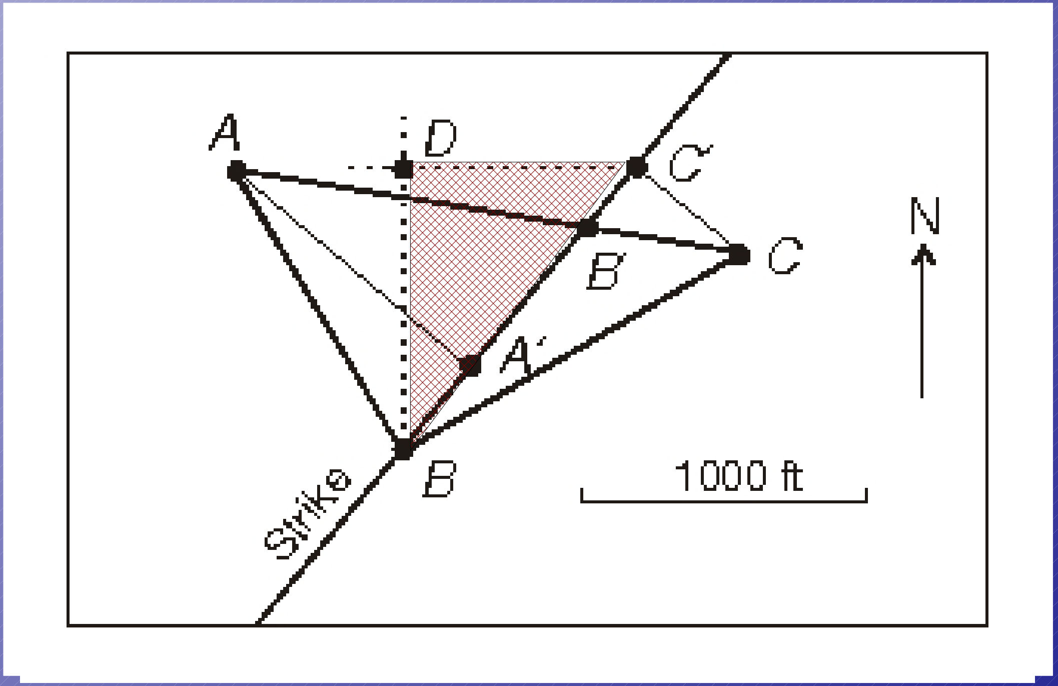

The Three-Point Problem: Computational Solutions

- There are several ways to calculate the strike and dip of a surface (for a 3 point problem) without measuring anything.

- With these techniques, you can solve 50 or more 3 point problems in the time it takes you to enter the data.....but it will require a little math...

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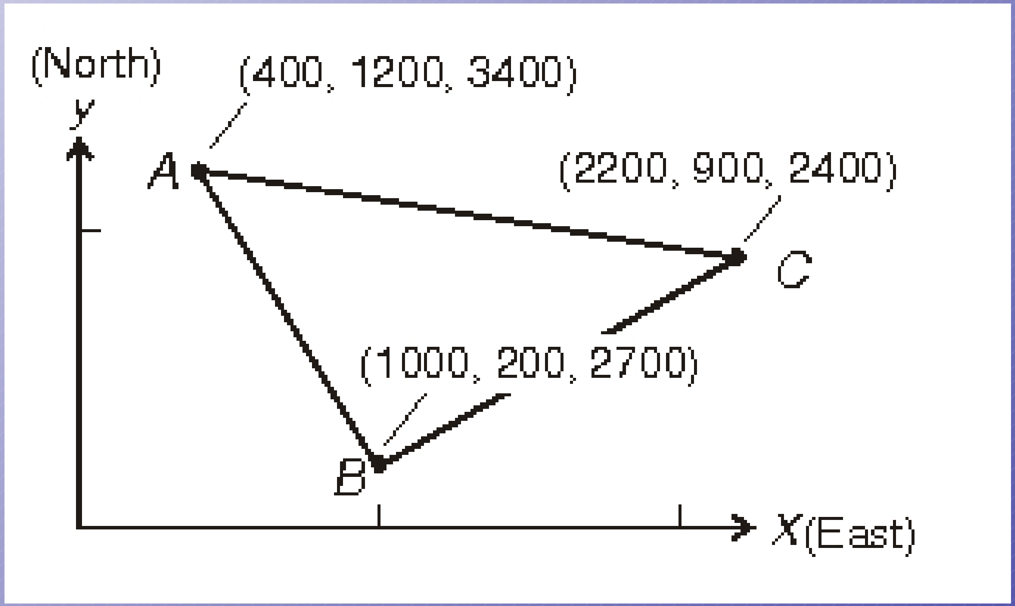

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If you are not given this info in the original format, you can easily convert using sines and cosines.

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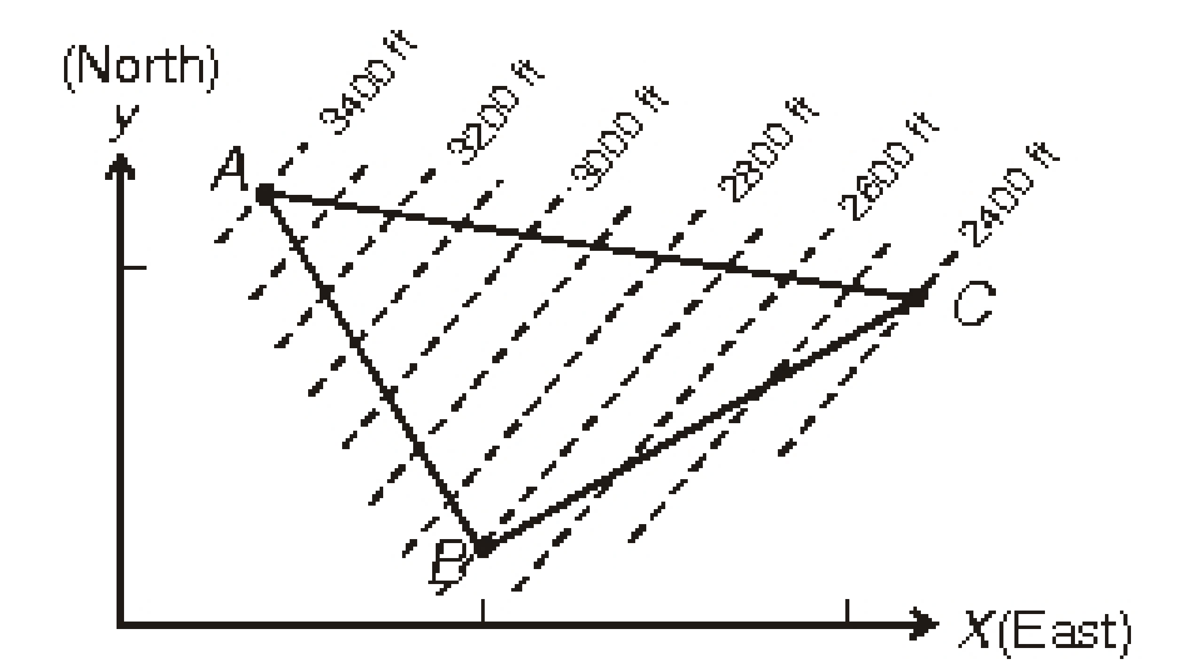

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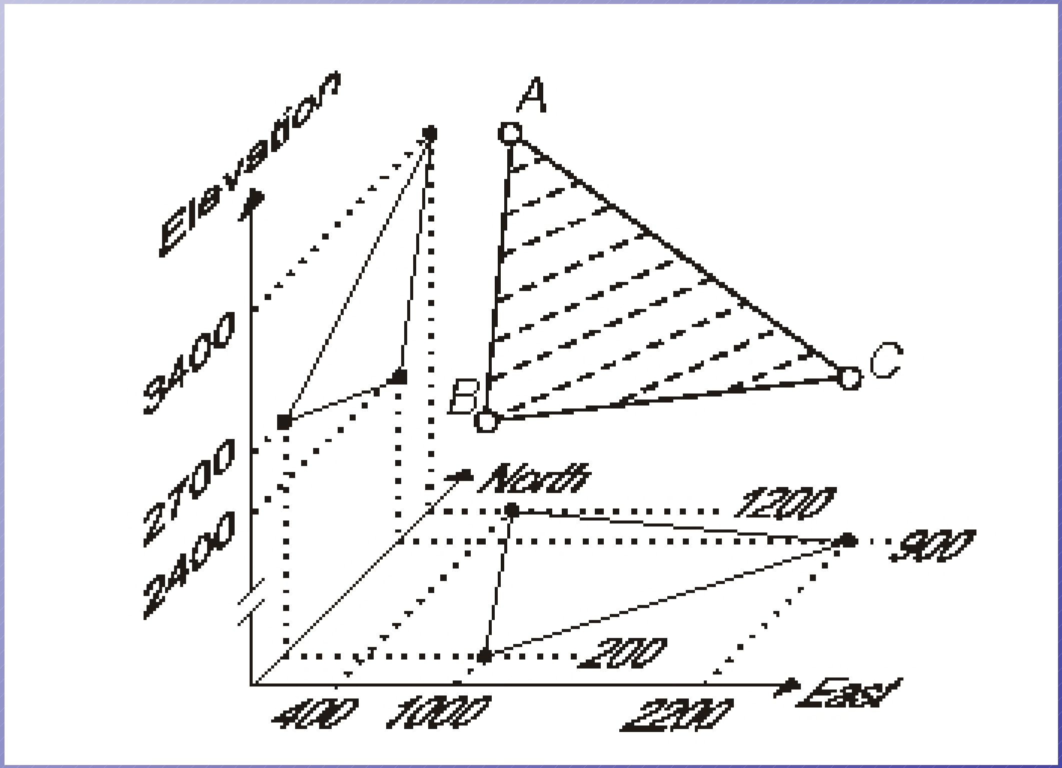
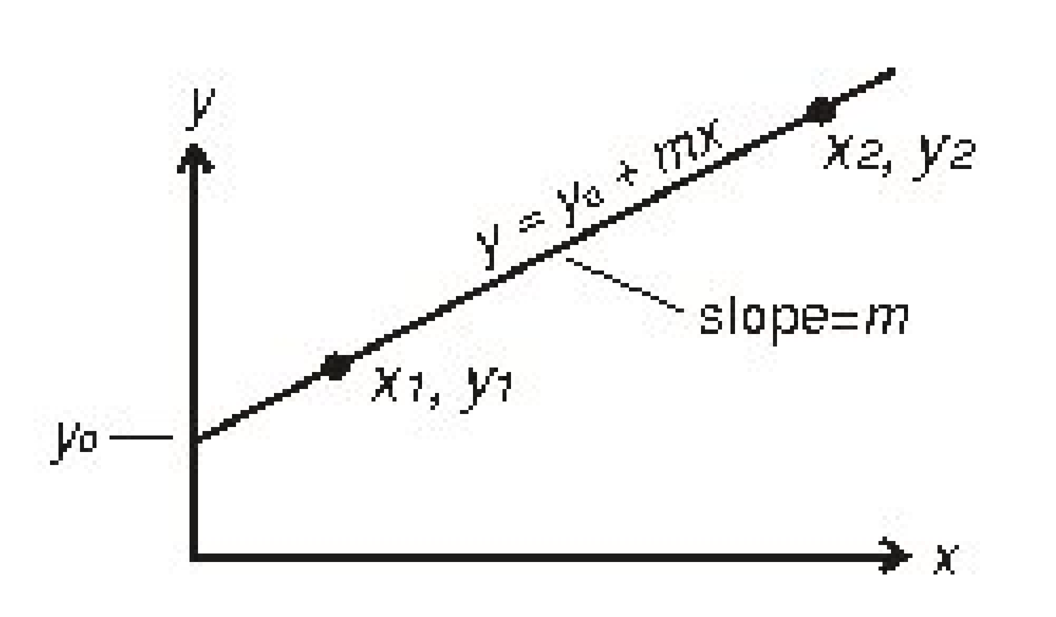


- In 3-D space, the plane may look like this
- Triangles for the plane can be projected onto each axes

The Three-Point Problem: Start with the 2 Point Problem

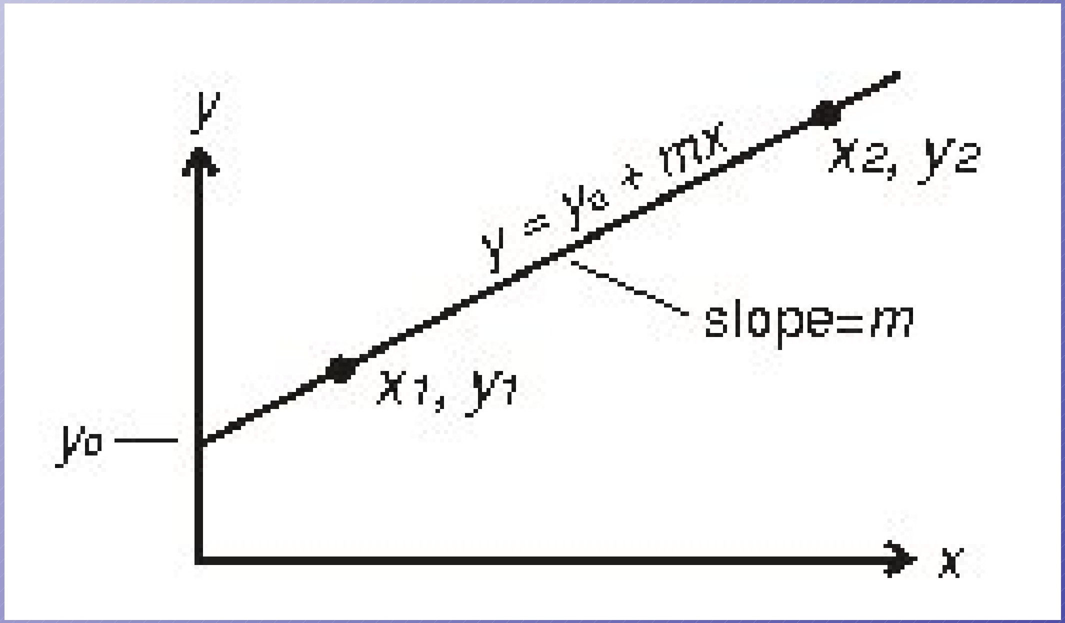


- Given 2 points, how to find the slope of a line ?

$$y = y_o + mx$$

$$m = \text{slope} = (y_1 - y_2) / (x_1 - x_2)$$

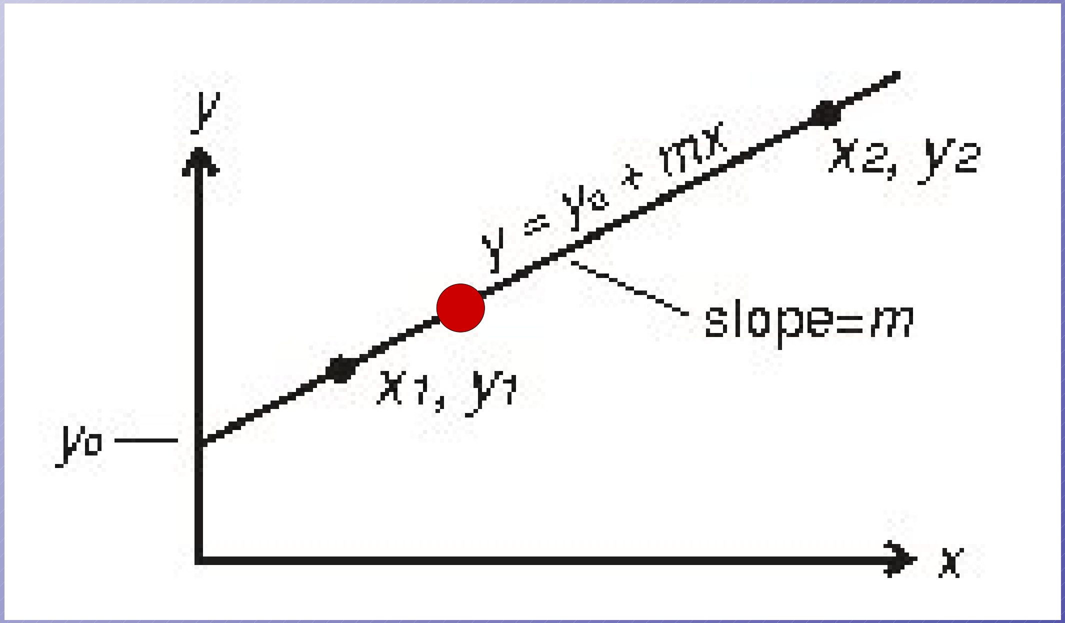
The Three-Point Problem:
Start with the 2 Point Problem



- The slope can also be obtained by ***differentiating*** the equation

$$\frac{dy}{dx} = \frac{d}{dx}(y_o + mx) = m$$

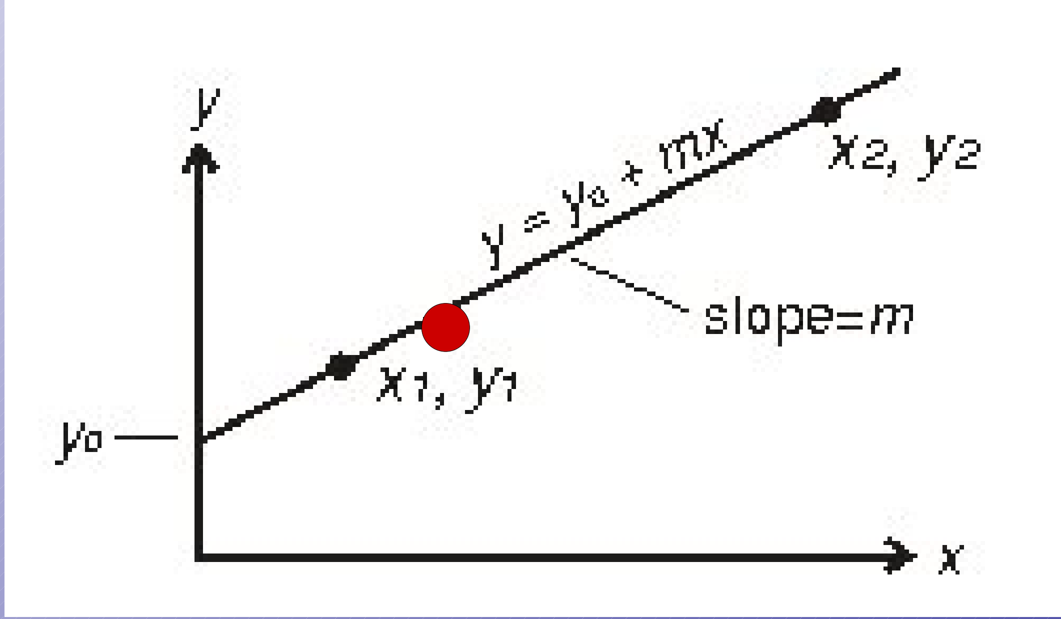
The Three-Point Problem: Start with the 2 Point Problem



- You can find the equation for the slope and y intercept also
- Consider an arbitrary point (x,y) on the line
- The slope, m is

$$\text{Slope} = m = (y - y_1) / (x - x_1)$$

The Three-Point Problem: Start with the 2 Point Problem



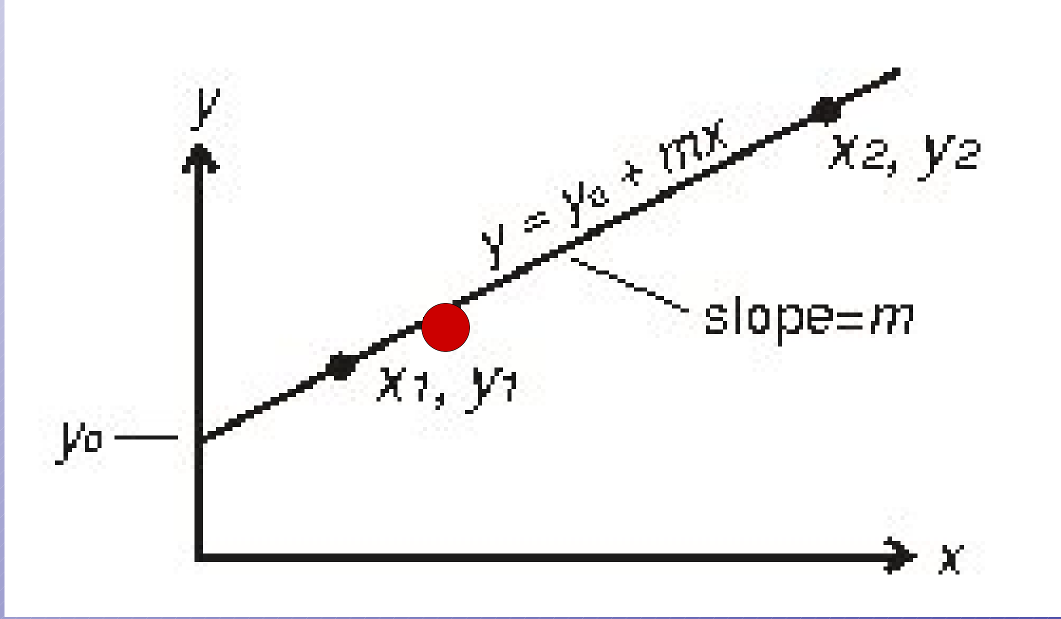
- The slope between either of 3 points on this line will be the same

$$m = (y - y_1) / (x - x_1) = (y_2 - y_1) / (x_2 - x_1)$$

- Solve for y and identify the ***slope*** and ***y intercept***:

$$y = [\, y1 - (y_2 - y_1) / (x_2 - x_1) * x_1 \,] + (y_2 - y_1) / (x_2 - x_1) * x$$

The Three-Point Problem: Start with the 2 Point Problem



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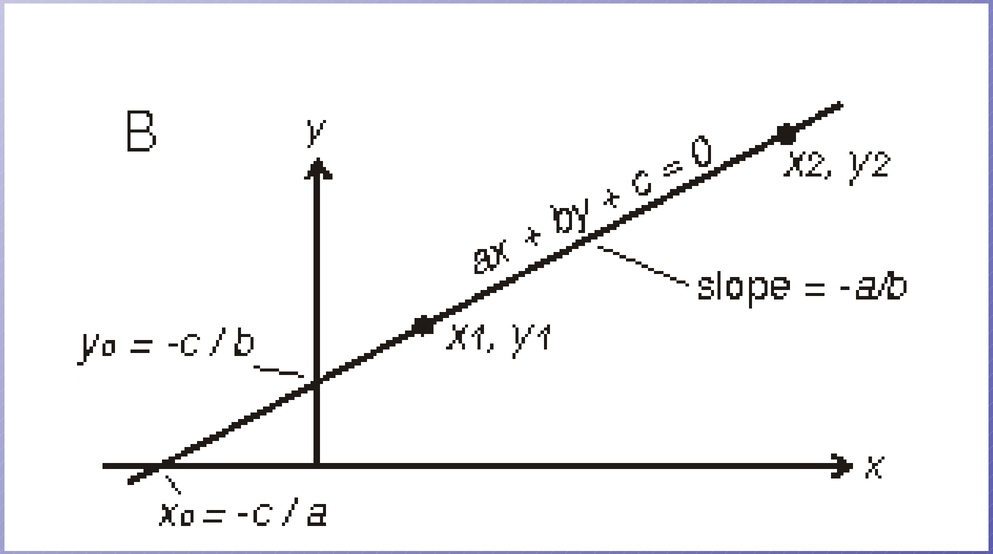
The Three-Point Problem:
The 2 Point Problem: yet Another Aproach



- Write an equation for this same line with linear-coefficients

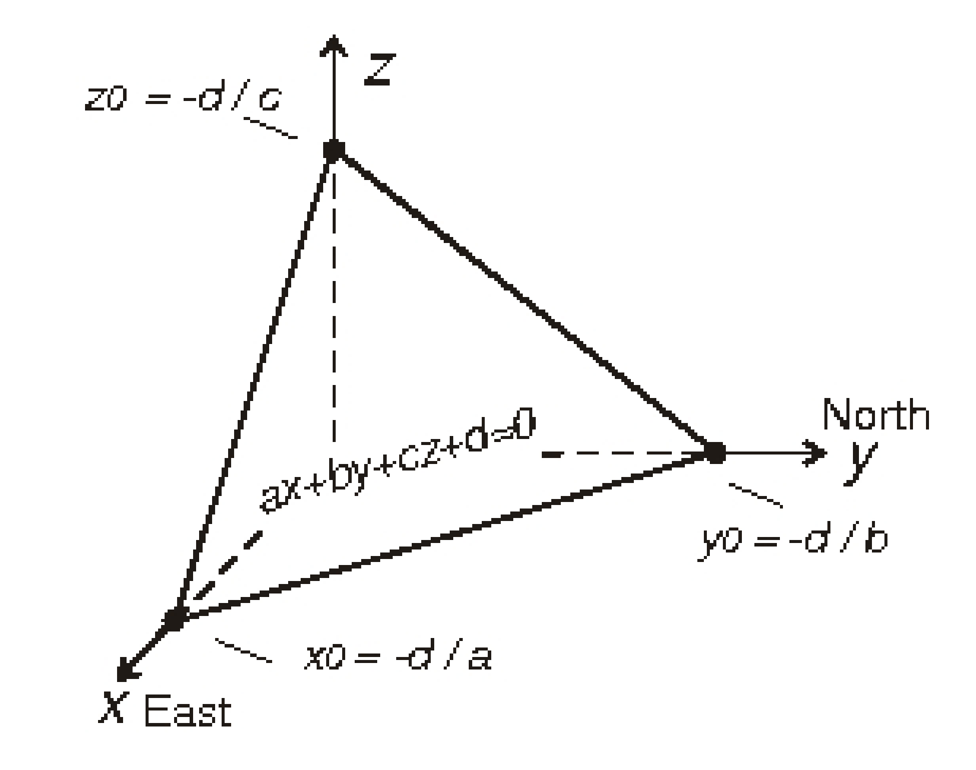
$$ax + by + c = 0$$

- Rearrange and solve for y

$$y = -c/b - (a/b)x + c$$

- What is the slope and intercept ? ....the ratio of coefficients

The Three-Point Problem:
The 2 Point Problem – 3 Point Problem



- The 3-D analog of the line (from last figure) is shown here

$$ax + by + cz + d = 0$$

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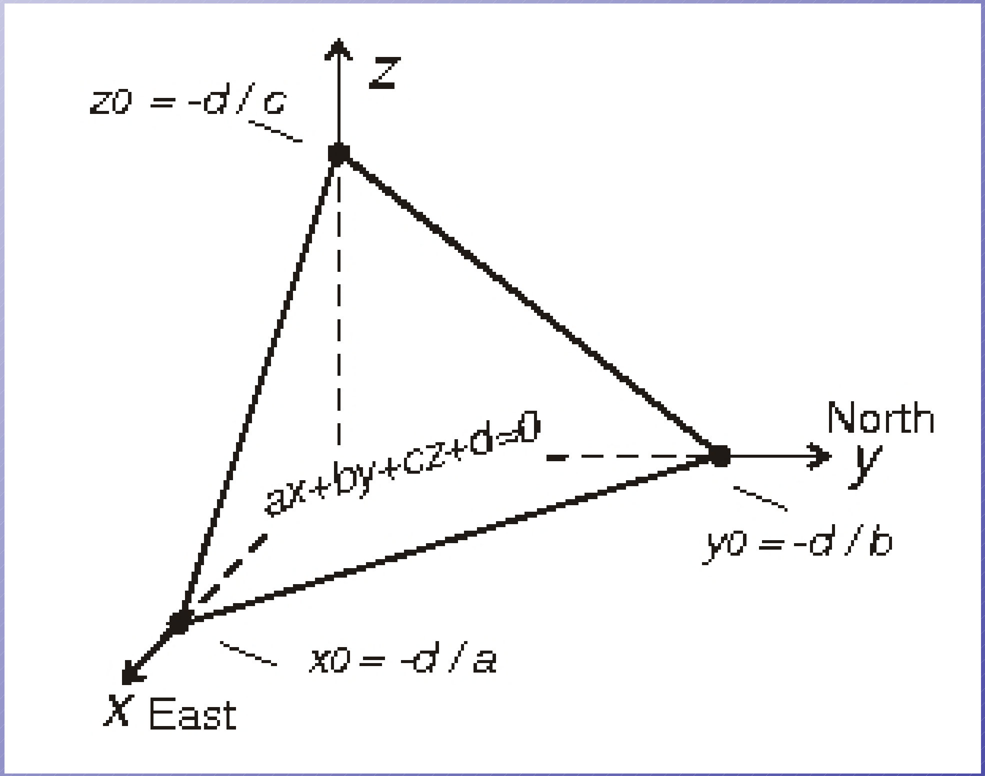
The 2 Point Problem – 3 Point Problem



- The three intercepts (x,y,z axes) can be obtained by setting 2 of the 3 variables (x,y,z) to zero.

$$x_o = -d/a$$
$$y_o = -d/b$$
$$z_o = -d/c$$

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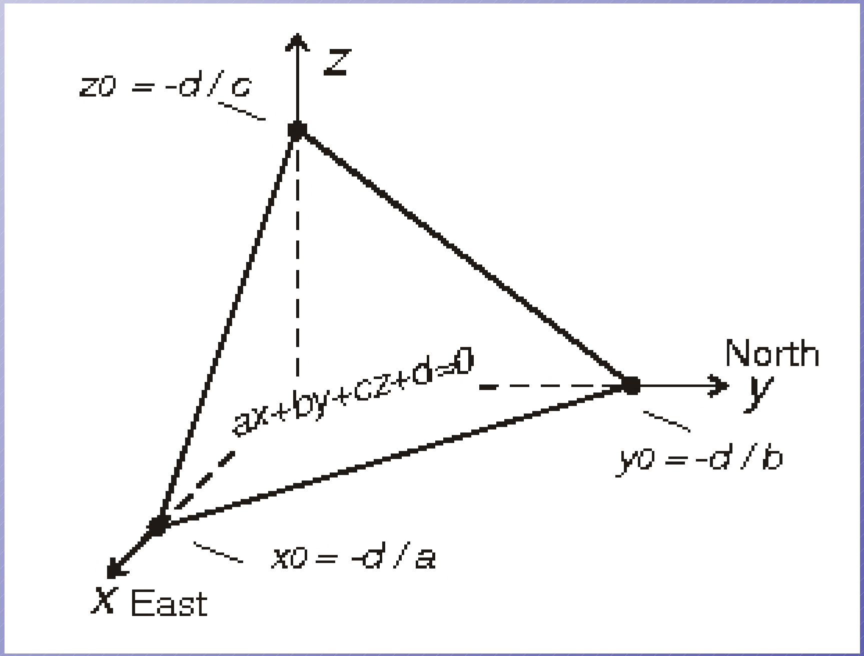

- By setting 1 of the 3 variables (x,y,z) to zero one at a time you can obtain the slope: for the xz-plane, y = 0

$$ax + cz + d = 0$$

$$z = -d/c - (a/c)x$$

$dz/dx = -a/c$ = slope in that plane *(partial derivative)*

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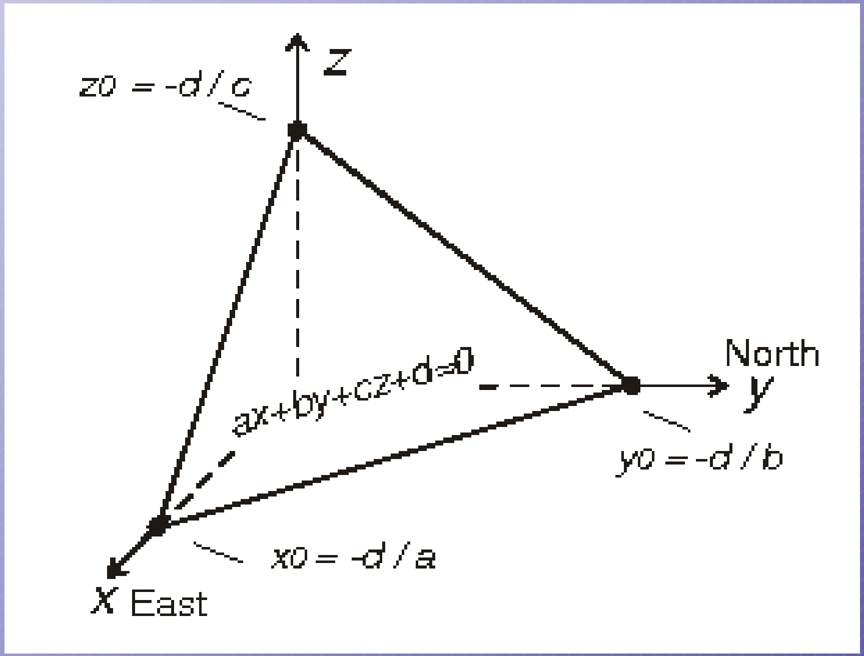
The 2 Point Problem – 3 Point Problem



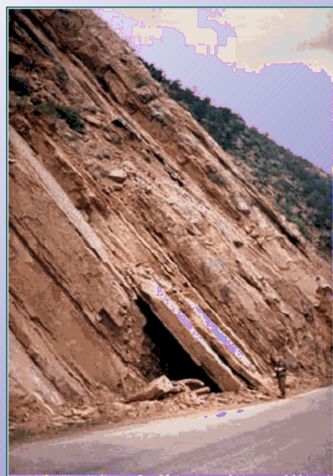
- The xy plane is horizontal.
- The equation where in this plane, z = 0, is a ***line of strike***

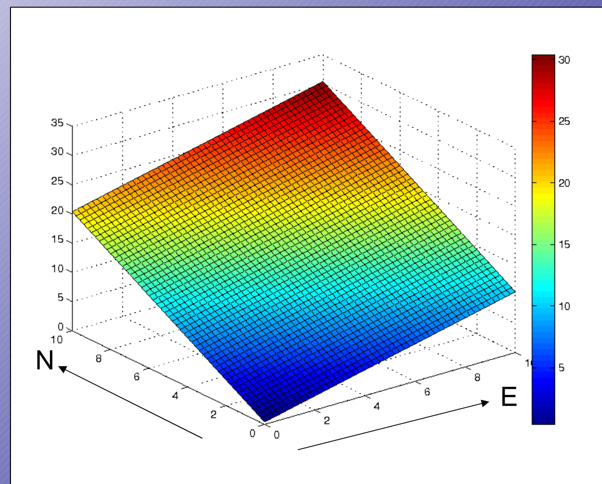
$$a\boldsymbol{x} + b\boldsymbol{y} + d = 0$$

$$\Theta_{strike} = \arctan(dx/dy) = \arctan(-b/a)$$

$$\Theta_{dip} = \arctan(dy/dx) = \arctan(-a/b)$$

The Three-Point Problem





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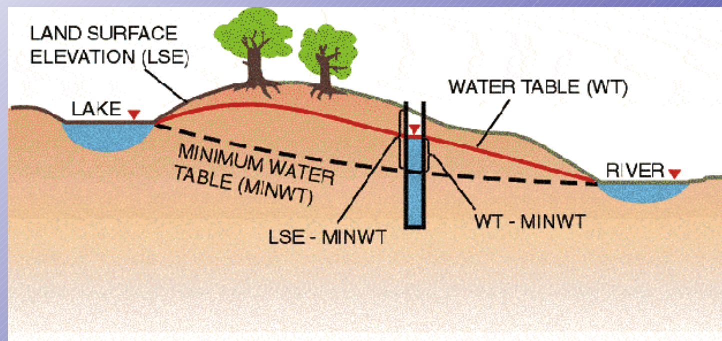

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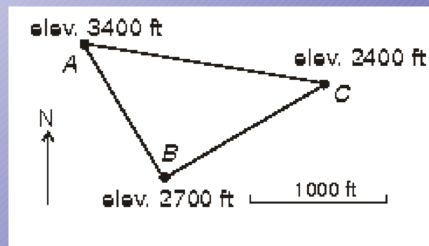

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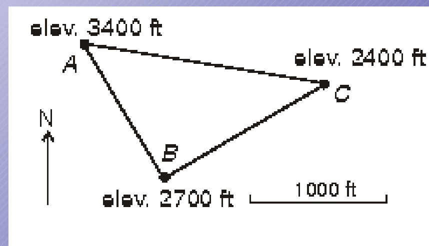

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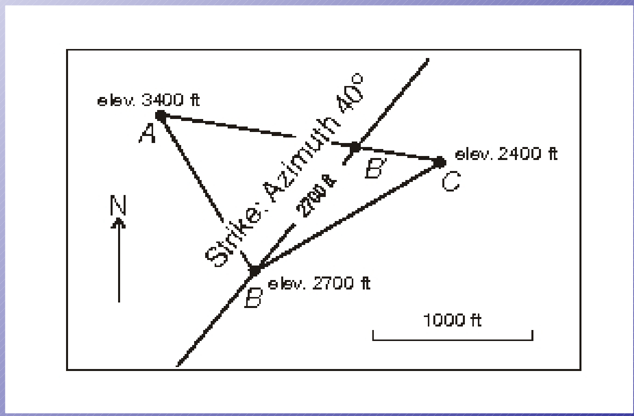

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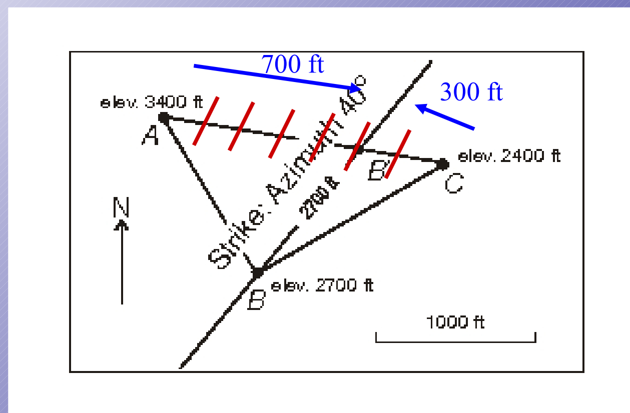

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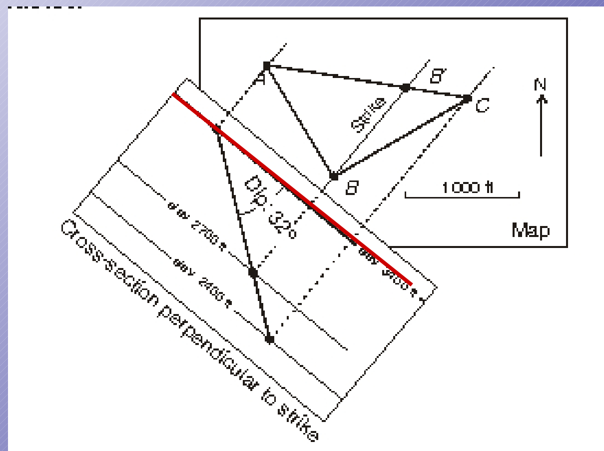

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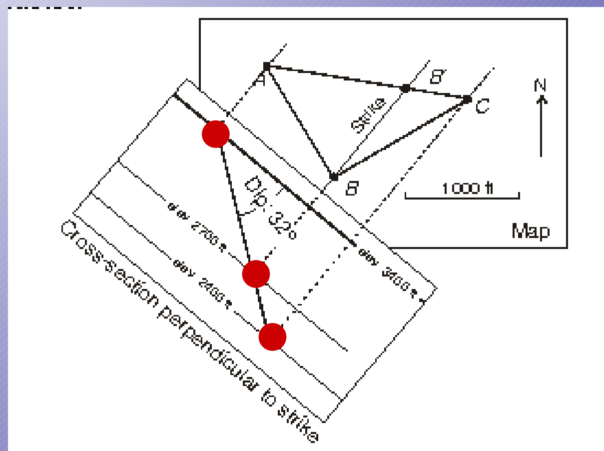

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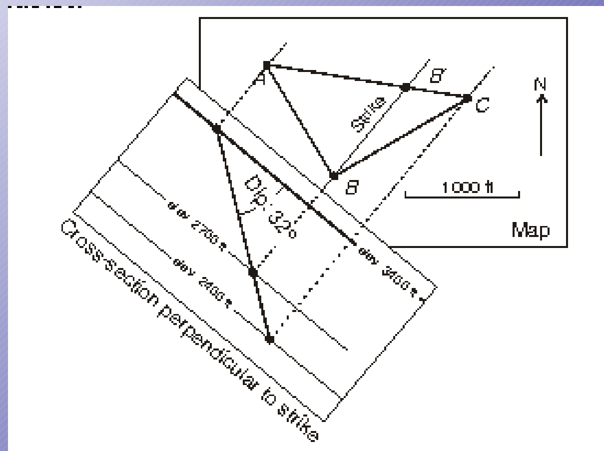

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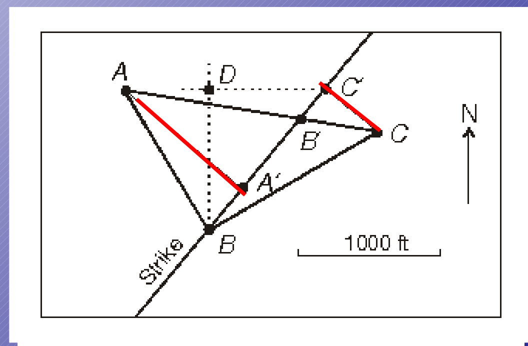

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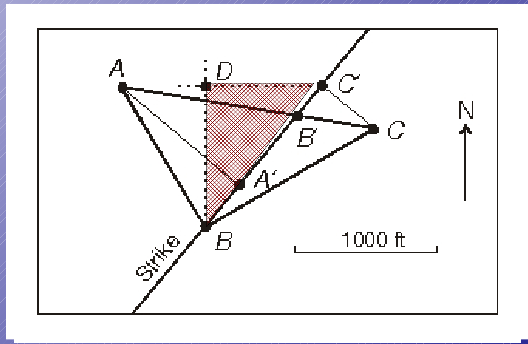

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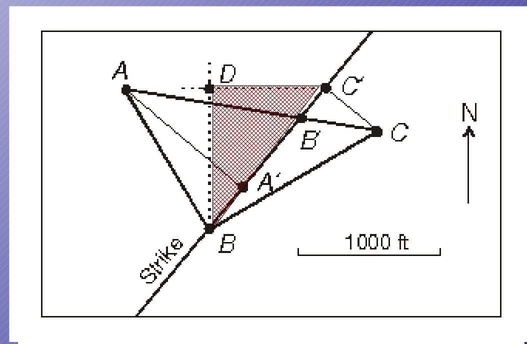

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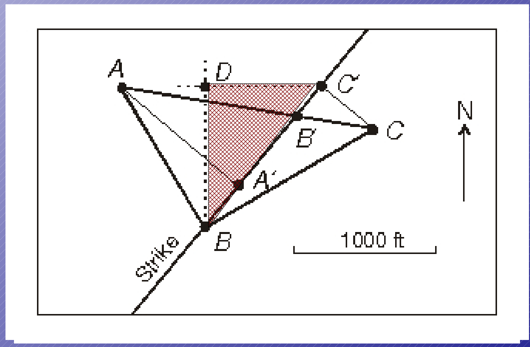

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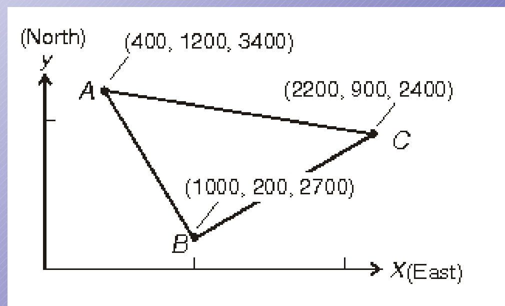

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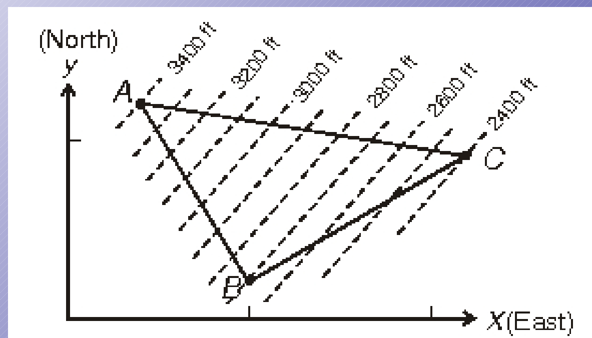

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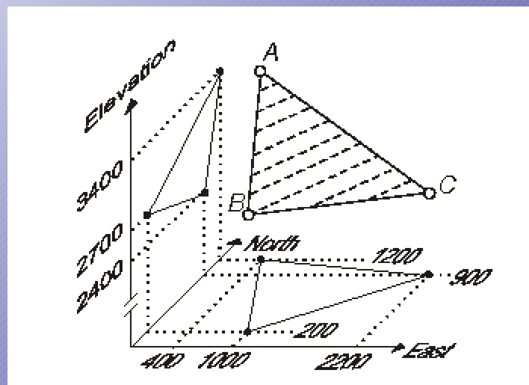

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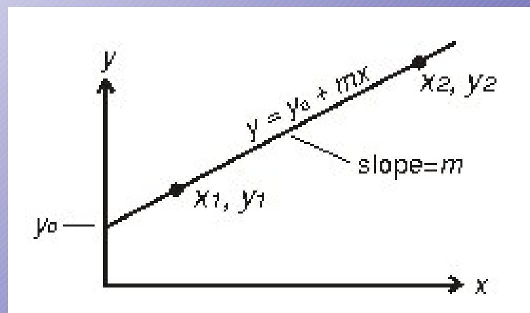

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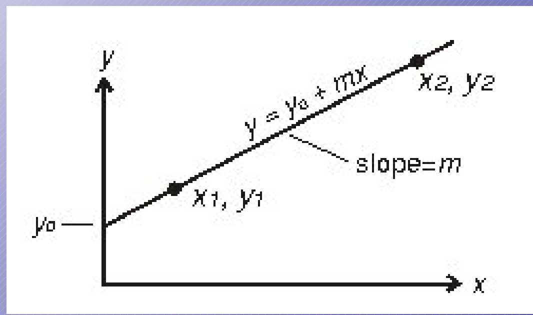

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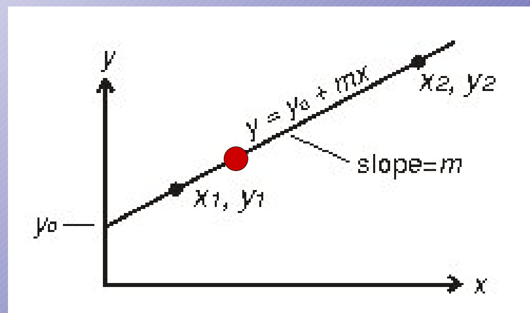

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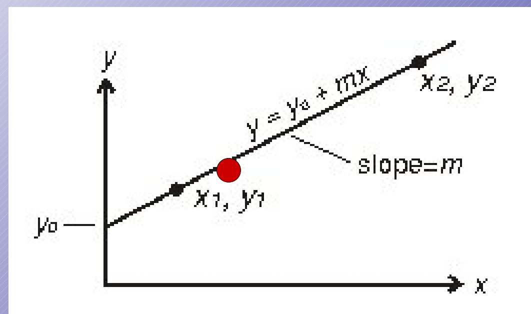

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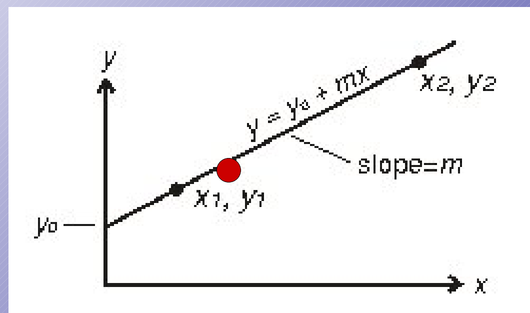
$$m = (y - y_1) / (x - x_1) = (y_2 - y_1) / (x_2 - x_1)$$

- Solve for y and identify the ***slope*** and ***y intercept***:

$$y = [\, y1 - (y_2 - y_1) / (x_2 - x_1)*x_1 \,] + (y_2 - y_1) / (x_2 - x_1)*x$$

The Three-Point Problem:
Start with the 2 Point Problem



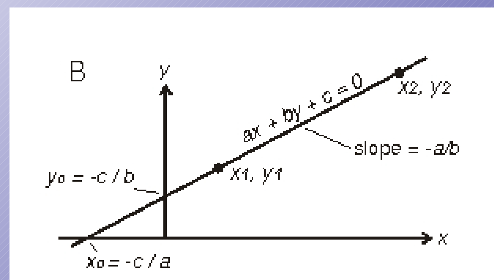
- The slope between either of 3 points on this line will be the same

$$m = (y - y_1) / (x - x_1) = (y_2 - y_1) / (x_2 - x_1)$$

- Solve for y and identify the ***slope*** and ***y intercept***:

$$y = [\, y1 - (y_2 - y_1) / (x_2 - x_1)*x_1 \,] + (y_2 - y_1) / (x_2 - x_1)*x$$

The Three-Point Problem:
The 2 Point Problem: yet Another Aproach



- Write an equation for this same line with linear-coefficients

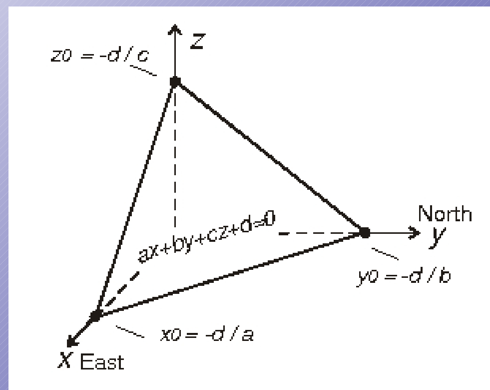
$$a\boldsymbol{x} + b\boldsymbol{y} + c = 0$$

- Rearrange and solve for y

$$\boldsymbol{y} = -c/b - (a/b)\boldsymbol{x} + c$$

- What is the slope and intercept ? ....the ratio of coefficients

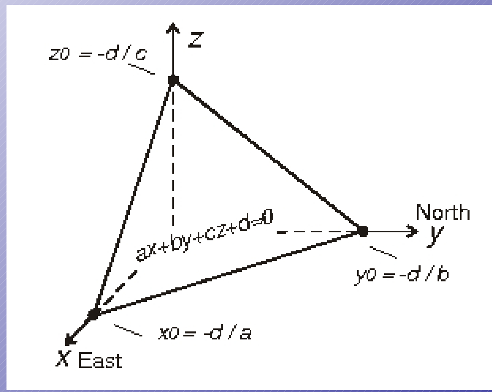
The Three-Point Problem:
The 2 Point Problem – 3 Point Problem



- The 3-D analog of the line (from last figure) is shown here

$$a\boldsymbol{x} + b\boldsymbol{y} + c\boldsymbol{z} + d = 0$$

The Three-Point Problem:
The 2 Point Problem – 3 Point Problem



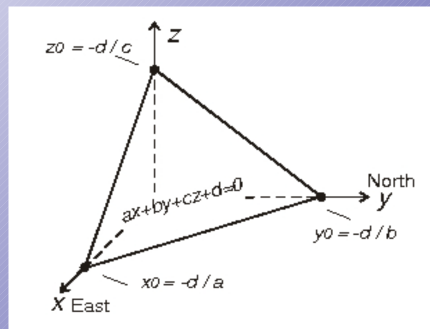
- The three intercepts (x,y,z axes) can be obtained by setting 2 of the 3 variables (x,y,z) to zero.

$x_o = -d/a$

$y_o = -d/b$

$z_o = -d/c$

The Three-Point Problem:
The 2 Point Problem – 3 Point Problem



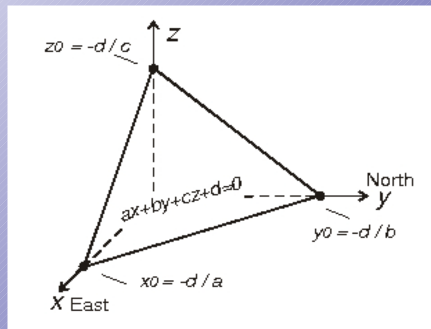
- By setting 1 of the 3 variables (x,y,z) to zero one at a time you can obtain the slope: for the xz-plane, y = 0

$$a\boldsymbol{x} + c\boldsymbol{z} + d = 0$$

$$\boldsymbol{z} = -d/c - (a/c)x$$

$dz/dx = -a/c$ = slope in that plane *(partial derivative)*

The Three-Point Problem:
The 2 Point Problem – 3 Point Problem



- The xy plane is horizontal.
- The equation where in this plane, z = 0, is a ***line of strike***

$$a\boldsymbol{x} + b\boldsymbol{y} + d = 0$$

$$\Theta_{strike} = \arctan(dx/dy) = \arctan(-b/a)$$

$$\Theta_{dip} = \arctan(dy/dx) = \arctan(-a/b)$$