Problem 5 – CHP Efficiency

Question:
From Problem 2, η_{elec} = 50.6%. If 80% of the waste heat is recovered, calculate the CHP efficiency.

Solution:
Heat fraction = 1 - 0.506 = 0.494
Recovered = 0.80 × 0.494 = 0.395
η_{CHP} = 0.506 + 0.395 = 0.901 = 90.1%

Final Answer:
CHP efficiency = 90.1%

Problem 6 – Nernst Correction with Gas Pressures

Question:
At 25 °C, the reversible voltage is corrected for partial pressures: $E = E° + (RT/2F) \ln(pH2 \times pO2^{0.5})$. If pH2 = 0.80 atm, pO2 = 0.21 atm, E° = 1.229 V, RT/2F = 0.01285 V, calculate E_{rev} and efficiency at V = 0.75 V.

Solution:
$\ln(0.8\sqrt{0.21}) = \ln(0.3666) = -1.003$
ΔE = 0.01285 × (-1.003) = -0.0129 V
E_{rev} = 1.229 - 0.0129 = 1.216 V
η = (0.75 / 1.216) × 0.8296 = 0.6167 × 0.8296 = 51.2%

Final Answer:
E_{rev} = 1.216 V, Efficiency = 51.2%

Problem 7 – Temperature Effect

Question:
The reversible voltage decreases with temperature as dE/dT ≈ -0.000847 V/K. From 25 °C to 80 °C (ΔT = 55 K), calculate the new reversible voltage and efficiency at V = 0.70 V.

Solution:
ΔE = -0.000847 × 55 = -0.0466 V
E_{rev} = 1.229 - 0.0466 = 1.182 V
η = (0.70 / 1.182) × 0.8296 = 0.592 × 0.8296 = 49.1%

Final Answer:
E_{rev} = 1.182 V, Efficiency = 49.1%

Problem 8 – Efficiency with Upstream Reformer

Question:
A fuel processor converts methane to hydrogen with 75% efficiency. The fuel cell stack itself operates at 50% efficiency. Calculate the overall system efficiency.

Solution:
$\eta_{sys} = 0.75 \times 0.50 = 0.375 = 37.5\%$

Final Answer:
System efficiency = 37.5%

Problem 9 – Net AC Efficiency

Question:
A stack operates at 48% efficiency (HHV). The inverter efficiency is 95%, and parasitic loads consume 8% of the DC output. Calculate the net AC efficiency.

Solution:
Net DC after parasitics = 0.92 × 0.48 = 0.4416
Net AC = 0.4416 × 0.95 = 0.419 = 41.0%

Final Answer:
Net AC efficiency = 41.0%

Problem 10 – HHV vs LHV Reporting

Question:
At a given operating point, $V/E_{rev} = 0.60$. Compare efficiency on HHV and LHV bases.

Solution:
$\eta_{HHV} = 0.60 \times 0.8296 = 49.8\%$
$\eta_{LHV} = 0.60 \times 0.9449 = 56.7\%$

Final Answer:
HHV = 49.8%, LHV = 56.7%

Problem 11 – Efficiency Drop with Voltage Loss

Question:
A fuel cell runs at 0.75 V initially and drops to 0.71 V after 5000 h. Calculate the efficiency drop (HHV basis).

Solution:

$\eta_{initial}$ = (0.75 / 1.229) × 0.8296 = 50.6%
η_{final} = (0.71 / 1.229) × 0.8296 = 47.8%
Drop = 50.6 - 47.8 = 2.8 points (~5.5% relative)

Final Answer:
Efficiency drop = 2.8 percentage points

Problem 12 – 10 kW Stack Efficiency and Fuel Flow

Question:
A 60-cell stack operates at 0.72 V/cell, giving V_{stack} = 43.2 V. The net power output is 10 kW. Calculate the required hydrogen flow rate and overall efficiency.

Solution:
Current: I = P/V = 10000/43.2 = 231.5 A
From voltage-based formula: $\eta \approx (0.72 / 1.229) \times 0.8296 = 48.6\%$
Fuel power: Pfuel = 10000 / 0.486 = 20.6 kW
Hydrogen flow: $\dot{n}H2$ = 20600 / 285830 = 0.072 mol/s (~97 SLPM)

Note: Current of ~14,000 A would be required for this flow unless parallelization is used. The efficiency remains ~48.6%.

Final Answer:
Efficiency ≈ 48.6%, Hydrogen flow ≈ 0.072 mol/s (~97 SLPM)

2. Voltage-Current Characteristics

Fuel cell polarization curves describe how voltage declines as current density increases. There are three characteristic regions:

- Activation region: steep voltage drop at low current due to slow oxygen reduction reaction (ORR).
- Ohmic region: nearly linear drop caused by resistive losses in the membrane and electrodes.
- Concentration region: sharp voltage fall at high currents when mass transport cannot supply sufficient reactants.

For a PEMFC operating at 0.7 V and 0.5 A/cm^2, the cell produces 0.35 W/cm^2. At higher currents, performance falls due to losses.

4. Power Density

Power density (P) is defined as:

$$P = V \times i$$

where V is the voltage and i the current density. It indicates how much power can be produced per electrode area.

Example: At V = 0.65 V and i = 1.2 A/cm^2, P = 0.78 W/cm^2. PEMFCs typically range 0.5–1.0 W/cm^2, while SOFCs may exceed 2.0 W/cm^2 at high temperatures.

High power density is crucial for compact systems, especially in transportation.

Hydrogen Fuel Cell – Current, Voltage & Power
Detailed Math Problems with Step-by-Step Solutions

Key Relations (used throughout)

Power density: $p = i \times V$ (W/cm^2)

Stack power: $P = I \times V_{stack}$ (W), where $I = i \times A_{cell}$ (A)

Linear polarization: $V = E - iR$ (E in V, R in Ω·cm^2, i in A/cm^2)

Semi-empirical example: $V(i) = E - a \ln(i) - b\, i$ (natural log unless noted)

Ohmic drop from ASR: $\Delta V_{ohmic} = i \times ASR$ (V)

Hydrogen consumption (Faraday's law): $\dot{n}_{H2} = I / (2F)$ (mol/s), F = 96485 C/mol

Oxygen consumption: $\dot{n}_{O2} = I / (4F)$ (mol/s)

STP conversion: 1 mol = 22.414 L; 1 L/s = 60 L/min (SLPM)

Problem 1 – Direct Power Density

Question:
A PEM hydrogen fuel cell operates at a current density i = 0.60 A/cm^2 with a cell voltage V = 0.70 V. Compute the power density p, expressing units clearly.

Solution:
Given: i = 0.60 A/cm^2, V = 0.70 V.
Relation: $p = i \times V$ (W/cm^2).
Calculation: $p = 0.60 \times 0.70 = 0.42$ W/cm^2.
Final Answer: p = 0.42 W/cm^2.

Problem 2 – Maximum Power for Linear Polarization ($V = E - iR$)

Question:
A cell shows an approximately linear polarization curve $V = E - iR$ with E = 0.95 V and area-specific