

## **Chapter 4**

### **The second and third laws of thermodynamics**

#### **8.1 The Carnot cycle**

The Carnot cycle is an idealized heat cycle developed by Sadi Carnot. This cycle represents the maximum theoretical efficiency possible for any heat engine operating between two temperatures, regardless of the nature of the working material. No real engine can exceed the Carnot efficiency.

#### **The Carnot efficiency formula:**

$$\eta_{\text{Carnot}} = 1 - T_C / T_H$$

#### **What does this formula mean?**

This equation tells us that the efficiency of a heat engine depends only on the temperatures of the hot and cold sources. To increase efficiency, we must:

1- Raise the temperature of the hot source ( $T_H$ ): This is why solar thermal power plants and modern engines strive to operate at the highest possible temperatures.

2- Decrease the temperature of the cold source ( $T_c$ ): This is why cooling towers are used in power plants to efficiently dissipate waste heat.

## **Conclusion**

The second law of thermodynamics imposes natural limits on the efficiency of heat engines. This law prevents 100% of the heat from being converted into work and makes it necessary to have a cold source to dissipate waste heat.

The Carnot cycle offers the ultimate in this, providing a tool for engineers to further refine power systems and drive electric motors.

### **8.1.1 Applying the Carnot Cycle to Solar Thermal Power Plants**

Solar thermal power plants (CSP) operate as heat engines. These plants use mirrors to concentrate sunlight to heat a fluid (the hot source) to very high temperatures (up to 500-1000°C). This heat is then used to generate steam that drives a turbine (the heat engine).

- **Hot source ( $T_H$ ):** It is the temperature of a hot liquid heated by the sun.

**Cold source ( $T_c$ ):** It is the temperature of the environment in which the steam is cooled, such as a cooling tower or a nearby river.

**Example (4):**

Suppose a solar thermal power plant operates between a hot temperature ( $T_H = 600^\circ\text{C}$ ) and a cold temperature ( $T_C = 30^\circ\text{C}$ ). What is the maximum possible efficiency of this station?

$$T_H = 600 + 273 = 873 \text{ K}$$

$$T_C = 30 + 273 = 303 \text{ K}$$

$$\eta_{\text{Carnot}} = 1 - T_C/T_H$$

$$\eta_{\text{Carnot}} = \text{Eff}$$

$$T_1 = T_C$$

$$T_2 = T_H$$

$$\text{Eff} = 1 - T_C/T_H$$

$$\text{Eff} = 1 - 303/873$$

$$\text{Eff} = 1 - 0.347$$

$$\text{Eff} = 0.653$$

This result means that the maximum possible efficiency for this plant, regardless of the quality of the turbines or materials used, is 65.3%. The plant's actual efficiency will always be lower due to heat loss, friction, and other factors.

### 8.1.2 Explanation of the Carnot cycle and its laws

It is a closed loop consisting of four successive thermodynamic processes:

**First:** The gas expands inversely and isothermally at  $T_2$  from  $B \leftarrow A$  (the temperature is constant and does not change),  $\Delta E=0$

$$w_2 = -q_2 = -2.303 mRT \log \frac{V_2}{V_1} \quad (1)$$

$$w_2 = -2.303 mRT \log \frac{V_b}{V_a}$$

**Second:** Every adiabatic expansion (accompanied by a decrease in temperature) is an inverse and adiabatic expansion of the gas ((  $C \leftarrow B$  )) ( $T_1 \leftarrow T_2$  ) and the relationship between  $T$  and  $V$  is (inverse).

$$C_v \ln \frac{T_2}{T_1} = R \ln \frac{V_1}{V_2} \left( \frac{V_b}{V_c} \right) \quad (2)$$

**Third:** Reversible and isothermal contraction of gas ( $T_1$ ),  $\Delta E=0$ , ( $D \leftarrow C$ )

$$w_1 = -q_1 = -2.303 mRT_1 \log \frac{V_2}{V_1} \left( \frac{V_d}{V_c} \right) \quad (3)$$

**Fourth:** The reversible and adiabatic contraction of gas  $q=0$ , ( $T_2 \leftarrow T_1$ )( $A \leftarrow D$ )

$$C_v \ln \frac{T_2}{T_1} = R \ln \frac{V_d}{V_a} \quad (4)$$

**The final result (total)**

$$\Delta E = 0$$

$$w = q_1 + q_2$$

As we know, the law of machine efficiency is:

$$\text{Eff} = \frac{w}{q_2}$$

$$\text{Eff} = \frac{q_1 + q_2}{q_2} = 1 + \frac{q_1}{q_2}$$

**The above law is the law of efficiency in terms of the amount of heat.**

There is no way to measure the amount of heat, but the temperature in this enclosed space can be measured.

Therefore, the law of efficiency in terms of the amount of heat must be converted to the law of efficiency in terms of temperature ( $T_1$  &  $T_2$ ) because measuring temperature is easier than measuring the amount of heat

**The efficiency law for heating machines is as follows:**

$$\text{Eff} = \frac{T_2 - T_1}{T_2}$$

$$\% \text{Eff} = \frac{T_2 - T_1}{T_2} \times 100$$

$$T_1 = T_C$$

$$T_2 = T_H$$

$$\eta_{\text{Carnot}} = 1 - T_C/T_H$$

$$\eta_{\text{Carnot}} = \text{Eff}$$

$$\text{Eff} = 1 - T_C/T_H$$

**The efficiency law for refrigeration machines is as follows:**

$$\% \text{Eff} = \frac{T_2 - T_1}{T_1} \times 100$$

**Example (5):** Compare the maximum efficiency of two heating machines, one containing water and the other containing mercury, if you know that the temperature of the water is ( $T_2 = 100^\circ\text{C}$ ), the temperature of the mercury is ( $T_2 = 357^\circ\text{C}$ ) and the temperature of the cold tank is ( $T_1 = 25^\circ\text{C}$ ).

**Note:** The question mentions maximum efficiency because the four processes are reversible, and each process gives maximum efficiency.

**Note:** The Celsius temperature must be converted to absolute temperature.

$$\% \text{Eff}_{\text{H}_2\text{O}} = \frac{T_2 - T_1}{T_2} \times 100$$

$$\% \text{Eff}_{\text{H}_2\text{O}} = \frac{373 - 298}{373} \times 100 = 20\%$$

$$\% \text{Eff}_{\text{Hg}} = \frac{630 - 298}{630} \times 100 = 52.7\%$$

We note that the efficiency of mercury is greater than the efficiency of water, but from an economic and health perspective, we use water with lower efficiency.

**Example (6):** Calculate the percentage of maximum efficiency for a commercial refrigerator operating between temperature ( $-10^{\circ}\text{C}$ ) (internal temperature) and temperature ( $25^{\circ}\text{C}$ ) (room temperature). Also, calculate the minimum amount of work that must be done to extract (100 joules) of heat from inside the refrigerator.

$$\% \text{Eff} = \frac{T_2 - T_1}{T_1} \times 100$$

$$\% \text{Eff} = \frac{298 - 263}{263} \times 100 = 13.3\%$$

$$\text{Eff} = \frac{w}{q} = \frac{13.3}{100}$$

$$\frac{13.3}{100} = \frac{w}{100}$$

$$w = 13.3 \text{ Joule}$$

**Example (7):** Calculate the percentage of efficiency of the machine in which the temperature of the hot tank is (4) times that of the cold tank.

$$T_2 = 4T_1$$

$$\%Eff = \frac{T_2 - T_1}{T_2} \times 100$$

$$\%Eff = \frac{4T_1 - T_1}{4T_1} \times 100$$

$$\%Eff = \frac{3T_1}{4T_1} \times 100$$

$$\%Eff = 75\%$$

If the temperature of the hot tank = twice the temperature of the cold tank, then:

$$\%Eff = \frac{T_2 - T_1}{T_2} \times 100$$

$$\%Eff = \frac{2T_1 - T_1}{2T_1} \times 100 = \frac{2T_1}{2T_1} - \frac{1T_1}{2T_1} = 1 - 1/2 = 0.5$$

$$\%Eff = 50\%$$

## **8.2 The relationship of entropy to the second law of thermodynamics**

Entropy changes of the Carnot cycle

$$\Delta S_T = \Delta S_{A-B} + \Delta S_{B-C} + \Delta S_{C-D} + \Delta S_{D-A}$$

$$\therefore \Delta S_T = q_2/T_2 + 0 + q_1/T_1 + 0$$

$$\Delta S_T = q_2/T_2 + q_1/T_1 = 0$$

$$q_2/T_2 + q_1/T_1 = 0 \therefore$$

$$q_2/T_2 = -q_1/T_1$$

$$q_1/q_2 = -T_1/T_2$$

**\*The above relationship is between the amount of heat (q) and the temperature (T). It is used to find the efficiency law.**

$$\text{Eff} = w / q_2 = (q_1 + q_2)/q_2$$

$$\text{Eff} = 1 + q_1/q_2$$

$$\text{Eff} = 1 - T_1/T_2 \therefore \quad \text{By unifying the positions} \quad \text{Eff} = 1 - T_C/T_H$$

$$T_1 = T_C$$

$$T_2 = T_H$$

**$\therefore$  We conclude that entropy is one of the formulas of the second law and is identical to Kelvin's law.**